Photodeformation Based Distributed Opto-electromechanical Shell Actuators: Modeling and Analysis

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Abstract

Distributed opto-electromechanical actuators represent a new class of precision distributed actuator based on the photodeformation process and controlled by high energy lights. Fundamental opto thermo-electromechanical constitutive relations are discussed, which are further applied to formulation of optically induced control forces and moments. Detailed mathematical modeling and analysis of distributed opto-electromechanical shell actuators are presented. A generic distributed photo-actuation theory is proposed and the closed-loop opto-thermo-electromechanical equations of circular cylindrical shells are derived. The systems equations reveal the couplings among elasticity, photodeformation, pyro-electricity, and thermo-elasticity. Active distributed control of flexible cylindrical shells using segmented distributed opto-electromechanical shell actuators are investigated and the control effectiveness is evaluated. Analysis results show that the distributed optical actuators are effective to controlling low-frequency oscillations. Actuator placements and sizes also affect the overall control effectiveness of the shell natural modes. Time history analyses of selected shell natural oscillations suggest that the Lyapunov control is more effective than the proportional feedback control.

1 Introduction

Shell-type structures and components ranging from micro-nozzles to aerospace structures usually represent complicated. Distributed control of flexible shell structures represents an challenging issue for many advanced structural systems and components, e.g., nozzles, rockets, pipes, antenna dishes, fuel tanks, etc. [1]. Conventional control practices can be divided into two major categories: 1) passive techniques and 2) active techniques. The passive techniques usually involve energy dissipation devices, e.g., viscoelastic dampers, dynamic absorbers, constraint-layer dampers, etc. The active control techniques usually apply actuators and/or counteracting devices, e.g., electromagnetic actuators, piezoelectric actuators, proof-mass dampers, active constraint-layer dampers, etc. [2]. Conventional actuators require hard-wire connections to transmit energy sources and commands to activate the actuator mechanisms. The hard-wire signal transmission busses can easily attract electric noises influenced by electrical and/or magnetic fields, long distance transmissions, etc. Accordingly, control commands may not be accurately executed. In this study, non-contact distributed opto-mechanical actuators activated by high-energy lights are investigated and their distributed control characteristics are evaluated.

Light driven opto-mechanical actuators have many advantages over conventional electromechanical actuators, such as 1) non-contact activation, 2) immune from electromagnetic disturbances, 3) light weight and small size, etc. One-dimensional (beam-type) opto-mechanical actuators were studied [3-7], and two-dimensional planar (plate-type) opto-mechanical actuators with applications to distributed vibration control...
were investigated [8,9]. However, distributed opto-mechanical shell actuators and their control electromechanics or thermo-electromechanics [10] have not been investigated.

Studies of piezoelectric shell actuators reveal that the control actions of shell actuators involve 1) membrane control forces and 2) countering control moments [11]. In this paper, detailed control opto-thermo-electromechanics of distributed opto-mechanical shell actuators are investigated. A generic distributed shell actuation theory including the photodeformation and thermo-electromechanics is developed and governing opto-thermo-electromechanical equations are derived. Observability and controllability of the sensor/actuator systems are discussed. Optimization procedures for sensor/actuator placements are proposed. Application of the theory to a circular cylindrical shell is demonstrated and detailed sensing/control effectivenesses are studied.

2 Photodeformation Induced Control Effects

High-energy lights, e.g., lasers, ultraviolet lights, etc., irradiated on the distributed optical actuator induces a photodeformation process. With appropriate layouts and configurations, this photodeformation can be used to induce control forces and moments and ultimately applied to precision manipulation and control of structural and mechatronic systems [9]. The photodeformation process involves two fundamental effects: 1) the photovoltaic effect and 2) the converse piezoelectric effect. The light irradiated on the optical actuator first induces a charge flowing, opposite to the polarized direction, usually from the lighted surface to the dark surface. This charge flow causes a voltage generation between the surface electrodes. This process is called the photovoltaic effect. This voltage consequently induces mechanical strains due to the converse effect. In addition, the high-energy light also heats up the optical actuator, and the temperature change triggers the pyroelectric effect from which a secondary pyroelectric voltage further enhances the strain generation due to the converse effect [9].

The control forces and moments are imposed by the spatially distributed optical actuator placed from coordinate $a_0$ to $a_2$ in the $a_1$ direction and from $a_3$ to $a_5$ in the $a_2$ direction of a generic shell continuum, Figure 1.

![Figure 1: A distributed optical actuator laminated on a generic shell.](image)

The control forces $F^i(a)$ and moments $M^i(a)$ of the optical actuator are defined by the double step (rectangular-shape) functions.

$$F^i(a) = \frac{1}{2}(h^2 \delta^i \delta^j) U(a_1) U(a_2) \left( \alpha \right) \ U(a_3) U(a_4) \ U(a_5), \ i,j = 1,2.$$ (1)

$$M^i(a) = 0.5(h^2 \delta^i \delta^j) \left( \frac{\partial^2}{\partial a_1 \partial a_2} \right) U(a_1) U(a_2), \ i,j = 1,2.$$ (2)

where $\gamma^i_5$ is Young's modulus of the optical actuator, $h^2$ is the actuator thickness, $s_5$ is the piezoelectric strain constant, $E^i_5$ and $E^j_5$ are respectively the photovoltaic voltage and the body temperature induced by the high-energy illumination at the time $t$; and $\lambda_i$ is the thermal stress constant. The double step function $U(a_i)$ (rectangular shape) is defined by the coordinates of the optical actuator patch laminated on the shell.

$$U(a_i) = 1 \text{ when } a_i = a_i \text{ and } 0 \text{ when } a_i = a_i, \ i,j = 1,2.$$ (3)

Note that the control force acts as an in-plane membrane control force controlling the membrane oscillation and the control moment defined by the control force multiplied by the moment arm. The distance measured from the shell neutral surface to the mid-plane of the actuator - counteracts the bending oscillations.

3 Vibration Control of Circular Cylindrical Shells

Modeling of a simply supported circular cylindrical shell laminated with a distributed optical actuator patch is studied and its control effects are evaluated in this section. The cylindrical shell is defined in a cylindrical coordinate system, i.e., $a_1 = r$, $a_2 = \theta$, and $a_3 = z$.

![Figure 2: A circular cylindrical shell tube with an optical actuator.](image)

Figure 2: A circular cylindrical shell tube with an optical actuator.

The system equations in the three axial directions of the circular cylindrical shell can be derived from the generic shell equations using the Lame parameters and the radii of curvature [11].

$$\frac{\partial N}{\partial t} - \frac{1}{R} \frac{\partial M}{\partial \beta} + \rho a \beta = F_1 + \frac{\partial N_1}{\partial t},$$ (5)

$$\frac{\partial M}{\partial t} - \frac{1}{R} \frac{\partial M}{\partial \beta} + \frac{1}{R} \frac{\partial M}{\partial \beta} + \rho a \beta = F_2 + \frac{1}{R} \frac{\partial M_2}{\partial \beta} + \frac{1}{R} \frac{\partial M_2}{\partial \beta},$$ (6)
\[
\frac{\partial^3 M_{xx}}{\partial \tau^3} - \frac{2}{R} \frac{\partial^2 M_{xx}}{\partial \tau^2} + \frac{1}{R^3} \frac{\partial^2 M_{xx}}{\partial \tau^2 \partial \phi^2} + \frac{N_{xx}}{R} = \rho \ddot{u}_T,
\]

where \(N_4\) and \(M_4\) respectively denote the elastic forces and moments; \(N_5\) and \(M_5\) denote the optical control forces and moments; \(R\) is the radius of the cylinder; \(\rho\) is the shell mass density; \(h\) is the shell thickness; \(\ddot{u}_T\) are the accelerations; and \(F_1, F_2, F_3\) are the external excitations. Detailed photodeformation process and the optical control forces and moments are discussed next.

3.1 Modal Expansion Method

It is assumed the dynamic response induced by the external forces and moments can be synthesized by the participating mode shape functions. The amount of each modal participation in the total dynamic response is defined by a modal participation factor. Thus, the total dynamic response can be represented by the summation of all participating natural modes multiplied by their respective modal participation factors:

\[
u_{in}(\tau, \alpha, \alpha_T) = \sum \eta_i(\tau) U_{in}(\alpha, \alpha_T), i = 1, 2, 3,
\]

where \(\eta_i(\tau)\) is the modal participation factor; \(U_{in}(\alpha, \alpha_T)\) is the mode shape function; and \(k\) denotes the k-th mode. Substituting the modal expansion equation into the shell equations, integrating over the whole shell surface and applying the modal orthogonality, one can derive the k-th modal equation:

\[
\ddot{\eta}_i + 2\omega_n^2 \dot{\eta}_i + \omega_n^2 \eta_i = \ddot{F}_n,
\]

where \(\omega_n\) is the k-th modal natural frequency; \(\zeta_\alpha\) is the k-th modal damping ratio and \(\zeta_\alpha = c(2\zeta_\alpha h \omega_n)\); and \(c\) is the damping constant. When the external mechanical forces \(F_1 = F_2 = F_3 = 0\), the distributed modal (control) force \(\ddot{F}_n\) is defined as

\[
\ddot{F}_n = \frac{1}{\rho N_{\infty}} \int \left[ \frac{\partial^2 M_{xx}}{\partial \tau^2 \partial \phi} + \frac{1}{R} \frac{\partial^2 M_{xx}}{\partial \tau^2 \partial \phi} \right] U_{in} d\phi d\tau
\]

\[
\left[ \frac{\partial^2 M_{xx}}{\partial \tau^2 \partial \phi} + \frac{1}{R} \frac{\partial^2 M_{xx}}{\partial \tau^2 \partial \phi} \right] U_{in} d\phi d\tau
\]

\[
N_m = \frac{1}{R} \int \sum \int U_{in} \text{d}a d\alpha d\tau. \quad \text{Detailed modal control forces/moments} \ \ddot{F}_n \ \text{are defined by the control algorithms, i.e., the Lyapunov control and the proportional feedback control.}
\]

3.2 Lyapunov Control

In this case, the control force is defined by signum functions and it is reflected in induced strains and controlled by the light illuminations.

\[
\dot{F}_{\text{max}} = \frac{V_{Th}^*}{\rho N_{\infty}} \left[ \frac{A_5 S_{41} + B_5 L S_{51}}{n mR_\infty} + \frac{L S_{51}}{mR_\infty} \right] \left[ \text{cos}(m_{\alpha} x_1 / L) - \text{cos}(m_{\alpha} x_1 / L)[\text{sin} m_{\alpha} x_1 - \text{sin} m_{\alpha} x_1] \right]
\]

where \(A_5, B_5\) are the normalized modal amplitudes; the strains \(S_m\) are the induced control strains resulting from the photovoltaic electric field and the temperature effect, i.e.,

\[
S_m = d_1 E_m^n (t) + \delta_1 \theta_1 (t) / n_{\alpha}^p
\]

and the electric field \(E_m^n(t)\), temperature \(\theta_1(t)\), and light intensity \(l(t)\) are defined as

\[
E_m^n(t) = E_m^n (t_0) + \left[ E_m^n (t_0) - E_m^n (t) \right] e^{-\gamma_1 (t_0 - t)}, \quad \Delta t
\]

\[
\theta_1(t) = \theta_1 (t_0) + \left[ \theta_1 (t_0) - \theta_1 (t) \right] e^{-\gamma_1 \Delta t} \Delta t
\]

\[
l(t) = l(t_0) + \left[ l(t_0) - l(t) \right] e^{-\gamma_1 \Delta t} \Delta t
\]

where \(E_m^n\) is the saturation voltage; \(\alpha_0\) is the optical actuator constant; \(\gamma_1\) is the voltage leakage constant; \(\gamma_1\) is the heat transfer rate; \(W\) is the power of absorbed heat; \(\Delta t\) is the time step; \(\lambda\) is the light intensity gain; \(g\) is the feedback gain; and \(\text{sign}(\cdot)\) is a signum function. Detailed nominal values are summarized in a case study presented later.

3.3 Proportional Feedback Control

It is assumed that the control force/moment generated by the optical actuator is proportional to the vibration velocity in the proportional velocity feedback control. The control effect is reflected in the light intensity control and the modal control force takes the same form as above. Furthermore, the \(j\)-th step force, strain, electric field, temperature equations are similar to the Lyapunov control; the light intensity, however, is defined by the proportional control algorithm:

\[
\dot{F}_j = \lambda_0 \left[ \text{sign}(\text{sgn}(\eta_j) - g_{\text{max}} \eta_j) - \text{sgn}(\eta_j) \right] \eta_j
\]

3.4 Membrane and Bending Control Effects

The overall control effectiveness is introduced by the membrane control forces and the control moments. The \(k\)-th modal control force is divided into two parts: the membrane control component \(\dot{F}_{\text{mem}}\) and the control moment component \(\dot{F}_{\text{mom}}\):

\[
\begin{align}
\dot{F}_{\text{mem}} &= \frac{1}{\rho N_{\infty}} \int \frac{\partial^2 M_{xx}}{\partial \tau^2 \partial \phi} U_{in} d\phi d\tau
\end{align}
\]

\[
\begin{align}
\dot{F}_{\text{mom}} &= \frac{V_{Th}^*}{\rho N_{\infty}} \left[ \frac{A_5 S_{41} + B_5 L S_{51}}{n mR_\infty} + \frac{L S_{51}}{mR_\infty} \right] \left[ \text{cos}(m_{\alpha} x_1 / L) - \text{cos}(m_{\alpha} x_1 / L)[\text{sin} m_{\alpha} x_1 - \text{sin} m_{\alpha} x_1] \right]
\end{align}
\]
\[
(\tilde{\hat{F}}_{\text{mem}})_{\text{mem}} = \frac{1}{\phi \eta N_{\text{mem}}} \sum_{j} \left[ \frac{1}{R_j} \frac{dM_{\text{mem}}^j}{d\beta} U_{\text{mem}}^j + \left( \frac{\partial M_{\text{mem}}^j}{\partial \beta} \right) \right] \text{d}x_j \beta
\]

\[
= \frac{(h + h')}{\phi \eta N_{\text{mem}}} \sum_{j} \left[ \frac{m \alpha_j}{2n!} \sum_{j} \frac{\partial S_{j1}^2}{\partial \beta} \right] \left[ \cos(m \alpha_j / l) - \cos(m \alpha_j / l) \right] \left[ \sin n \beta \right] - \sin n \beta \right]
\]

It is observed that the spatial distribution part of these force components are identical.

Thus, the modal force of the k-th mode can be written as

\[
\tilde{F}_{\text{mem}} = (\tilde{\hat{F}}_{\text{mem}})_\text{mem} + (\tilde{\hat{F}}_{\text{mem}})_\text{load}.
\]

As discussed previously, \( S_{j1} \) and \( S_{j2} \) are respectively the control strains in the x and y directions induced by the photo deformation process. The original process involves three strain components due to the opto-electricity, the opto-thermo-electricity and the opto-thermo-elastostaticity [9]. For simplicity, one can lump all three induced fields as \( \tilde{E}_j \) and define the induced strains \( S_{j1} \) and \( S_{j2} \) as

\[
S_{j1} = \tilde{\alpha}_{j1} \tilde{E}_j \quad \text{and} \quad S_{j2} = \tilde{\alpha}_{j2} \tilde{E}_j.
\]

where \( \tilde{\alpha}_{j} \) are the resultant optical strain constants. Substituting strain expressions into the membrane and moment control effects yields

\[
(\tilde{\hat{F}}_{\text{mem}})_\text{mem} = \frac{Y^* \alpha_k^j \tilde{E}_j}{\phi \eta N_{\text{mem}}} \left( A_j^k \frac{\alpha_{j1}}{n} + \frac{B_j \alpha_{j2}}{m \alpha_j} \right) \left( \frac{\partial \beta}{\partial \beta} \right)
\]

\[
= \left[ \cos(m \alpha_j / l) - \cos(m \alpha_j / l) \right] \left[ \sin n \beta \right] - \sin n \beta \right]
\]

\[
= \frac{(h + h')}{\phi \eta N_{\text{mem}}} \sum_{j} \left[ \frac{m \alpha_j}{2n!} \sum_{j} \frac{\partial S_{j1}^2}{\partial \beta} \right] \left[ \cos(m \alpha_j / l) - \cos(m \alpha_j / l) \right] \left[ \sin n \beta \right] - \sin n \beta \right]
\]

\[
(\tilde{\hat{F}}_{\text{mem}})_\text{load} = \left( l + h' \right) \frac{Y^* \alpha_k^j \tilde{E}_j}{\phi \eta N_{\text{mem}}} \left( \frac{m \alpha_j}{2n!} \sum_{j} \frac{\partial S_{j1}^2}{\partial \beta} \right) \left[ \cos(m \alpha_j / l) - \cos(m \alpha_j / l) \right] \left[ \sin n \beta \right] - \sin n \beta \right]
\]

4 Distributed Control Effectiveness

Dynamics and control of a simply supported cylindrical shell coupled with two distributed optical actuator patches ranging from \( x_1 \) to \( x_2 \) and from \( y_1 \) to \( y_2 \) is investigated. The optical actuators are respectively laminated on the shell top and bottom surfaces, one induces a positive control action and the other induces a negative control action. All optical actuator parameters have been calibrated based on the laboratory experiments [7,9]. Vibration control and time history analyses are presented next. Detailed membrane and bending control contributions are studied.

4.1 Comparison of Membrane and Bending Control Effects

Recall that \( (\tilde{\hat{F}}_{\text{mem}})_{\text{mem}} \) and \( (\tilde{\hat{F}}_{\text{mem}})_{\text{load}} \) are regarded as the control coefficients of the k-th mode, which respectively indicate the modal membrane and bending control effectiveness of the optical actuator. The control coefficients of membrane control forces and those of control moments of the \( (m=1, n=1) \) mode are compared in Figure 3. Figure 4 shows the control coefficients (modes \( m=1, n=1 \) and \( m=2, n=1 \)) vary with respect to the mode number \( n \) for a thicker cylindrical shell (\( h=0.4m \)).

![Figure 3: Membrane and bending control coefficients of a thin shell (h=0.4m).](image)

It is observed that the control coefficients of membrane forces are much larger than those of control moments for most of modes \( (n=1, 2, \ldots, 15) \) and the absolute value decreases as the mode \( n \) increases. The variations are affected by the individual mode shape function and oscillation for the given opto-mechanical actuator patch. The bending control effect is relatively insignificant for lower natural modes. Summing the membrane control effect and the bending control effect gives the total modal control effect. The bending control effect is relatively insensitive as compared with the membrane control effect. The control equation reveals that the bending control effect increases as the moment arm increases. Accordingly, the bending control effect increases when the shell becomes thicker, if the optical actuator is surface laminated. Note that the membrane control effect remains identical as before. The control effect variations for the modes \( m=1 \) and \( m=2 \) are about the same, however, the \( m=2 \) control effect is smaller than that of the \( m=1 \) mode. As discussed previously, the variations are affected by the individual mode shape function and oscillation for the specified opto-mechanical actuator patch (size and location).
4.2 Time History Analysis

Membrane and bending control effects are investigated previously, transient time history responses with and without control are calculated and compared in this section. The controlled natural modes of the cylindrical shell are limited to lower natural modes. A velocity signal of a reference point obtained by the optimization procedures is used in the velocity feedback control - the proportional velocity feedback and the Lyapunov feedback. It is assumed that the optical actuator is placed from 0.2m to 0.5m and from -5n/11 to 5n/11. An initial displacement of 1.0x10^-7m is imposed and the closed back responses with and without control are investigated. Note that only the modal coordinate responses are plotted. The physical modal displacement response can be obtained by multiply the modal response by the mode shape function. The resultant cylinder response is obtained by summing all participating modal responses. Transient time history responses of the cylindrical shell of the free oscillation and with the Lyapunov control and the negative velocity proportional feedback control are calculated and presented. For demonstration, two low natural modes, the (1,5) mode with the lowest frequency and an arbitrary (1,9) mode, are presented in Figures 5 and 6.

Free response and control responses are compared in each figure. Note that the damped frequencies change due to the enhanced controlled damping ratios. Settling times of the time history responses can also be used to compare the control effectiveness. For example, the (1,5) mode's 10% settling times corresponding to the proportional feedback control, the Lyapunov control and the free response are respectively 0.28s, 0.10s and 0.44s. Apparently, the Lyapunov control provides better control effect than the proportional control. (This behavior also complies with a number of earlier studies of distributed systems with piezoelectric actuators [9, 11, etc.])

5 Summary and Conclusions

Shell-type structures and components are widely used in many advanced mechanical and structural systems, e.g., nozzles, fuel tanks, antenna dishes, rockets, etc. Non-contact light-activated distributed opto-mechanical actuators represent a new class of distributed actuator which can induce distributed actuation without hard-wire connections. Active distributed control of flexible shells using distributed opto-mechanical shell actuators are,
investigated in this study. Fundamental photodeformation behavior was defined first and induced control forces and moments were defined next. Modelling of a circular cylindrical shell laminated with distributed optical actuator patches was presented and the closed-loop opto-thermoelectromechanical equations were derived. The system equations revealed the couplings among elasticity, photodeformation, pyroelectricity, and thermoelasticity. Solution procedures based on the modal analysis technique were outlined and detailed mechanism of the optical actuator patches was investigated. The resultant control effect includes 1) the membrane control effect and 2) the control moment effect in which the membrane control force contributes the in-plane control effect and the control moment counteracts the bending oscillations. Formulations of the proportional and the Lyapunov control algorithms were also presented. Numerical results indicated that the membrane control force dominates the control of lower (membrane-type) natural modes and the control moment contributes that higher (bending-type) mode control. Time-history analyses illustrated the control effectiveness of the distributed optical actuator and suggested that the Lyapunov control is more effective than the proportional control. Note that all results were based on theoretical analysis and numerical simulations using experimentally calibrated data. Further experiments or finite element analysis should be carried out to validate the results.

References