PARAMETRIC STUDY OF SEGMENTED TRANSUDCERS LAMINATED ON CYLINDRICAL SHELLS, PART 2: ACTUATOR PATCHES

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The spatial actuation and control effectiveness of distributed segmented actuator patches laminated on a piezoelectric laminated cylindrical shell are studied. Modal control forces of an arbitrary actuator patch and also quarterly segmented patches are derived and evaluated. The modal actuation factor, the modal feedback factor and the controlled damping ratio are derived and their detailed membrane and bending actuations are evaluated with respect to actuator design parameters: actuator thickness, shell lamina thickness, shell curvatures, shell sizes and natural modes. Analytical and simulation results suggest that the membrane control action dominates the lower natural modes and it increases as the shell curvature increases—for deep shells. The bending control action is effective for higher modes and also for shallow or zero-curvature continua. The modal control effect increases when the actuator becomes thicker; however, it drops significantly as the shell lamina becomes thicker.

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1. INTRODUCTION

The actuation effect of distributed piezoelectric actuators depends on a number of factors, such as actuator materials, actuator shapes, placements, spatial distribution, and locations. In general, the control action of distributed actuators laminated on zero-curvature structures, e.g., beams and plates, is derived from the counteracting control moments. However, there are two control actions: (1) membrane control forces; and (2) counteracting control moments induced by distributed actuators laminated on shell-type structures, e.g., cylindrical shells, spherical shells, ring shells, etc. [1]. Various studies of distributed piezoelectric actuators laminated on zero-curvature planar structures have been conducted and reported in recent years [2–8]. However, studies of non-zero curvature shells have not been fully explored [1, 9, 10]. The mechanical properties, e.g., mass and stiffness, of piezoelectric sensor/actuator layers have been mostly neglected in previous studies. These mechanical effects can be crucial if piezoceramics are used in the laminated structures. In this study, the mechanical properties of piezoelectric layers are incorporated in the formulation and analysis.

This is the second part of the study of segmented shell transducers, which concentrates on the spatial actuation effects and design parameter study of distributed segmented actuators laminated on cylindrical shells. The electromechanics of segmented piezoelectric
actuator patches are studied and detailed contributions of membrane and bending control effects are analyzed. The design parameters and control effectiveness are evaluated.

2. PIEZOELECTRIC LAMINATED CYLINDRICAL SHELL

A simply supported laminated cylindrical shell (with radius $R$, length $L$ and curvature angle $\beta^*$) consisting of three elastic laminae (with thicknesses $h_2$, $h_3$ and $h_4$) and two piezoelectric laminae (with thicknesses $h_1$ and $h_5$) is used in this study. The bottom (first) layer serves as a sensor layer and the top (fifth) layer serves as an actuator layer. Segmenting of the sensor layer into sensor patches was investigated in Part 1 of this paper; segmenting of the actuator layer into actuator patches is studied here in Part 2. Detailed modelling and the governing electromechanical equations were presented in Part 1. This part only concentrates on spatial characteristics and the control effectiveness of the segmented actuator patches. Open loop control is discussed first, followed by closed loop control. Control effects of an arbitrary actuator patch and quarterly segmented actuator patches are analyzed.

3. SEGMENTED DISTRIBUTED ACTUATOR (OPEN LOOP)

It is assumed that an arbitrary $i$th surface electrode—a segmented patch—on the piezoelectric actuator layer is defined from $x_i^*$ to $x_i^*$ in the longitudinal direction and from $\beta_i^*$ and $\beta_i^*$ in the circumferential direction, where $x_i^*$, $x_i^*$, $\beta_i^*$ and $\beta_i^*$ are specified co-ordinates in the cylindrical co-ordinate system (see Figure 1). The effective actuator area is defined by $R(x_i^* - x_i^*)(\beta_i^* - \beta_i^*)$. It is assumed that both piezoelectric layers are used as actuators to enhance the control action in the open loop control.

If the electrode resistance is neglected, the potential is constant over the effective electrode surface—the segmented patch. Thus, a transverse control potential $\phi_i(x, \beta, t)$ applied to the $i$th distributed piezoelectric actuator patch is defined as

$$\phi_i(x, \beta, t) = \phi_i(t)[u_s(x - x_i^*) - u_s(x - x_i^*)][u_s(\beta - \beta_i^*) - u_s(\beta - \beta_i^*)]$$  \hspace{1cm} (1)

where the superscript $c$ denotes the control action and $u_s(\cdot)$ is a unit step function; $u_s(x - x_i^*)$ is equal to 1 when $x \geq x_i^*$, and to 0 when $x < x_i^*$. The spatial derivatives are

$$\frac{\partial}{\partial x} (\phi_i) = \phi_i(t)[\delta(x - x_i^*) - \delta(x - x_i^*)][u_s(\beta - \beta_i^*) - u_s(\beta - \beta_i^*)],$$ \hspace{1cm} (2a)

$$\frac{\partial}{\partial \beta} (\phi_i) = \phi_i(t)[u_s(x - x_i^*) - u_s(x - x_i^*)][\delta(\beta - \beta_i^*) - \delta(\beta - \beta_i^*)],$$ \hspace{1cm} (2b)

where $\delta(\cdot)$ is the Dirac delta function: $\int \delta(x - x_i^*) \, dx = 1$, and $\delta(x - x_i^*) = 0$ when

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Figure 1. A piezoelectric laminated cylindrical shell with actuator patches.
Using the modal expansion technique [1], one can derive the \textit{mth} modal equation of the cylindrical shell as

\[
\ddot{\eta}_m + 2\zeta_m \omega_m \dot{\eta}_m + \omega_m^2 \eta_m = F_{m \alpha}(t) + F_{m \beta}(t) + F_{m \gamma}(t),
\]

where \( \eta_m \) is the modal participation factor; \( \zeta_m \) is the damping ratio; \( \omega_m \) is the \textit{mth} natural frequency; \( F_{m \alpha}(t) \) is the mechanical excitation; \( F_{m \beta}(t) \) is the temperature excitation; \( F_{m \gamma}(t) \) is the electric control force; and \( m \) is the half-wavenumber in the \( x \) direction and \( n \) is the half-wavenumber in the \( \beta \) direction. A generalized \textit{mth} modal electric control force \( F_{m \beta} \) induced by the piezoelectric actuator patch includes three effects: a control moment \( M_{m \beta} \) in the longitudinal (\( x \)) direction, a control moment \( M_{m \gamma} \) in the circumferential (\( \beta \)) direction, and a control force \( N_{m \beta} \) in the circumferential (\( \beta \)) direction:

\[
F_{m \beta} = \frac{1}{\rho h N_m} \int_0^L \int_0^{\pi} \left[ \frac{\partial^2 M_{m \beta}}{\partial x^2} + \frac{\beta^2}{R^2} M_{m \gamma} + \frac{1}{R} N_{m \beta} \right] R \, dx \, d\beta,
\]

where \( N_m = \int_0^L \int_0^{\pi} U_{m \beta}(x, \beta) R \, dx \, d\beta; U_{m \beta} \) is the mode shape function, which equals \( \sin \left( \frac{m \pi x}{L} \right) \sin \left( \frac{n \pi \beta}{\beta^*} \right) \) for a simply supported cylindrical shell; \( \rho h \) is the weighted mass density

\[ (\rho h) = \sum_{i=1}^{\phi_i} \rho_i h_i \text{ and } h_1 = h_5; h_2 = h_3 = h_4; \]

\( \rho \) is the mass density and \( h_i \) is the thickness of the \( i \)th layer. Assume that two control voltages are, respectively, injected into the two piezoelectric layers, i.e., the first layer \( \phi_i \) and the fifth \( \phi_5 \). (Both layers are used as actuator layers in this case). The generalized modal electric control force becomes

\[
F_{m \beta} = \frac{4}{\rho h L R \beta^*} \int_0^L \int_0^{\pi} \left\{ \left[ \frac{\epsilon_{ij}}{2} \left( h_1 + 3 h_3 \right) \frac{\partial}{\partial x} \left( \phi_i \phi_j \right) \right] 
+ \left[ \frac{\epsilon_{ij}}{2 R^2} \left( h_1 + 3 h_3 \right) \frac{\partial^2}{\partial \beta^2} \left( \phi_i \phi_j \right) \right] 
- \left[ \frac{\epsilon_{ij}}{R} \left( \phi_i \phi_j \right) \right] \sin \left( \frac{m \pi x}{L} \right) \sin \left( \frac{n \pi \beta}{\beta^*} \right) \right\} R \, d\beta \, dx
\]

\[
= \frac{4}{\rho h L R \beta^*} \int_0^L \int_0^{\pi} \left\{ \left[ \epsilon_{ij} r^p u_s(\beta - \beta^*) - u_s(\beta - \beta^*) \right] (\phi_i - \phi_j) 
\times \frac{\partial}{\partial x} [\delta(x - x^*) - \delta(x - x^*)] \right\}
+ \left[ \frac{\epsilon_{ij}}{R^2} \left[ u_s(x - x^*) - u_s(x - x^*) \right] (\phi_i - \phi_j) \frac{\partial}{\partial \beta} [\delta(\beta - \beta^*) - \delta(\beta - \beta^*)] \right]
- \left[ \frac{\epsilon_{ij}}{R} (\phi_i + \phi_j) \left[ u_s(x - x^*) - u_s(x - x^*) \right] \right]
\times [u_s(\beta - \beta^*) - u_s(\beta - \beta^*)] \right\} \sin \left( \frac{m \pi x}{L} \right) \sin \left( \frac{n \pi \beta}{\beta^*} \right) R \, d\beta \, dx.
\]
where \( e_{31} \) is the piezoelectric constant; \( r^* \) is the control moment arm, i.e., the distance from the reference surface to the middle surface of the actuator layer, and \( r^* = (h_i + 3h_2)/2 \). In this expression, the term inside the first square bracket is due to the control moment \( M_{31} \), that inside the second square bracket is due to the control moment \( M_{13} \), and that inside the third square bracket is due to the membrane control force \( N_{31} \). Carrying out the surface integrations gives

\[
F_{mn} = -\frac{4}{\rho \pi L^2} \left[ \left( \frac{\cos \frac{m \pi x^*}{L}}{\beta^*} - \frac{\cos \frac{n \pi y^*}{L}}{\beta^*} \right) \left( \frac{\cos \frac{n \pi y^*}{\beta^*}}{\beta^*} - \frac{\cos \frac{n \pi y^*}{\beta^*}}{\beta^*} \right) \right] \\
\times \left\{ e_{31} r^*(\phi_i^* - \phi_i^*) \left( \frac{m \beta^*}{nL} \right) \right\} + \left[ \frac{e_{31}}{R} r^*(\phi_i^* - \phi_i^*) \left( \frac{nL}{m \beta^*} \right) \right] \\
+ \left[ \frac{e_{31}}{R} (\phi_i^* + \phi_i^*) \left( \frac{\beta^* L}{m \pi} \right) \right].
\]  

(6)

Note that the cosine terms inside the first bracket represent the spatial actuation effects of the \( mn \)th mode and that the other control forces are grouped in the braces. The effects of the spatial distribution of actuators and segmented patches are discussed next.

3.1. FULLY DISTRIBUTED ACTUATOR

For a fully distributed actuator layer. \( \beta^* = 0 \), \( \beta^* = \beta^* \), \( x^* = 0 \), \( x^* = L \) and \( \phi_i^* = \phi_i^* = \phi(t) \). The modal control force becomes

\[
F_{mn} = -\frac{4}{\rho \pi L^2} \left[ 1 - \cos (n \pi) \right] \left[ 1 - \cos (m \pi) \right] \left( \frac{e_{31}}{R} \frac{\beta^* L}{m \pi} \right) \\
= \left\{ -\frac{32 e_{31}}{\rho \pi n m R} \phi(t), \quad \text{when } m \text{ and } n \text{ are odd}; \right. \\
\left. 0, \quad \text{when } m \text{ or } n \text{ are even}. \right. 
\]  

(7)

The above equations reveal that the fully distributed piezoelectric actuator layer is insensitive to all even modes of the simply supported cylindrical shell. Control moment effects are cancelled out due to equal amplitude and opposite moment effects on the top and bottom actuator layers. The generalized modal electric control force \( F_{mn} \) depends on the mode numbers, the spatial distribution, the actuator material properties, and the locations and dimensions of the actuator patches.

3.2. ARBITRARY \( \rho \)TH SEGMENTED PATCH

Assume that the distributed actuator layer is segmented into \( k \) patches (\( k > 1 \)). The \( \rho \)th patch is defined from \( x_\rho \) to \( x_\rho \) in the \( x \)-direction and from \( \beta_{\rho i} \) to \( \beta_{\rho i} \) in the \( \beta \)-direction. The resultant modal control force \( F_{mn} \) is a summation of all control forces, \( (F_{mn}) \), contributed by all actuator patches; i.e.,

\[
F_{mn} = \sum_{\rho=1}^{k} (F_{mn})_ho.
\]
where the subscript \( p \) denotes the \( p \)th actuator patch. The modal control force of the \( p \)th segmented patch is
\[
(F^*_m)_p = -\frac{4}{\rho h L \beta^*} \left\{ \left( \cos \frac{\pi \beta_1}{L} - \cos \frac{\pi \beta_2}{L} \right) \left( \cos \frac{\pi \beta_1}{\beta^*} - \cos \frac{\pi \beta_2}{\beta^*} \right) \right\} \\
\times \left\{ e^2_i r^2(\phi_i - \phi_i^*) \left( \frac{m \beta^*}{n L} \right) + e^2_i r^2(\phi_i^* - \phi_i) \left( \frac{n L}{m \beta^*} \right) \right\} \\
+ \left\{ e^2_i R (\phi_i + \phi_i^*) \left( \frac{\beta^* L}{m \pi^2} \right) \right\}.
\]
(8)

The resultant control force contributed by all actuator patches becomes
\[
F^*_m = -\frac{4}{\rho h L \beta^*} \left\{ \left[ e^{2}_i r^2(\phi^*_i - \phi^*_i) \left( \frac{m \beta^*}{n L} \right) + e^{2}_i r^2(\phi^*_i - \phi^*_i) \left( \frac{n L}{m \beta^*} \right) \right] \\
+ \left\{ \frac{e^2_i}{R} (\phi^*_i + \phi^*_i) \left( \frac{\beta^* L}{m \pi^2} \right) \right\} \sum_{\rho=1}^{k} \left( \cos \frac{\pi \beta_1}{L} - \cos \frac{\pi \beta_2}{L} \right) \\
\times \left( \cos \frac{\pi \beta_1}{\beta^*} - \cos \frac{\pi \beta_2}{\beta^*} \right) \right\}.
\]
(9)

Again, the resultant electric control force is composed of two control moment effects and one membrane control force effect. The spatial characteristics and modal contributions are also observed.

3.3. FOUR SEGMENTED ACTUATOR PATCHES

Furthermore it is assumed that the distributed piezoelectric actuator layer, with an effective area \( R(x^* - x^*) (\beta^* - \beta^*) \), is divided into four segments (\( k = 4 \)) such that \( x_1 = x_2 = x^*_1, x_2 = x_3 = x^*_2, x_3 = x_4 = x^*_3, \beta_1 = \beta_2 = \beta^*_1, \beta_2 = \beta_3 = \beta^*_2, \beta_3 = \beta_4 = \beta^*_3 \). (These four arbitrary divided actuator patches are then assumed to be equally divided patches in later derivations). Thus, the summation part becomes
\[
\sum_{\rho=1}^{4} \left( \cos \frac{\pi \beta_1}{L} - \cos \frac{\pi \beta_2}{L} \right) \left( \cos \frac{\pi \beta_1}{\beta^*} - \cos \frac{\pi \beta_2}{\beta^*} \right) \\
= \left( \cos \frac{\pi \beta_1}{L} - \cos \frac{\pi \beta_2}{L} \right) \left( \cos \frac{\pi \beta_1}{\beta^*} - \cos \frac{\pi \beta_2}{\beta^*} \right). \]
(10)

It is observed that the generalized modal electric control force \( F^*_m(\mathbf{i}) \) remains unchanged regardless of the number of actuator patches as long as the same control voltage is applied to all actuator patches. Different control voltages applied to various actuator patches would change the resultant control force effect. In addition, different shaped actuator electrodes would also influence the resultant control force \( F^*_m \) and, consequently, it is possible to excite or suppress other natural modes. The magnitude of the control forces
and/or moments depends on the control voltages and the actuator patch sizes and locations.

It is further assumed that the fully distributed actuator layer is equally divided into four equal-sized segments (the electrode cut along the centerline); i.e., $x = L/2$ and $\beta = \beta^*/2$. Then the first segment is defined by $(\beta^*/2 - 0)/(L/2 - 0)$, the second by $(\beta^*/2 - 0)/(L - L/2)$, the third by $(\beta - \beta^*/2)/(L/2 - 0)$, and the fourth by $(\beta - \beta^*/2)/(L - L/2)$. Control voltages $(\phi_{i,1} + \phi_{i,2})_1 = 2\phi_1$, $(\phi_{i,1} + \phi_{i,2})_2 = 2\phi_2$, $(\phi_{i,1} + \phi_{i,2})_3 = 2\phi_3$, and $(\phi_{i,1} + \phi_{i,2})_4 = 2\phi_4$ are, respectively, input to these four patches. (Note that the subscript $i$ with the $\phi_i$ on the right-hand side denotes the $i$th patch). The resultant control force can be evaluated by

$$F_{m} = -\frac{8\varepsilon_{11}}{\rho h^2 n m R} \sum_{j=1}^{4} \left( \phi_j \left( \cos \frac{m \pi \beta x_{j,1}}{L} - \cos \frac{m \pi x_{j,2}}{L} \right) \right) \left( \cos \frac{n \pi \beta x_{j,1}}{\beta^*} - \cos \frac{n \pi \beta x_{j,2}}{\beta^*} \right). \tag{11}$$

More specifically, the resultant modal control forces for low even modes ($k < 4$) are defined as follows:

$$F_{i,2} = -\frac{8\varepsilon_{11}}{\rho h^2 m R} (\phi_1 + \phi_2 - \phi_3 - \phi_4), \quad F_{i,1} = -\frac{8\varepsilon_{11}}{\rho h^2 n R} (\phi_1 - \phi_2 + \phi_3 - \phi_4), \quad \tag{12a, b}$$

$$F_{i,2} = -\frac{8\varepsilon_{11}}{\rho h^2 n R} (\phi_1 - \phi_2 + \phi_3 - \phi_4), \quad F_{i,1} = -\frac{8\varepsilon_{11}}{3\rho h^2 n R} (\phi_1 - \phi_2 + \phi_3 - \phi_4). \quad \tag{12c, d}$$

$$F_{i,2} = -\frac{8\varepsilon_{11}}{3\rho h^2 n R} (\phi_1 + \phi_2 - \phi_3 - \phi_4), \quad \ldots \ldots \quad \tag{12e}$$

Accordingly, the corresponding low even modes ($m, n < 4$) can be excited or suppressed. Note that when a single control voltage is input to all four actuator patches, i.e., $\phi_1 = \phi_2 = \phi_3 = \phi_4$, control forces corresponding to the even modes are zero, and these modes are uncontrollable. For higher even modes, more segmented actuator patches, regularly or spatially shaped, should be considered.

4. SEGMENTED DISTRIBUTED SENSOR AND ACTUATOR (CLOSED LOOP)

In the closed-loop feedback control, the top layer serves as an actuator layer, the bottom layer serves as a sensor layer, and the other three middle layers are elastic laminae in the five-layer laminated cylindrical shell configuration. Note that when segmenting distributed sensor and actuator layers into patches, both sensor and actuator layers are segmented into the same patterns such that each actuator patch has a corresponding sensor patch with the same co-ordinates in $x$ and $\beta$. Each sensor signal is processed and fed back to the corresponding colocated distributed actuator patch. As discussed in the segmented distributed sensor patches (Part I), the output signal $(\phi^s)_p$ of the $p$th sensor patch is contributed by the direct piezoelectric effect and the pyroelectric effect:

$$(\phi^s)_p = \sum_{m=1}^{x} \sum_{n=1}^{\varepsilon} (\phi^m^s)_p + (\phi^n^s)_p = \sum_{m=1}^{x} \sum_{n=1}^{\varepsilon} \left[ (\phi^m^s)_{mem,\beta} + (\phi^n^s)_{mem,\beta} \right] + (\phi^n^s)_p, \tag{13}$$

where $(\phi^m^s)_p$ is the signal contributed by the direct piezoelectric effect of the $mn$th mode; $(\phi^n^s)_p$ is the signal contributed by the temperature induced pyroelectric effect. The direct effect signal depends on strains, which can be further divided into (1) the membrane contribution $(\phi^m^s)_{mem,\beta}$ and (2) the bending contribution $(\phi^n^s)_{bend,\beta}$. Note that the subscript $p$ denotes the $p$th sensor patch; the superscript $s$ denotes the sensor, $d$ denotes the direct effect, and $p$ denotes the pyroelectric effect. In dynamic analysis and vibration control, the
temperature related pyroelectric effect is not considered. Dynamic modal electric control forces of various actuator patches incorporating collocated sensor signals are defined next.

4.1. ARBITRARY pTH ACTUATOR PATCH AND MODAL ACTUATION FACTOR

It is assumed that the actuator layer is segmented into a total of \(k\) actuator patches. The \(mn\)th modal electric control force from an arbitrary \(p\)th distributed segmented actuator patch is defined as

\[
(F_{mn})_p = -\frac{4\epsilon_{33}}{\rho h R \beta^* L} \left(\frac{mR \beta^*}{mn\pi^2} + r\left(\frac{mL}{mR \beta^*} + \left(\frac{nL}{nR \beta^*}\right)\right)\right)
\times \left[\left(\cos \frac{m\pi X_p}{L} - \cos \frac{m\pi x_p}{L}\right)\left(\cos \frac{n\pi \beta_1}{\beta^*} - \cos \frac{n\pi \beta_2}{\beta^*}\right)\right],
\]

where \((\phi')_p\) is the feedback voltage from the \(p\)th sensor patch to the collocated actuator patch, and this control voltage will be specifically defined in feedback algorithms presented later. Define an \(mn\)th modal actuation factor \(A^e_{mn}\) of the laminated cylindrical shell:

\[
A^e_{mn} = \frac{4\epsilon_{33}}{\rho h R \beta^* L} \left(\frac{mR \beta^*}{mn\pi^2} + r\left(\frac{mL}{mR \beta^*} + \left(\frac{nL}{nR \beta^*}\right)\right)\right).
\]

Note that the \(mn\)th modal actuation factor \(A^e_{mn}\) is independent of time and spatial distribution, and represents the \(mn\)th modal force magnitude per unit control voltage [N/(kg/V)]. Thus, the \(mn\)th control force of the \(p\)th actuator patch can be simplified to

\[
(F_{mn})_p = -A^e_{mn} (\phi')_p \left(\cos \frac{m\pi X_p}{L} - \cos \frac{m\pi x_p}{L}\right)\left(\cos \frac{n\pi \beta_1}{\beta^*} - \cos \frac{n\pi \beta_2}{\beta^*}\right).
\]

The resultant control force is a summation of all \(k\) control forces resulting from the actuator patches; i.e.,

\[
F_{mn}(t) = -A^e_{mn} \sum_{p=1}^{k} (\phi')_p \left(\cos \frac{m\pi X_p}{L} - \cos \frac{m\pi x_p}{L}\right)\left(\cos \frac{n\pi \beta_1}{\beta^*} - \cos \frac{n\pi \beta_2}{\beta^*}\right).
\]

Again, the modal control force depends on the spatial distribution, the modal actuation factor and the control voltage applied to the actuator pitch. In the cylindrical shell case, the modal actuation factor can be further divided into the membrane actuation factor \((A^e_{mn})_{mem}\) and the bending actuation factor \((A^e_{mn})_{bend}\), respectively representing the membrane and bending control actions:

\[
A^e_{mn} = (A^e_{mn})_{mem} + (A^e_{mn})_{bend},
\]

where

\[
(A^e_{mn})_{mem} = \frac{4\epsilon_{33}}{\rho h R \beta^* L} \left(\frac{\beta^* L}{mn\pi^2}\right), \quad (A^e_{mn})_{bend} = \frac{4\epsilon_{33}}{\rho h R \beta^* L} r\left(\frac{mL}{nR \beta^*} + \left(\frac{nL}{mR \beta^*}\right)\right).
\]

4.2. QUARTERLY SEGMENTED ACTUATOR PATCHES

Furthermore, it is assumed that the distributed piezoelectric actuator is equally segmented into four equal-sized actuator patches. (Note that the sensor layer is also
segmented into the same sensor patches). Substituting the modal control force into the modal equation (3) gives

\[ \ddot{\eta}_{mn} + 2\zeta_{mn}\omega_{mn}\dot{\eta}_{mn} + \omega_{mn}^2 \eta_{mn} = F_{mn}^v(t) + \sum_{p=1}^{4} F_{mn}^c \]

\[ = F_{mn}^v(t) + \left( \phi^t \right) \left( \frac{1 - \cos \frac{m\pi}{2}}{1 - \cos \frac{n\pi}{2}} \right) \left( \frac{\cos \frac{m\pi}{2}}{1 - \cos \frac{m\pi}{2}} \right) \]

\[ + \left( \phi^t \right) \left( \frac{\cos \frac{n\pi}{2}}{1 - \cos \frac{n\pi}{2}} \right) \left( \cos \frac{m\pi}{2} - \cos m\pi \right) \]

\[ + \left( \phi^t \right) \left( \cos \frac{m\pi}{2} - \cos \frac{m\pi}{2} \right) \left( \cos \frac{n\pi}{2} - \cos \frac{n\pi}{2} \right) \]

\[ + \left( \phi^t \right) \left( \cos \frac{n\pi}{2} - \cos \frac{n\pi}{2} \right) \left( \cos \frac{m\pi}{2} - \cos \frac{m\pi}{2} \right) \]  \hspace{1cm} (20)

where \((\phi^t)\) is the control voltage fed into the \(i\)th actuator patch and \(i = 1, 2, 3, 4\). Note that all quadrupiles of the \(m\) or \(n\) modes cannot be controlled by the quarterly segmented actuators. (To control these modes, other segmentation or shaping techniques need to be implemented.) The positive or negative signs of the modal voltages resulting from the sensor patches depend on their respective patch locations and mode numbers \(m\) and \(n\). Accordingly, the signs of the feedback voltages to the actuator patches need to be carefully monitored. It is also assumed that the temperature variation frequencies differ from the shell natural frequencies, and modal filters are used such that each modal equation can be considered independently—i.e., neither observation nor control spillover. Three generic feedback algorithms (i.e., displacement, velocity and acceleration) and detailed modal feedback factors are discussed next. (Note that combination of the displacement and velocity feedback yield the conventional proportional plus derivative feedback.)

4.2.1. Displacement feedback

In the displacement feedback control, the actuator voltage to the arbitrary \(p\)th actuator patch is proportional to the sensor signal of a collocated sensor patch.

\[ (\phi^t)_p = \mathcal{G}_d(\phi^t(x, \beta))_p = \sum_{\gamma=1}^{\infty} \sum_{\phi=1}^{\infty} \mathcal{G}_d(\phi^\gamma)_p + \mathcal{G}_d(\phi^\gamma)_p \]

\[ = \mathcal{G}_d(\phi^\gamma)_p + \sum_{\gamma=1}^{\infty} \sum_{\phi=1}^{\infty} \mathcal{G}_d(\phi^\gamma)_p + \mathcal{G}_d(\phi^\gamma)_p, \quad p = 1, 2, \ldots, k. \]  \hspace{1cm} (21)

where \(\mathcal{G}_d\) is the displacement feedback gain and \(k\) denotes a total \(k\) actuator patches. Invoking modal filters gives only the \(m\)th modal signal, and all other signals are filtered out. Thus, the feedback voltage is defined by

\[ (\phi^t)_p = \mathcal{G}_d(\phi^\gamma)_p. \]  \hspace{1cm} (22)
and the resultant modal control force becomes

\[ F_{m}(t) = -A_{mn}^u \sum_{r=1}^{L} G_r(\phi_m^u_r) \left( \cos \frac{m_1 x_{1r}}{L} - \cos \frac{m_2 x_{2r}}{L} \right) \left( \cos \frac{n_1 \beta_{n1}}{\beta^*} - \cos \frac{n_2 \beta_{n2}}{\beta^*} \right) \]

\[ = -\eta_{mn} A_{mn}^u S_n^u \sum_{r=1}^{L} \frac{R \beta^* L}{S_n} \left( \cos \frac{m_1 x_{1r}}{L} - \cos \frac{m_2 x_{2r}}{L} \right) \]

\[ \times \left( \cos \frac{n_1 \beta_{n1}}{\beta^*} - \cos \frac{n_2 \beta_{n2}}{\beta^*} \right) \]

\[ = -\eta_{mn} A_{mn}^u S_n^u \sum_{r=1}^{L} \frac{R \beta^* L}{S_n} \left( \cos \frac{m_1 x_{1r}}{L} - \cos \frac{m_2 x_{2r}}{L} \right) \]

\[ \times \left( \cos \frac{n_1 \beta_{n1}}{\beta^*} - \cos \frac{n_2 \beta_{n2}}{\beta^*} \right). \]  

(23)

where \( S_n^u \) is the modal sensitivity (see Part 1); \( A_{mn}^u \) denotes the spatial effects of the segmented actuator patch and

\[ A_{mn} = \sum_{r=1}^{L} \left( R \beta^* L / S_n^L \right) \left( \cos \left( m_1 x_{1r} / L \right) - \cos \left( m_2 x_{2r} / L \right) \right) \]

\[ \times \left( \cos \left( n_1 \beta_{n1} / \beta^* \right) - \cos \left( n_2 \beta_{n2} / \beta^* \right) \right). \]

Then, the modal equation becomes

\[ \eta_{mn} + 2\zeta_{mn} \omega_{mn} \eta_{mn} + (\omega_{mn} + \mathcal{F}_{mn}^d) \eta_{mn} = F_{m}^u(t) + F_{m}^c(t), \]  

(24)

where \( \mathcal{F}_{mn}^d \) is the \( mnth \) modal displacement feedback factor, defined by the modal actuation factor, modal sensitivity, gain and spatial effect:

\[ \mathcal{F}_{mn}^d = A_{mn}^u S_n^u \mathcal{G}_d A_{mn}^s, \]  

(25)

which has a direct influence on the frequency change.

For the quarterly segmented actuator patches, i.e., \( k = 4 \), the spatial effect \( A_{mn}^s \) is expressed as follows:

\[ A_{mn}^s = 4 \left( 1 - \cos \frac{n \pi}{2} \right) \left( 1 - \cos \frac{m \pi}{2} \right) \left( \cos \frac{m \pi}{2} - \cos m \pi \right) \]

\[ + \left( \cos \frac{m \pi}{2} - \cos m \pi \right) \left( 1 - \cos \frac{n \pi}{2} \right) \left( \cos \frac{n \pi}{2} - \cos n \pi \right) \left( \cos \frac{m \pi}{2} - \cos m \pi \right) \left( \cos \frac{n \pi}{2} - \cos n \pi \right). \]  

(26)

The modal actuation factor \( A_{mn}^u \) and the modal sensitivity \( S_{mn}^u \) can be separated into a membrane component and a bending component, i.e., \( A_{mn}^u = (A_{mn}^u)_{mem} + (A_{mn}^u)_{bend} \) and \( S_{mn}^u = (S_{mn}^u)_{mem} + (S_{mn}^u)_{bend} \), and four possible feedback schemes can be assumed for the \( mnth \) mode:

1. membrane signal to membrane control force,

\[ (F_{mn})_{mem} = (A_{mn}^u)_{mem} (S_{mn}^u)_{mem}; \]  

(27a)

2. bending signal to control moment,

\[ (F_{mn})_{bend} = (A_{mn}^u)_{bend} (S_{mn}^u)_{bend}; \]  

(27b)
(3) bending signal to membrane control force,
\[ (F_{mn})_{n,m} = (A_{mn}^w)_{n,m} (S_{mn})_{n,m}; \]  \hspace{1cm} (27c)

(4) membrane signal to control moment,
\[ (F_{mn})_{h,m} = (A_{mn}^m)_{n,m} (S_{mn})_{n,m}. \]  \hspace{1cm} (27d)

Including all four possible effects, one can rewrite the modal displacement feedback factor as
\[ \mathcal{F}_{mn}^\prime = [(F_{mn})_{n,m} + (F_{mn})_{h,b} + (F_{mn})_{n,b} + (F_{mn})_{h,m}] \mathcal{G}_{n,m} \]  \hspace{1cm} (28)

The controlled frequency \( \omega_{mn}' \) and the damping ratio \( \zeta_{mn}' \) of the shell feedback system are written as follows
\[ \omega_{mn}' = \sqrt{\omega_{mn}^2 + \mathcal{F}_{mn}^\prime}, \quad \zeta_{mn}' = \zeta_{mn} \omega_{mn}/\omega_{mn}'. \]  \hspace{1cm} (29)

4.2.2. Velocity feedback

In the velocity feedback (derivative feedback) control, the \( p \)th actuator voltage is proportional to the derivative of the collocated sensor signal:
\[ (\phi_i)',_p = \mathcal{G}_i (\phi_i)(x, \beta)_p = \sum_{\ell=1}^\infty \sum_{\gamma=1}^\infty \mathcal{G}_i (\phi_i^\ell) + \mathcal{G}_i (\phi_i^\gamma)_p \]
\[ = \mathcal{G}_i (\phi_i^\omega) + \sum_{\ell=1}^\infty \sum_{\gamma=1}^\infty \mathcal{G}_i (\phi_i^\ell) + \mathcal{G}_i (\phi_i^\gamma)_p, \quad p = 1, 2, \ldots, k, \]  \hspace{1cm} (30)

where \( \mathcal{G}_i \) is the velocity feedback gain. Imposing modal filters to isolate the \( mn \)-modal signal, one can write the modal feedback voltage as
\[ (\phi_i)_p = \mathcal{G}_i (\phi_i^\omega)_p. \]  \hspace{1cm} (31)

Thus, the resultant modal control force from all \( k \) actuator patches is
\[ F_{mn}^\prime(t) = -A_{mn}^w \sum_{\ell=1}^\infty \mathcal{G}_i (\phi_i^\ell)_p \left( \cos \frac{m\pi x_i}{L} - \cos \frac{n\pi x_i}{L} \right) \left( \cos \frac{n\pi \beta_{i\ell}}{\beta^*} - \cos \frac{m\pi \beta_{i\ell}}{\beta^*} \right) \]
\[ = -\eta_{mn}(t) A_{mn}^w S_{mn} \mathcal{G}_i \sum_{\ell=1}^\infty \frac{R_{\beta_{i\ell}}}{S_{i\ell}} \left( \cos \frac{m\pi x_i}{L} - \cos \frac{n\pi x_i}{L} \right)^2 \]
\[ \times \left( \cos \frac{n\pi \beta_{i\ell}}{\beta^*} - \cos \frac{m\pi \beta_{i\ell}}{\beta^*} \right) = -\eta_{mn}(t) A_{mn}^w S_{mn} \mathcal{G}_i A_{mn}. \]  \hspace{1cm} (32)

Then, the \( mn \)th modal equation becomes
\[ \dot{\eta}_{mn} + (2\zeta_{mn} \omega_{mn} + \mathcal{F}_{mn}) \eta_{mn} + \omega_{mn} \eta_{mn} = F_{mn}^\prime(t) + F_{mn}'(t), \]  \hspace{1cm} (33)

where \( \mathcal{F}_{mn} \) is the \( mn \)th modal velocity feedback factor and
\[ \mathcal{F}_{mn}^\prime = A_{mn}^w S_{mn} \mathcal{G}_i A_{mn}. \]  \hspace{1cm} (34)
For the four equal-sized segmented actuator, i.e., \( k = 4 \), the spatial effect \( \Delta_{\text{m.m.}} \) is

\[
\Delta_{\text{m.m.}} = 4 \left( \left( 1 - \cos \frac{m\pi}{2} \right)^2 \left( 1 - \cos \frac{n\pi}{2} \right)^2 + \left( \cos \frac{m\pi}{2} - \cos m\pi \right)^2 \right)
+ \left( \cos \frac{n\pi}{2} - \cos n\pi \right)^2 \left( 1 - \cos \frac{m\pi}{2} \right)^2 + \left( \cos \frac{m\pi}{2} - \cos m\pi \right)^2 \left( \cos \frac{n\pi}{2} - \cos n\pi \right)^2.
\]

(35)

The modal velocity feedback factor \( \Phi_{\text{m.m.}} \) including all four feedback possibilities can be written as

\[
\Phi_{\text{m.m.}} = \left[ (F_{\text{m.m}})_{m.m} + (F_{\text{m.m}})_{b.b} + (F_{\text{m.m}})_{a.a} + (F_{\text{m.m}})_{b.m} \right] \Phi_{\text{m.a.}} \Delta_{\text{m.m.}}
= (\Phi_{\text{m.m}})_{m.m} + (\Phi_{\text{m.m}})_{b.b} + (\Phi_{\text{m.m}})_{a.a} + (\Phi_{\text{m.m}})_{b.m}.
\]

(36)

Note that the subscripts \( m.m, b.b, m.b \) and \( b.m \) are defined similarly to those in equations (27a-d). The first letter denotes the actuator component effect and the second letter denotes the sensor component effect; \( m \) is for the membrane effect and \( b \) is for the bending effect. There are membrane and bending contributions to the sensor signals in sensor patches, and also membrane control force and control moment effects in actuator patches. Accordingly, the controlled modal damping ratio \( \zeta_{\text{m.m.}} \) includes the original damping ratio \( \zeta_{\text{m.m.}} \) and the modal velocity feedback factor \( \Phi_{\text{m.m.}} \):

\[
\zeta_{\text{m.m.}} = (2\zeta_{\text{m.m.}}\omega_{\text{m.m.}} + \Phi_{\text{m.m.}})/2\omega_{\text{m.m.}} = \zeta_{\text{m.m.}} + \Phi_{\text{m.m.}}/2\omega_{\text{m.m.}}
= \zeta_{\text{m.m.}} + (\zeta_{\text{m.m.}})_{m.m} + (\zeta_{\text{m.m.}})_{b.b} + (\zeta_{\text{m.m.}})_{m.b} + (\zeta_{\text{m.m.}})_{b.m}.
\]

(37)

The \( m.m \)th damping ratio components due to various membrane/bending combination of sensor signals and actuator actions are defined below:

\begin{align*}
(\zeta_{\text{m.m.}})_{m.m} &= (\Phi_{\text{m.m}})_{m.m.}/2\omega_{\text{m.m.}} = [(A_{\text{m.m.}})_{m.m.}(S_{\text{m.m.}})_{m.m.}\Phi_{\text{m.m.}}]\Delta_{\text{m.m.}}/2\omega_{\text{m.m.}} \\
&= \left( h_{e_{11}}\frac{4e_{11}}{mne_{11}} \right) \left( \frac{4e_{11}}{\rho h R \beta^* L R_{\pi}^2} \right) \left[ \frac{m R \beta^*}{n L} + \left( \frac{n L}{m R \beta^*} \right) \right] \Phi_{\text{m.m.}}/2\omega_{\text{m.m.}}; \quad (38a)
\end{align*}

\begin{align*}
(\zeta_{\text{m.m.}})_{b.b} &= (\Phi_{\text{m.m}})_{b.b.}/2\omega_{\text{m.m.}} = [(A_{\text{m.m.}})_{b.b.}(S_{\text{m.m.}})_{b.b.}\Phi_{\text{m.m.}}]\Delta_{\text{m.m.}}/2\omega_{\text{m.m.}} \\
&= \left( h_{e_{11}}\frac{4e_{11}}{mne_{11}} \right) \left( \frac{4e_{11}}{\rho h R \beta^* L R_{\pi}^2} \right) \left[ \frac{m R \beta^*}{n L} + \left( \frac{n L}{m R \beta^*} \right) \right] \Phi_{\text{m.m.}}/2\omega_{\text{m.m.}} \\
&\times \left( \frac{m}{L} \right) + \left( \frac{n}{m R \beta^*} \right) \Phi_{\text{m.m.}}/2\omega_{\text{m.m.}}; \quad (38b)
\end{align*}

\begin{align*}
(\zeta_{\text{m.m.}})_{m.b} &= (\Phi_{\text{m.m}})_{m.b.}/2\omega_{\text{m.m.}} = [(A_{\text{m.m.}})_{m.b.}(S_{\text{m.m.}})_{m.b.}\Phi_{\text{m.m.}}]\Delta_{\text{m.m.}}/2\omega_{\text{m.m.}} \\
&= \left( h_{e_{11}}\frac{4e_{11}}{mne_{11}} \right) \left( \frac{4e_{11}}{\rho h R \beta^* L R_{\pi}^2} \right) \left[ \frac{m R \beta^*}{n L} + \left( \frac{n L}{m R \beta^*} \right) \right] \Phi_{\text{m.m.}}/2\omega_{\text{m.m.}}; \quad (38c)
\end{align*}
(38d)  
\[
(\zeta_{n})_{h,m} = \left(\mathcal{F}_{n}^{r}\right)_{h,m}/2\omega_{m} = \left[(\mathcal{A}_{n}^{\infty})_{h,m}(S_{n})_{h,m} A_{n}^{m} A_{n}^{dab} \right]/2\omega_{m} \\
= \left[\frac{h\epsilon_{31}}{m\epsilon_{31}}\left(\frac{4\epsilon_{31}}{m\epsilon_{31}}\right) \frac{r^4}{R^5} \left[(mR\beta^*)/nL + nL / mR\beta^*\right] \right] A_{n}^{m} A_{n}^{dab}/2\omega_{m}.
\]

If the sensor and the actuator layers are symmetrically laminated on the bottom and top surfaces of the cylindrical shell, the moment arms \(r' = -r\). Thus, the controlled coupling damping ratio components \((\zeta_{n})_{h,n}\) and \((\zeta_{n})_{h,m}\) are of equal magnitudes and opposite signs, and these coupling effect cancel out each other in the damping expression.

4.2.3. Acceleration feedback

In the acceleration feedback control, the actuator voltage is proportional to the double time derivative of the collocated sensor signal:

\[
(\dot{\phi})_{p} = \mathcal{G}_{s}(\ddot{\phi}(x,\beta))_{p} = \sum_{\gamma=1}^{\infty} \sum_{\lambda=1}^{\infty} \mathcal{G}_{s}(\ddot{\phi}_{\gamma,\lambda})_{p} + \mathcal{G}_{s}(\ddot{\phi}_{0})_{p} \\
= \mathcal{G}_{s}(\ddot{\phi}_{e0})_{p} + \sum_{\gamma=1}^{\infty} \sum_{\lambda=1}^{\infty} \mathcal{G}_{s}(\ddot{\phi}_{\gamma,\lambda})_{p} + \mathcal{G}_{s}(\ddot{\phi}_{0})_{p}, \quad p = 1, 2, \ldots, k \tag{39}
\]

where \(\mathcal{G}_{s}\) is the acceleration feedback gain. Imposing modal filtering, one can rewrite the feedback voltage, modal control force and modal equation as

\[
(\dot{\phi})_{p} = \mathcal{G}_{s}(\ddot{\phi}_{e0})_{p}, \quad F_{e0}(t) = -\mathcal{G}_{s}(t)A_{e0}^{m} S_{n} A_{n}^{dab}. \tag{40}
\]

\[
1 + \mathcal{F}_{e0} \tilde{\eta}_{e0} + 2\zeta_{e0} \omega_{m} \tilde{\eta}_{e0} + \omega_{m}^{2} \tilde{\eta}_{e0} = F_{e0}(t) + F_{e0}(t). \tag{41}
\]

where \(F_{e0}\) is the \(m\)th modal acceleration feedback factor and

\[
\mathcal{F}_{e0} = A_{e0}^{m} S_{n} A_{n}^{dab}. \tag{42}
\]

As discussed in the above feedback sections, the modal acceleration feedback factor can be written as a summation of four possible components:

\[
\mathcal{F}_{e0} = [(F_{e0})_{h,m} + (F_{e0})_{h,n} + (F_{e0})_{h,b} + (F_{e0})_{h,b}][A_{s} A_{m}^{dab}]
\]

\[
= (\mathcal{F}_{e0})_{h,m} + (\mathcal{F}_{e0})_{h,n} + (\mathcal{F}_{e0})_{h,b} + (\mathcal{F}_{e0})_{h,b}. \tag{43}
\]

Recall that the subscripts \(a, s\) denote the sensing signal component fed back to the actuator control component; \(m\) denotes the membrane component effect and \(b\) denotes the bending component effect. The controlled frequency \(\omega_{m}\) and damping ratio \(\zeta_{m}\) are as follows:

\[
\omega_{m} = \sqrt{\omega_{m}^{2}/(1 + \mathcal{F}_{e0})}, \quad \zeta_{m} = \zeta_{m} \omega_{m}^{2}/(1 + \mathcal{F}_{e0}) \omega_{m}. \tag{44}
\]

Note that the modal feedback factors for the displacement \((\mathcal{F}_{e0})_{h}\), velocity \((\mathcal{F}_{e0})_{v}\) and acceleration \((\mathcal{F}_{e0})_{a}\) are identical, except that the feedback gain \(\mathcal{G}\) has a different physical significance. Although three generic feedback algorithms have been discussed and their modal feedback factors derived, the velocity feedback is usually used in practical applications. Accordingly, only the velocity feedback and control damping ratios are evaluated in the parametric studies presented next.

5. PARAMETRIC STUDIES AND ACTIVE VIBRATION CONTROL

A five-layer piezoelectric and graphite/epoxy laminated cylindrical shell is used in parametric studies of active vibration control. The standard dimensions of the piezoelectric...
laminated shell are shell length \( L = 0.1 \) m, shell curvature angle \( \beta^* = \pi/2 \), shell radius \( R = 0.05 \) m, elastic lamina thickness \( h_1 = 0.0005 \) m and piezoelectric lamina thickness \( h_2 = 20 \) \( \mu \)m. Other dimensional changes are compared with the standards. Detailed material properties are provided in Part 1 of this paper. Recall that the distributed sensor and actuator layers (the first and fifth, respectively) are quarterly segmented into the same patch patterns. The sensor signal is fed back to the corresponding actuator patch in the feedback control. Various design parameters related to the performance of distributed sensor patches are evaluated in Part 1, and those parameters related to the control effectiveness of actuator patches are evaluated in this section. It is assumed that the original modal damping ratio is 1%. Changes of modal actuation factor, modal velocity feedback factor and damping ratio in the velocity feedback are evaluated with respect to changes of design parameters: actuator thickness, shell thickness, curvature angle, shell dimension and feedback gain. Recall that the modal actuation factor denotes the magnitude of the resultant modal control forces and the modal velocity feedback factor represents the combined effect of the sensor and actuator patches. Detailed membrane and bending moment control contributions are studied. Note that the actuator performance should also include the spatial distribution effects, and that all quadruples of natural modes are not controllable by the quarterly segmented equal-sized actuator patches. Feedback gain enhances the control effects linearly, and thus it is not particularly presented in later studies.

5.1. CASE 1: ACTUATOR THICKNESS

Actuator thickness effects on the modal actuation factor, the modal velocity feedback factor and the damping ratio in the velocity feedback are evaluated. The actuator thickness is evaluated at \( h_1 = 10, 20, 30, 40 \) and 50 \( \mu \)m, respectively. (Natural frequency variations due to thickness changes were discussed in Part 1.) The modal (total) actuation factor and its membrane and bending component actuation factors of the first 16 natural modes \( (m = 1, 2, 3, 4 \) and \( n = 1, 2, 3, 4 \)) are plotted in Figure 2. It is observed that the major control action is derived from the membrane actuation, which decreases as the mode number increases. The bending actuation factors remain about the same for every actuator thickness, which becomes dominant for high natural modes. Since the actuator thickness changes very little and also since the actuator material was piezoelectric polyvinylidene fluoride, the effect of thickness on the total modal actuation factor is relatively insignificant. However, the increased mass does slightly outweigh the increased stiffness and consequently the natural frequency of a thicker actuator drops slightly; see also Part 1.

The modal velocity feedback factors—(1) the membrane signal to membrane actuation \( (F_m^m)_{1,m} \), (2) the membrane signal to bending actuation \( (F_m^b)_{1,m} \), (3) the bending signal to membrane actuation \( (F_b^m)_{1,m} \), (4) the bending signal to bending actuation \( (F_b^b)_{1,m} \), and (5) the total effect \( F_m^m \)—are plotted with respect to the actuator thickness, from 10 \( \mu \)m to 50 \( \mu \)m, in Figure 3. Note that the coupling terms \( (F_m^m)_{1,m} \) and \( (F_m^b)_{1,m} \) are of equal magnitudes and opposite signs, and they cancel each other out in the symmetrically laminated shell. Again, this shows that the membrane control action dominates the total control action of the cylindrical shell. The controlled damping ratios of the corresponding natural modes are plotted in Figure 4. The relationship between the modal feedback factor and the controlled modal damping ratio is \( \zeta_m = (\zeta_m = F_m^m/2\omega_m) \). Thus, only the controlled damping ratios are presented in later parameter studies, due to restrictions in the length of the paper. Since the frequency drops and the modal velocity feedback increases as the actuator becomes thicker, the resultant modal damping ratio increases due to the enhanced actuation effect, as shown in Figure 3. This effect becomes relatively insignificant for higher modes.
Figure 2. Modal actuation factors for various actuator thicknesses: □, m = 1; ■, m = 2; △, m = 3; ▲, m = 4. (a) h₁ = 0.01 mm; (b) h₁ = 0.02 mm; (c) h₁ = 0.03 mm; (d) h₁ = 0.04 mm; (e) h₁ = 0.05 mm.
Figure 3. Modal velocity feedback factors for various actuator thicknesses: •, modal feedback factor from membrane sensor to membrane actuator; □, modal feedback factor from bending sensor to bending actuator; △, modal feedback factor from membrane sensor to bending actuator; ■, modal feedback factor from bending sensor to membrane actuator; ▲, modal feedback factor for the controlled system. Values of m.n: (a) 1, 1; (b) 2.1; (c) 3.1; (d) 1.2; (e) 2.2; (f) 3.2; (g) 1.3; (h) 2.3; (i) 3.3.
Figure 4: Controlled modal damping ratios for various actuator thicknesses. Symbols and sub-figures as in Figure 3.
5.2. CASE 2: SHELL THICKNESS

The effect of thickness variations of elastic lamina, \( h = 0.2, 0.3, 0.4, 0.5 \) and 0.6 mm on control effectiveness is evaluated. Modal actuation factors, including membrane, bending and total effects, are plotted in Figure 5, and damping ratio variations are plotted in Figure 6. The analytical results suggest that the modal actuation factors and modal damping ratio decrease as the shell thickness increases, due to the increased shell bending modulus, which is a cubic function of shell thickness. Although the moment control effect increases linearly, the increased shell rigidity outweighs the increased moment control effect, and thus thicker (or stiffer) shells are much more difficult to control.

5.3. CASE 3: SHELL CURVATURE ANGLES

Continuous curvature transformations are of importance in adaptive (geometry) shells and structures [11]. Curvature effects to modal actuation factors and modal damping ratios are investigated. It is assumed that the total shell size remains the same, and that only the shell curvature changes. The total effective arc (circumferential) length is constant, i.e., \( R\beta^* = 0.05\pi/2 \) m and the curvature angles are \( \beta^* = 30^\circ, 60^\circ, 90^\circ, 120^\circ \) and \( 150^\circ \). The modal actuation factors and modal damping ratios are calculated and presented in Figures 7 and 8, respectively.

Analytical results indicate that the membrane actuation effect increases significantly as the shell curvature increases, due to an increased membrane effect in curved shells. The bending control effect remains about the same, since the moment arm remains unchanged. Again, the membrane control effect dominates the lower natural modes as well as the total control effect. Recall that the membrane strain energy dominates in the lower natural modes of deep shells and the bending strain energy dominates in shallow or zero-curvature shells; see Part 1. For lower natural modes, the natural frequency of shallow shells keeps increasing, while the frequency of highly curved shells drops as the mode increases; see Part 1. The frequency variation due to the curvature changes also affects the relatively irregular variations of the controlled modal damping ratios; i.e., \( \zeta_{cn} = (\zeta_{mn} + \mathcal{F}_{mn}/2\omega_{mn}) \).

5.4. CASE 4: SHELL SIZES

Next, all sensor, actuator and elastic lamina thicknesses are kept the same and the curvature remains unchanged, at \( \beta^* = \pi/2 \). The shell length is evaluated at \( L = 0.100, 0.125, 0.150, 0.175 \) and \( 0.200 \) m, the corresponding arc lengths being defined by \( R = L/\beta^* \). Accordingly, these shells are of the same shape and they only vary in size. The modal actuation factors and controlled damping ratios of these cylindrical shells are calculated and plotted in Figures 9 and 10. Since the shell length, radius and effective area appear in the denominator of the modal actuation factor and the controlled damping ratio, increasing both dimensions causes drops in both situations and is insignificant for higher modes. It should be noted that in order to keep a good comparison among all design parameters and natural modes, the same or similar units and/or scales on the vertical axis were used, which unavoidably causes some curves go beyond the scales, especially for higher modes.

6. SUMMARY AND CONCLUSIONS

The spatial actuation and control effectiveness of segmented actuator patches laminated on a simply supported piezoelectric laminated cylindrical shell were studied. Modal control forces of open and closed loop vibration control effects were, respectively, evaluated for an arbitrary actuator patch and also for quarterly segmented actuator patches. The modal
Figure 5. Modal actuation factors for various elastic lamina thicknesses: □, \( m = 1 \); ■, \( m = 2 \); △, \( m = 3 \); ▲, \( m = 4 \). (a) \( h_1 = 0.2 \) mm; (b) \( h_1 = 0.3 \) mm; (c) \( h_1 = 0.4 \) mm; (d) \( h_1 = 0.5 \) mm; (e) \( h_1 = 0.6 \) mm.
Figure 6. Controlled modal damping ratios for various elastic lamina thicknesses: Symbols and sub-figures as in Figure 3.
Figure 7. Modal actuation factors for various curvature angles: □, $m = 1$; ■, $m = 2$; △, $m = 3$; ▲, $m = 4$. (a) $\beta = 30^\circ$; (b) $\beta = 60^\circ$; (c) $\beta = 90^\circ$; (d) $\beta = 120^\circ$; (e) $\beta = 150^\circ$. 
Figure 8. Controlled modal damping ratios for various curvature angles. Symbols and subfigures as in Figure 3.
Figure 9. Modal actuation factors for various shell sizes: (a) $m = 1$; ■, $m = 2$; ▲, $m = 3$; ▲, $m = 4$. (a) $L = 0.100$ m; (b) $L = 0.125$ m; (c) $L = 0.150$ m; (d) $L = 0.175$ m; (e) $L = 0.200$ m.
Figure 10: Controlled modal damping ratios for various shell sizes. Symbols and subfigures as in Figure 3.
actuation factor and its membrane and bending components were defined. Modal feedback factors and controlled damping ratios were derived, and their membrane and bending contributions were evaluated with respect to actuator parameters: actuator thickness, shell lamina thickness, shell curvatures, shell sizes and natural modes. Analytical and simulation results suggested that the membrane control action is of importance for lower natural modes and that bending control action becomes dominant for higher natural modes. Studies of design parameters also revealed the following:

1. The modal actuation factor decreased slightly as the actuator became thicker. Both the modal velocity factor and the controlled damping ratio increased due to the enhanced actuation effect.

2. The shell rigidity increases as the shell lamina becomes thicker. Although the control moment arm is linearly proportional to the shell thickness, the area moment of inertia is a cubic function of the shell thickness. Thus, both the modal action factor and the modal damping ratio dropped significantly as the shell lamina became thicker.

3. Increasing curvature significantly enhances the membrane effect. Since the dominating control action is derived from the membrane control action, curvature increase significantly enhances the modal actuation factor and also the controlled damping ratio.

4. The modal actuation factor and controlled damping ratio decreased as the shell size enlarged, provided that the thickness remained unchanged.

Note that all of the results were evaluated based on constant piezoelectric coefficients, without hysteresis or temperature effects. In conclusion, there are two control actions introduced by the shell actuators: a membrane control action and a bending moment control action. The membrane control action dominates the lower natural modes and it increases as the shell curvature increases—for deep shells. The bending control action is effective for higher modes and also for shallow or zero-curvature continua.

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