Collocated independent modal control with self-sensing orthogonal piezoelectric actuators (theory and experiment)

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Abstract. Distributed self-sensing piezoelectric actuators provide perfect collocations of sensors and actuators in closed-loop structural controls. To achieve independent control of various natural modes, spatially distributed self-sensing orthogonal piezoelectric actuators are proposed in this study. A generic spatially shaped orthogonal sensor/actuator theory is derived first, followed by an application to a Bernoulli–Euler beam. Spatially distributed orthogonal sensors/actuators are designed based on the modal strain functions and they are fabricated using a 46 μm piezoelectric polymer. A cantilever beam laminated with these self-sensing orthogonal piezoelectric actuators combined with a self-sensing feedback control circuit is tested. Collocated independent modal control of the cantilever beam with spatially distributed self-sensing orthogonal actuators is demonstrated and control effectiveness studied.

1. Introduction

A perfect sensor/actuator collocation usually provides a stable performance in closed-loop feedback controls. A self-sensing piezoelectric actuator is a single piezoelectric device simultaneously used for both sensing and control. (The sensor signal is separated from the control signal by using a differential amplifier; this signal is then amplified and fed back to induce control actions.) Self-sensing piezoelectric actuators have been proposed in recent years. Dosch and co-workers (1992) proposed a self-sensing piezoelectric actuator for collocated control of a cantilever beam. Anderson and co-workers (1992) presented an analytical modeling of the self-sensing actuator system, and studied its applications to beam and truss structures. Rectangular piezoelectric devices attached near the fixed end were used in both studies.

It is known that the spatially distributed orthogonal sensors and actuators are sensitive to a mode or a group of natural modes (Tzou 1993, Lee 1992). Spatially distributed piezoelectric sensors and actuators were investigated in a number of recent studies, such as beams, plates, rings and shells. (Lee and Moon 1990, Lee 1992, Anderson and Crawley 1991, Collins et al 1991, Hubbard and Burke 1992, Tzou and Fu 1993a, b, Tzou and Tseng 1990, Tzou et al 1993, Tzou 1993, Tzou et al 1994). Based on the modal orthogonality, a spatially shaped self-sensing orthogonal modal actuator is effective on only a single mode; consequently, each vibration mode can be independently controlled — independent modal control (Meirovitch 1988), while the feedback control system is kept simple. This paper aims to investigate the sensing and control characteristics of self-sensing orthogonal modal actuators. A generic theory for a spatially distributed self-sensing orthogonal actuator is proposed first, followed by an experimental study of self-sensing orthogonal piezoelectric actuators. Independent modal control with the self-sensing orthogonal actuators is demonstrated. (Note that the emphasis is placed on the experimental aspect.)

2. Theory

It is assumed that a spatially distributed piezoelectric layer is laminated on a one-dimensional (1D) structure, such as arches, rings, beams, or rods (figure 1). The shape of the piezoelectric layer is defined by a 1D shape function $W_5(x_1)$. Both the piezoelectric layer and the elastic continuum have constant thickness, i.e., $h_0$ and $h$, respectively. It is assumed that the piezoelectric material is hexagonally symmetrical such that the piezoelectric constants $e_{31} = e_{12}$. An open-circuit sensor signal $S_0$ from a 1D spatially distributed orthogonal sensor can be estimated from its strains, which can be further separated into two
The distributed velocity (strain-rate) feedback can be derived using the modal expansion method and a spatially distributed modal feedback force (Tzou et al. 1994). The kth modal equation can be written as

\[ \ddot{\eta}_k + \frac{c}{\rho h} \dot{\eta}_k + \omega_n^2 \eta_k = \frac{1}{\rho h N_k} \sum_{j=1}^{\infty} \sum_{m=-1}^{m+1} \int_{S^2} U_{jk}(\alpha_1, \alpha_2) A_1 A_2 d\alpha_1 d\alpha_2 \left( \mathcal{G}_j \eta_j(\alpha_1, \alpha_2) \right) \]

(2)

where \( \eta_k \) is a modal coordinate; \( c \) is the damping constant; \( \rho \) is the mass density; \( \omega_n \) is the natural frequency; \( N_k = \sum_{m=-1}^{m+1} \int_{S^2} U_{jk}(\alpha_1, \alpha_2) A_1 A_2 d\alpha_1 d\alpha_2 \); and \( \mathcal{G}_j \) is the distributed velocity feedback function. Using the modal orthogonality, one can write the distributed velocity feedback function as a product of a velocity weighting factor \( \mathcal{G}_j \eta_j \) (gain constant) and an orthogonal function \( U_{jk}(\alpha_1, \alpha_2) \):

\[ \mathcal{G}_j \eta_j(\alpha_1, \alpha_2) = \mathcal{G}_j \eta_j U_{jk}(\alpha_1, \alpha_2) \]  

(3)

Based on the modal orthogonality, all \( n \neq k \) modes are filtered out when substituting (3) into (2). Imposing the modal orthogonality and considering the transverse oscillation only, one can derive the nth modal equation (independent modal control equation) with the velocity modal feedback control force as

\[ \ddot{\eta}_n + \frac{1}{\rho h} \left( c - \frac{\mathcal{G}_j \eta_j}{N_n} \int_{S^2} U_{jk} A_1 A_2 d\alpha_1 d\alpha_2 \right) \dot{\eta}_n + \omega_n^2 \eta_n = 0 \]  

(4)

Note that the subscript \( k \) is replaced by \( n \) when modal orthogonality is considered. For \( \alpha_1 \) continua, the transverse mode shape \( U_{jk} \) is only a function of one coordinate \( \alpha_1 \), e.g., the circumferential direction in rings and arches, the longitudinal direction in beams and rods. If electrode areas of the actuators are designed as a \( \alpha_1 \) shape function of \( W(\alpha_1) \), the modal control force for a \( \alpha_1 \) spatially shaped actuator can be rewritten as

\[ \ddot{\eta}_n + \frac{1}{\rho h} \left( c - \frac{\mathcal{G}_j \eta_j}{N_n} \int_{S^2} W(\alpha_1) U_{jk} A_1 A_2 d\alpha_1 d\alpha_2 \right) \dot{\eta}_n + \omega_n^2 \eta_n = 0 \]  

(5)

Note that the modal coupling and the spillover from all other natural modes are eliminated. The shape functions can be defined by the modal functions. These modal filtering characteristics will be demonstrated in an experimental study on a cantilever beam laminated with orthogonal sensors/actuators presented later.

2.1. An orthogonal sensor/actuator for a cantilever beam

A \( \alpha_1 \) cantilever Bernoulli–Euler beam usually exhibits transverse oscillations only. (The in-plane longitudinal oscillation is neglected.) The Lamé parameters for a flat uniform beam are \( A_1 = 1, A_2 = 1 \); the radii are \( R_1 = \infty \) and \( R_2 = \infty \). In addition, \( \partial(\cdot)/\partial\alpha_2 = 0 \). Accordingly, the closed-loop equation of motion of a cantilever beam can be derived:

\[ p h \ddot{u}_1 + Y h^2 \frac{\partial^2 u_1}{\partial x^2} - k h^2 \frac{\partial^2 \left( M^1_1 \right)}{\partial x^2} = F_1 \]  

(6)

where \( Y \) is Young's modulus; \( I \) is the area-moment of inertia; \( b \) is the beam width; \( M^1_1 \) is the induced control moment; and \( F_1 \) is the external mechanical force. As discussed previously, the orthogonal modal sensors/
actuators are designed based on the modal functions $U_{\lambda_m}(x)$:

$$U_{\lambda_m}(x) = \frac{1}{\lambda_m^2} \frac{d^2U_{\lambda_m}(0)}{dx^2} \left[ C(\lambda_m x) - \frac{A(\lambda_m x)}{B(\lambda_m x)} D(\lambda_m x) \right]$$  \hspace{1cm} (7)

where

$$A(\lambda_m x) = 0.5[\cosh(\lambda x) + \cos(\lambda x)]$$  \hspace{1cm} (8a)

$$B(\lambda_m x) = 0.5[\sinh(\lambda x) + \sin(\lambda x)]$$  \hspace{1cm} (8b)

$$C(\lambda_m x) = 0.5[\cosh(\lambda x) - \cos(\lambda x)]$$  \hspace{1cm} (8c)

$$D(\lambda_m x) = 0.5\sinh(\lambda x) - \sin(\lambda x)$$  \hspace{1cm} (8d)

and $x$ defines the distance measured from the fixed end. The eigenvalue $\lambda_m$ is determined by its characteristic equation

$$\cos(\lambda L)\cosh(\lambda L) + 1 = 0$$  \hspace{1cm} (9)

where $\lambda_1 L = 1.875$; $\lambda_2 L = 4.694$; $\lambda_3 L = 7.855$; $\lambda_4 L = 10.996$; $\lambda_5 L = 14.137$; etc. $L$ is the beam length. The first derivative $(d/dx)(U_{\lambda_m}(x))$ is the modal slope function and the second derivative $((d^2/dx^2)(U_{\lambda_m}(x))$ is the modal strain function. The modal strain function is used to define the shapes of orthogonal modal sensors/actuators:

$$U_{\lambda_m}''(x) = \frac{e^{i\lambda x} + \cos(\lambda L) + \sin(\lambda L)}{2 \left[e^{i\lambda L} + 2e^{i\lambda x}\sin(\lambda L) - 1 \right]}$$

$$+ \left[ e^{i\lambda x} \left[ 0.5\cos(\lambda x) - 0.5\sin(\lambda x) \right] + 0.5 \right]$$

$$+ e^{i\lambda L} \left[ 0.5\cos(\lambda x) - 0.5\sin(\lambda L) \right]$$

$$+ e^{i\lambda L} \left[ 0.5\cos(\lambda x)\sin(\lambda L) - \sin(\lambda x)\cos(\lambda L) \right]$$

$$- 0.5 \cos(\lambda x) - 0.5 \sin(\lambda x))$$

$$\left[ e^{i\lambda L} + 2e^{i\lambda L}\sin(\lambda L) - 1 \right] / (\lambda L)^2.$$  \hspace{1cm} (10)

Note that each orthogonal modal sensor/actuator has a distinct shape based on its modal strain function and eigenvalue. Detailed layouts of the spatially shaped orthogonal sensors/actuators are presented next.

### 3. Modal fabrication and experimental set-up

The shapes of distributed orthogonal sensors/actuators follow the definitions of modal strain functions defined by their eigenvalues. The first four modal functions are plotted in figure 2, and their modal strain functions are plotted in figure 3. Note that the effective regions are from zero to one, since they are normalized in the length direction.

Polymeric piezoelectric polyvinylidenefluoride (PVDF) is flexible and easy to cut into various shapes in a laboratory environment. In this study, a 40 $\mu$m PVDF sheet is used for the orthogonal sensors/actuators and the effective axis is aligned with the beam length. These sensor/actuator layers are cut according to their strain functions and then glued on a Plexiglas beam ($15 \times 1 \times \frac{1}{4}$ in).

Patterns of surface electrodes are first laid out on the Plexiglas beam using a thin-film silver paste to ensure a good electrical conductivity. Individual silver electrodes are connected by either thin silver-paste lines (internal connections) or 0.5 mm Teflon coated surgical wires (external connections). Polarity changes are achieved by reversing the cut PVDF sheets. The finished PVDF/Plexiglas beam is shown in figure 4.

#### 3.1. Apparatus

A self-sensing feedback control circuit is set up with two current amplifiers and a differential circuit (Anderson et al. 1992; Dosch et al. 1992). Three operational amplifiers (AD-711JN), six resistors (24.9 kΩ, 8 MΩ, and 16 MΩ), and a capacitor (14 nF) are used to build the circuits for the first and second orthogonal modal sensors/actuators. Figure 5 shows the circuit. The capacitor is used to match the capacitive of the orthogonal piezoelectric sensor/actuator. A power amplifier (BK-1651) supplies 30 V to the operational amplifiers, and a signal amplifier is used to amplify the sensor signal to induce control actions in the piezoelectric layers. A reference accelerometer (Kistler...
5205) is mounted at the free end to provide a reference signal. All signals are input into an HP data acquisition system (HP3566A) for signal processing and recording.

3.2. Experimental procedures

There are two sets of experiments carried out in this study. The first set is to test the modal orthogonality of orthogonal modal sensors; the second set is to evaluate the control effectiveness of the self-sensing orthogonal actuators.

The first set involves two tests: (i) strain signals and (ii) strain-rate signals. The strain signal is contributed by elastic bending strains of the cantilever beam; it is ultimately related to the transverse deflection u_t. Thus, the strain signal is often referred as the ‘displacement’ signal; the strain rate can be regarded as the ‘velocity’ signal. The signs of these signals are individually checked to ensure correct feedback signals in the self-sensing feedback control.

A self-sensing feedback control circuit, discussed previously, is used in the second set of experiments. Controlled time histories from the accelerometer are acquired; modal damping ratios are calculated using the eigensystem realization algorithm (ERA) method (Juang and Pappa 1985).

4. Results and discussion

Test results acquired from the modal signals of orthogonal modal sensors and the modal control effectiveness of the self-sensing orthogonal piezoelectric actuators are presented in this section.

4.1. Modal sensing

In order to evaluate the sensing effectiveness of orthogonal modal sensors, the spatially shaped piezoelectric layers were subjected to external excitations and their dynamic responses recorded. Strain and strain-rate responses were also tested using a strain-rate circuit (Lee 1992). Figure 6 shows the spectra of the accelerometer (top), the first modal sensor (bottom, spectrum 1), and the second modal sensor (bottom spectrum 2). It is observed that the accelerometer senses multiple modes of the cantilever beam, and the modal sensors only respond to their respective modes. Sixty hertz line noises also appear in these spectra.

4.2. Self-sensing and feedback control

As discussed previously, a self-sensing piezoelectric actuator provides a perfect collocation of sensor and actuator. In this section, free-oscillation and controlled time histories are presented and their respective damping ratios are calculated.

4.2.1. Free oscillations. For the first mode, an initial displacement was applied to the free end and the snap-back
response recorded. The free-oscillation time histories of strain and strain-rate signals of the first modal sensor/actuator are plotted in figures 7 and 8, and those of the second sensor/actuator are plotted in figures 9 and 10, respectively. Note that the time histories of the second sensor/actuator were obtained via impulse excitations. Damping ratios of the first and second modes were calculated using the free-vibration time histories. The first modal damping ratio is 4.0% and the second modal damping ratio is 3.4%. (Note that these data were calculated using more than five sample time histories.) It should be pointed out that there was an accelerometer cable taped on the Plexiglas beam, which caused a higher damping for the first natural mode.

4.2.2. Self-sensing control—independent modal control. Control effectiveness of the self-sensing orthogonal actuators were evaluated when the self-sensing feedback control circuit was turned on. The sensing (strain-rate) signal was separated from the actuating signal via the circuit shown in figure 5. The controlled responses of the Plexiglas beam are plotted in figures 11 and 12.
It was noted that the strain-rate signals are rather noisy, which is probably introduced by the electrical line noises, the circuit, the amplifier, etc. However, the Plexiglas beam is controlled well via the self-sensing orthogonal piezoelectric actuator. The averaged damping ratios of the controlled responses are 7.1% for the first mode and 4.2% for the second mode. (Note that since the second mode was relatively difficult to excite by an impulse excitation, twelve samples were used to obtain the averaged data. Control of the second mode could be improved by changing the resistors in the circuit.) It should be pointed out that the convergence of a modal response is determined by the product of the damping ratio and the modal frequency, i.e., $e^{-\gamma t}$. Consequently, responses of the higher modes usually converge much faster than those of the lower modes.

5. Summary and conclusion

Distributed self-sensing piezoelectric actuators provide perfect collocations of sensors and actuators in closed-loop structural controls. To separate control actions for different natural modes (independent modal control),
spatially distributed self-sensing orthogonal piezoelectric actuators were proposed in this study.

A generic orthogonal sensor/actuator theory was presented first, followed by an application to a Bernoulli–Euler beam. Spatially distributed orthogonal sensors/actuators were designed based on the modal strain functions. A physical model was fabricated and its self-sensing control effectiveness tested. Polymeric piezoelectric PVDF sheets (40 μm) were cut into shapes and laminated on a Plexiglas beam. Surface electrodes were connected by either silver pastes or surgical wires. A self-sensing feedback control circuit was set up and tested.

Experimental results showed that the orthogonal modal sensors are sensitive to their respective modes. Free and controlled (via the self-sensing feedback control circuit) time histories were recorded and their modal damping ratios calculated. The calculated results suggested that the modal damping ratios were enhanced by 77.5% for the first mode and by 23.5% for the second mode. Note that the convergence of modal responses is

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**Figure 10.** Free oscillation of the Plexiglas beam (second strain rate).

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**Figure 11.** Controlled time history of the Plexiglas beam (first).

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**Figure 12.** Controlled time history of the Plexiglas beam (second).
determined by the product of the modal damping and the modal frequency. The independent modal control of continua can be effectively achieved by using spatially distributed self-sensing orthogonal piezoelectric actuators.

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