ELECTROMECHANICS OF A THICK HEXAGONAL PIEZOELECTRIC SHELL APPLIED TO ACTIVE STRUCTURES

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ABSTRACT

With the fast development of active piezoelectric structures and systems, a better understanding of piezo-electromechanics would facilitate the future development of piezoelectric sensors, actuators, structures, and systems. This paper is concerned with a theoretical development of an active thick piezoelectric shell continuum in which the effects of transverse shear deformation and rotary inertia are included. Five electromechanical equations, three in translations and two in rotations, are derived using Hamilton’s principle and variational procedures: electric forces and moments in these equations can be manipulated to control dynamic characteristics of the piezoelectric shell. In addition, mechanical and electric boundary conditions are derived, and the electric components can also be used to control the boundaries. Electromechanical equations of the active thick piezoelectric shell are further simplified to an active thin shell when Kirchhoff-Love’s thin shell assumptions are employed. Electromechanical equations of an active piezoelectric spherical shell are also derived using four system parameters, i.e., two Lamé’s parameters and two radii of curvatures. Active structural control is discussed.

1. INTRODUCTION

Electromechanical phenomena of piezoelectric crystals were first discovered by Curie brothers in 1880. Wave propagation and vibration of well-defined piezoelectric continua, such as thin rods and plates with finite and infinite dimensions, rings, disks, circular cylindrical shells, etc., were systematically studied over the years [1]. Toupin [2] derived the equation of equilibrium for a
structural control. The charge equation of statics can be used to estimate electric signals when subjected to mechanical excitations [22]. This set of system equations can also be written in generalized coordinates $u_1$, $u_2$, and $u_3$ by using force/moment-stress equations and strain-displacement relationships in Appendix C. (Note that the equations listed in Appendix C also need to be appropriately simplified using the four system parameters defined for the spherical piezoelectric shell.)

9. CONCLUDING REMARKS

There are significant activities on the research and development of piezoelectric sensors, actuators, structures, and systems in recent years. Due to the inherent direct and converse effects, piezoelectric materials can be used in both sensor and actuator applications. This paper is concerned with the electromechanics and control of a thick piezoelectric shell. The effects of transverse shear deformations and rotatory inertias are considered in the derivation since the shell is relatively thick.

Five electromechanical equations, three in translations and two in rotations, were derived using Hamilton's principle and variational procedures. In addition, mechanical and electric boundary conditions were also derived from the variational equation. All electric forces and moments were grouped and transferred to the right of system equations; they can be manipulated to control the system characteristics in open or closed feedback control systems. Consequently, system frequencies and damping can be controlled.

The active thick piezoelectric shell theory was further simplified using Kirchhoff-Love's thin shell assumptions; this simplification resulted in an active thin piezoelectric shell theory. As discussed previously, a large number of active piezoelectric shell and non-shell continua can be evolved from the generic active thick and thin shell continua; structural control can be accomplished in the same manner. This procedure was demonstrated in an active piezoelectric spherical shell case.

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REFERENCES


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**APPENDIX A**

A piezoelectric material with symmetrical hexagonal structure (Class $C_{6v}$) is isotropic in transverse direction $\alpha_3$ and is anisotropic in $\alpha_1$ and $\alpha_2$ directions. The matrices $[e]$, $[c]$, and $[\epsilon]$ are defined as

\[
[e] = \begin{bmatrix}
\epsilon_{11} & \epsilon_{12} & \epsilon_{13} & 0 & 0 & 0 \\
\epsilon_{12} & \epsilon_{22} & \epsilon_{23} & 0 & 0 & 0 \\
\epsilon_{13} & \epsilon_{23} & \epsilon_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \epsilon_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & \epsilon_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & \epsilon_{44}
\end{bmatrix}
\]  

(A1)

\[
[c] = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\
c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{44}
\end{bmatrix}
\]  

(A2)

\[
[\epsilon] = \begin{bmatrix}
\epsilon_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & \epsilon_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & \epsilon_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \epsilon_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & \epsilon_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & \epsilon_{44}
\end{bmatrix}
\]  

(A2)

**APPENDIX B**

Resultant forces $N_{ij}$ and moments $M_{ij}$, both mechanical and electric, are defined as follows.

**Mechanical Membrane Forces**

\[
N_{11}^{\alpha_1} = \int \sigma_{\alpha_1 \alpha_3} \, d\alpha_3, 
\]  

(B1)

\[
N_{12}^{\alpha_1} = \int \sigma_{\alpha_2 \alpha_3} \, d\alpha_3, 
\]  

(B2)

\[
N_{13}^{\alpha_1} = \int \sigma_{\alpha_3 \alpha_3} \, d\alpha_3, 
\]  

(B3)

**Mechanical Bending Moments**

\[
M_{11}^{\alpha_1} = \int \sigma_{\alpha_1 \alpha_3} \, d\alpha_3, 
\]  

(B4)

\[
M_{12}^{\alpha_1} = \int \sigma_{\alpha_2 \alpha_3} \, d\alpha_3, 
\]  

(B5)

\[
M_{13}^{\alpha_1} = \int \sigma_{\alpha_3 \alpha_3} \, d\alpha_3. 
\]  

(B6)
Mechanical Transverse Shear Forces

\[ Q_{1}^{H} = \int_{0}^{L} \sigma_{13} \, d\alpha, \quad (B7) \]

\[ Q_{2}^{H} = \int_{0}^{L} \sigma_{23} \, d\alpha. \quad (B8) \]

Electric Membrane Forces

\[ N_{1}^{T} = \int_{0}^{L} \varepsilon_{1} E_{3} \, d\alpha, \quad (B9) \]

\[ N_{2}^{T} = \int_{0}^{L} \varepsilon_{2} E_{3} \, d\alpha, \quad (B10) \]

\[ N_{3}^{T} = 0. \quad (B11) \]

Electric Bending Moments

\[ M_{1}^{T} = \int_{0}^{L} \varepsilon_{1} E_{2} \, d\alpha, \quad (B12) \]

\[ M_{2}^{T} = \int_{0}^{L} \varepsilon_{2} E_{2} \, d\alpha, \quad (B13) \]

\[ M_{3}^{T} = 0. \quad (B14) \]

Electric Transverse Shear Forces

\[ Q_{1}^{T} = \int_{0}^{L} \varepsilon_{1} E_{5} \, d\alpha, \quad (B15) \]

\[ Q_{2}^{T} = \int_{0}^{L} \varepsilon_{2} E_{5} \, d\alpha. \quad (B16) \]

Note that the electric forces/moments induced by the converse effect can be externally controlled in control applications, e.g., open or closed control systems. Besides, all three electric fields \( E_{i} \) are considered in the system electromechanical equations in this case.

APPENDIX C

The membrane forces and moments of a thin shell can be defined as:

\[ N_{i}^{H} = K (S_{22}^{0} + S_{23}^{0}) \]

\[ (C1) \]

\[ N_{i}^{H} = K (S_{12}^{0} + S_{13}^{0}) \]

\[ (C2) \]

\[ N_{i}^{H} = K (1 - \mu) S_{11}^{0} \]

\[ (C3) \]

\[ M_{i1}^{H} = D (k_{11} + \mu k_{12}) \]

\[ (C4) \]

\[ M_{i2}^{H} = D (k_{21} + \mu k_{22}) \]

\[ (C5) \]

\[ M_{i3}^{H} = D (1 - \mu) k_{12} \]

\[ (C6) \]

where \( K = [Y h (1 - \mu^2)] \) is the membrane stiffness; \( D = [Y h^3/12(1 - \mu^2)] \) is the bending stiffness; \( Y \) is Young's modulus; and \( \mu \) is Poisson's ratio. The strains are divided into two components: 1) the membrane strains and 2) the bending strains.

Membrane Strains \( \varepsilon_{ij}^{0} \)

\[ \varepsilon_{11}^{0} = -\frac{1}{A_{1}} \frac{\partial u_{1}}{\partial \alpha_{1}} + \frac{u_{2}}{A_{2}} \frac{\partial A_{1}}{A_{1}} + \frac{u_{3}}{A_{3}} \frac{\partial A_{2}}{A_{2}} \]

\[ (C7) \]

\[ \varepsilon_{22}^{0} = -\frac{1}{A_{2}} \frac{\partial u_{2}}{\partial \alpha_{2}} + \frac{u_{1}}{A_{1}} \frac{\partial A_{2}}{A_{2}} + \frac{u_{3}}{A_{3}} \frac{\partial A_{2}}{A_{2}} \]

\[ (C8) \]

\[ \varepsilon_{12}^{0} = -\frac{1}{A_{1}} \frac{\partial u_{2}}{\partial \alpha_{1}} + \frac{A_{1}}{A_{2}} \frac{\partial \alpha_{1}}{A_{2}} \]

\[ (C9) \]

Bending Strains \( k_{ij} \)

\[ k_{11} = -\frac{1}{A_{1}} \frac{\partial A_{1}}{\partial \alpha_{1}} + \frac{1}{A_{1} A_{2}} \frac{\partial A_{1}}{A_{2}} \]

\[ (C10) \]

\[ k_{21} = -\frac{1}{A_{2}} \frac{\partial A_{2}}{\partial \alpha_{2}} + \frac{1}{A_{1} A_{2}} \frac{\partial A_{2}}{A_{1}} \]

\[ (C11) \]

\[ k_{12} = \frac{A_{2}}{A_{1}} \frac{\partial \alpha_{1}}{\partial \alpha_{2}} + \frac{A_{1}}{A_{2}} \frac{\partial \alpha_{2}}{A_{1}} \]

\[ (C12) \]

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