Design of a piezoelectric exciter/actuator for micro-displacement control: theory and experiment

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High-precision and high-speed machine operation is very much in demand in modern manufacturing industry. However, mechanical vibrations introduced by various sources can easily disrupt this operation. Thus, vibration isolation and control have become very important in high-precision machine operation. This paper presents an active micro-position control technique using a piezoelectric actuator. A general theory for a piezoelectric actuator subjected to mechanical excitations and feedback voltages is developed. Piezoelectric excitation induced by the converse piezoelectric effect via direct voltage injection is first studied. Then, the electromechanical phenomenon is applied to an active micro-position control by injecting a processed feedback voltage at variable feedback gains into the piezoelectric actuator counteracting the external excitation. The effectiveness of the piezoelectric micro-position attenuation is evaluated analytically and experimentally. The theoretical solutions are compared favourably with the laboratory experimental data.

Keywords: micro-displacement control, vibration isolation, piezoelectric actuators

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Effective surface area</td>
</tr>
<tr>
<td>C</td>
<td>Constant</td>
</tr>
<tr>
<td>d</td>
<td>Piezoelectric constant</td>
</tr>
<tr>
<td>D</td>
<td>Dielectric displacement</td>
</tr>
<tr>
<td>E</td>
<td>Electric field</td>
</tr>
<tr>
<td>f</td>
<td>Frequency (Hz) = ω/2π</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>G</td>
<td>Acceleration (g's)</td>
</tr>
<tr>
<td>I</td>
<td>Thickness</td>
</tr>
<tr>
<td>m</td>
<td>Total mass</td>
</tr>
<tr>
<td>m_s</td>
<td>Seismic mass</td>
</tr>
<tr>
<td>q_s</td>
<td>Transverse displacement</td>
</tr>
<tr>
<td>R</td>
<td>Percentage isolation of the seismic mass</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>V</td>
<td>Voltage</td>
</tr>
<tr>
<td>Y</td>
<td>Young's modulus</td>
</tr>
<tr>
<td>x_3</td>
<td>Direction normal to the surface at (x_1, x_2)</td>
</tr>
<tr>
<td>ω</td>
<td>Strain</td>
</tr>
<tr>
<td>ρ</td>
<td>Elastic compliance</td>
</tr>
<tr>
<td>ρ</td>
<td>Wave propagation</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
</tr>
<tr>
<td>σ</td>
<td>Stress</td>
</tr>
<tr>
<td>ω</td>
<td>Angular velocity ( (\text{rad s}^{-1}) = 2πf )</td>
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</table>

Subscripts:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>b</td>
<td>Base</td>
</tr>
<tr>
<td>exc</td>
<td>Excitation</td>
</tr>
<tr>
<td>fb</td>
<td>Feedback</td>
</tr>
<tr>
<td>r</td>
<td>Resultant component</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean square</td>
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</table>

There is a great demand for high-precision and high-speed machines for various industrial applications. In order to achieve high-precision and accuracy, a stable machine operation is required in all operational conditions and environments. However, this highly demanding machine operation can be easily disrupted due to mechanical oscillation introduced internally or externally. Thus, vibration isolation and control become essential in these conditions.
Vibration control of mechanical systems can be generally accomplished by either passive or active control techniques. Passive control techniques involve the use of energy absorption devices, e.g., dynamic absorbers, metal springs, elastomers, wire mesh, stranded wire springs etc. Active vibration controls utilize active forces artificially generated to counteract the disturbance, e.g., electromechanical, fluidic, electromagnetic, piezoelectric actuators etc. Although passive control schemes are relatively simple to incorporate with mechanical systems, their performance is rather restrictive since the design parameters are specified. On the other hand, active shock and vibration control devices have parameters which can be adjusted to effectively eliminate/reduce the disruptive frequencies and/or magnitudes and to maintain a desirable system operation or response characteristic. This paper presents a new active micro-position vibration control technique by using electromechanical behaviour, the converse piezoelectric effect, generated in an active control element - a piezoelectric actuator.

The converse piezoelectric effect is an electromechanical phenomenon observed in many piezoelectric materials in which a mechanical strain/stress is generated upon application of an electric charge. This phenomenon is the reverse electromechanical behaviour of the direct piezoelectric effect, i.e., an electric charge generated when an external mechanical stress is applied. There have been many piezoelectric transducers developed by using the direct effect, such as force/pressure transducers, accelerometers, microphones, hydrophones, tactile sensors etc., and a few oscillators or wave generators based on the converse piezoelectric effect. Most conventional piezoelectrics (e.g., quartz, barium titanate, lead zirconate titanate etc.) are brittle and are difficult to fabricate into complex geometries. However, the polymeric piezoelectric PVDF is a tough and pliant polymer resistant to most non-extreme standard environmental conditions and it can be easily formed into sheets and complex shapes. These unique properties have made it attractive for active micro-position control applications. In this study, the converse piezoelectric effect is applied to micro-position control for high-precision machine operations, and the material used for the active control element is a β-phase piezoelectric polyvinylidene fluoride (PVDF) polymer. However, the general solution procedure is valid for other piezoelectric actuators.

In this study, a theory on active vibration isolation using the piezoelectric is developed. A standard piezoelectric actuator subjected to general mechanical excitations and feedback voltages is studied. A dynamic equation for the piezoelectric actuator is formulated and its general solution is derived. This theoretical solution is validated by laboratory experiments. Excitation induced by the converse piezoelectric effect in a PVDF actuator and its effectiveness to active micro-position control are also evaluated.

Theoretical formulation

The formulation of a general theory on active position control using a piezoelectric actuator is derived in this section. The system equation of a piezoelectric actuator is formulated using Lagrange's equation. Analytical solutions for a piezoelectric system subjected to mechanical excitations and feedback voltages are derived. Applications of the general solution to a piezoelectric actuator and an active position feedback control (with variable gains) are discussed.

The linear piezoelectric constitutive equations coupling the elastic field and the electric field can be expressed in the forms of direct and converse effects, respectively:

\[ D_i = (\varepsilon^*)_{ij} [E_j] + (d_{ij}) [\sigma_j] \]

\[ E_i = (\kappa^*)_{ij} [\sigma_j] + (d_{ij}) [\varepsilon_i] \]

where \([D_i]\) is the electric displacement vector, \([\sigma^*]_{ij}\) is the dielectric matrix evaluated at constant strain, \([E_i]\) is the electric field vector, \([d_{ij}]\) is the piezoelectric constant matrix, \([\varepsilon_i]\) is the strain vector, and \([\kappa^*]_{ij}\) is the elastic compliance matrix measured at constant electric field. The first equation indicates the direct effect and the second is the converse effect. In our application, we only consider the converse effect in the transverse motion \(q_3\). Thus, the equation can be simplified to

\[ \varepsilon^*_{33} = d_{33} \varepsilon + \kappa \sigma_{33} \]

\[ \varepsilon_{33} = \frac{1}{\kappa} (\varepsilon^*_{33} - d_{33} \varepsilon) \]

where \(\varepsilon\) is the strain, \(\sigma\) is the stress, \(E\) is the electric field, \(d\) is the piezoelectric constant and \(\kappa\) is the elastic compliance. It is assumed that in the direct effect is neglected since the feedback voltage is much larger than the voltage generated by the piezoelectric actuator itself in the active vibration isolation application.

Consider a differential piezoelectric actuator with an effective surface area \(A\) and a thickness \(d\), as in the \(z_3\) direction, only the transverse motion \(q_3\) is considered in the theoretical derivation. Fig 1 illustrates a piezoelectric actuator with general mechanical and electrical boundary conditions. The mechanical boundary condition indicates the external excitations and the electrical boundary condition reflects the feedback voltages. It is intended to use a converse piezoelectric effect induced by the feedback voltages to counteract the mechanical excitation.
boundary conditions, where the electrical boundary conditions are determined by the feedback voltages and the mechanical boundary conditions are external excitations. Consider the general mechanical and electrical boundary conditions at \( x_3 = 0 \) and \( l/(l) \) is the thickness of the piezoelectric actuator) as shown in Fig 1.

\[
\begin{align}
  x_3 &= 0 \\
  \sigma_{33} &= \sigma_{33,0} \sin \omega t \\
  E &= E_0 \sin \omega t \\
  x_3 &= l \\
  \sigma_{33} &= \sigma_{33,0} \sin \omega t \\
  E &= E_0 \sin \omega t
\end{align}
\]

where \( \omega \) is the excitation frequency and \( E = V_{fb}/l \) and \( V_{fb} \) is the feedback voltage. The steady-state solution of \( q_3 \) can be derived by substituting Eqs (10) and (11) into Eq (9) and using Eq (7)

\[
q_3(t) = \int_0^l \frac{\dot{\varepsilon}}{\varepsilon_3} (x_3, t) dx_3
\]

\[
= l(d_{33}E_0 + \kappa \sigma_{33,0}) \frac{\tan(\phi/2)}{\omega^2} \sin \omega t
\]

where \( \phi = (n/l) \). The feedback acceleration \( G_{fb}(t) \) is produced by the piezoelectric polymer resulting from the converse piezoelectric effect, and the g-level can be expressed as

\[
G_{fb}(t) = -\frac{\omega^2}{g}(d_{33}E_0 + \kappa \sigma_{33,0}) \sin \omega t \frac{\tan(\phi/2)}{\omega^2}
\]

where \( g \) is the gravity. It is assumed that the piezoelectric actuator is used to isolate the motion of a seismic mass \( m_s \). After substituting \( E_0 = V_{fb}/l \) into Eq (13), the converse piezoelectric induced force \( F_{fb}(t) \) resulting from the feedback voltage \( V_{fb} \) (controlled by the feedback gain) becomes

\[
F_{fb}(t) = m_s g G_{fb}(t)
\]

\[
= -m_s \omega^2(d_{33}V_{fb} + \kappa \sigma_{33,0}) \sin \omega t \frac{\tan(\phi/2)}{\omega^2}
\]

Similarly, the equivalent force \( F_b(t) \) introduced by the base excitation \( G_b(t) \) is given by

\[
F_b(t) = m g G_b(t)
\]

where \( m \) is the total mass (including the piezoelectric actuator mass). The resultant acceleration \( G \), due to the combined effects of excitations \( G_b \) and feedbacks \( V_{fb} \), can be obtained
by balancing the forces,

\[ G_s = \frac{1}{m_c} \frac{F_a + F_{ib}}{m_c}. \]  

(16)

Substituting Eqs (14) and (15) into Eq (16) yields a general equation for a piezoelectric actuator subjected to base excitations and feedbacks.

\[ G_s(t) = \left[ G_{ba} - \omega^2 \left( \frac{d_{33}}{g} V_{ba} + \kappa G_{bs} m_s \right) \right] \tan(\phi/2) \frac{m_s}{m_c} \sin \omega t \]  

(17)

This is a general equation for a piezoelectric actuator with a seismic mass and it can also be used for other piezoelectric actuators. Integrating Eq (17) twice can result in a general stroke equation. In our case, we use a polymeric piezoelectric PVDF as the actuator.

Substituting the physical properties of the model, we found \([\tan(\phi/2)] \sim 1\). Thus,

\[ G_s(t) = \omega^2\left( \frac{d_{33}}{g} V_{ba} + \kappa G_{bs} m_s \right) \]  

(18)

This equation will be used to evaluate two dynamic effects in the piezoelectric actuator: a piezoelectric excitation induced by the external applied voltages and an active position control using feedback voltages.

**Piezoelectric exciter**

In this section, we evaluate a piezoelectric excitation due to the converse effect in the piezoelectric actuator. In this case, the base excitation is zero \((G_{ba} = 0)\) and an excitation voltage \(V_{exc} \) is applied to the piezoelectric layer. Note that \(V_{exc} \) takes the place of \(V_{ba} \) in Eq (18). The converse piezoelectric effect causes the seismic mass to vibrate and this motion can be detected by a mini-accelerometer.

Substituting \(G_{ba} = 0\) into Eq (18), we can estimate the resultant acceleration \(G_{exc} \) (in g)

\[ G_{exc} \sin \omega t = \left( \frac{\omega^2}{g} \frac{d_{33} V_{exc}}{m_c} \right) \sin \omega t \]  

(19)

The magnitude can be written as

\[ G_{exc} = \frac{\omega^2}{g} \frac{d_{33} V_{exc}}{m_c} \]  

(20)

The slopes of the induced piezoelectric excitation, Eq (20), with respect to excitation voltage and frequency are of interest. Thus, taking partial derivatives with respect to input voltage and the excitation frequency respectively yields

\[ \frac{\partial G_{exc}}{\partial V_{exc}} = \frac{d_{33} m_s}{g m_c} \omega^2 = (\text{constant}) \omega^2 \]  

(21)

\[ \frac{\partial G_{exc}}{\partial \omega} = \frac{2 d_{33} m_s}{g m_c} V_{exc} \omega = (\text{constant}) V_{exc} \omega \]  

(22)

Eq (21) shows that for constant excitation voltage, the resultant excitation acceleration (by the converse piezoelectric effect) varies with frequency squared. Eq (22) shows that for constant frequency, it varies linearly with excitation voltage.

**Active vibration isolation**

A major application of Eqs (17) and (18) is in the active micro-position feedback control in this study. If an excitation can be measured by an accelerometer, the accelerometer output can be processed and fed back into the piezoelectric actuator, counteracting the oscillation and eliminating the disturbance. The active vibration isolation due to the feedback-induced converse effect can then be defined as the difference between the resultant acceleration \(G_s(t)\) and the base excitation \(G_{ba}(t)\). Note that it is assumed the residual stress in the piezoelectric actuator is negligible in the active vibration isolation application. The isolation percentage \(R\) can be defined as

\[ R(\%) = \frac{G_{ba}(t) - G_s(t)}{G_{ba}(t)} \times 100 \]  

(23)

Substituting Eq (18) into Eq (23) gives an active isolation equation:

\[ R(\%) = \frac{\omega^2 m_s}{G_{ba} m_c} \left( \frac{d_{33}}{g} V_{ba} + \kappa G_{bs} m_s \right) \times 100 \]  

(24)

The gradient of the isolation surface with respect to the excitation frequency and the feedback voltage is evaluated when the base excitation is constant \((G_{ba} = G)\) and feedback gain \(\gamma\) varies \((V_{ba} = \gamma V_0)\) where \(V_0\) is the transducer output. When the base excitation is a constant \(G_{ba} = G\)

\[ R(\%) = \frac{\omega^2 m_s}{G m_c} \left( \frac{d_{33}}{g} V_0 + \kappa G m_s \right) \times 100 \]  

(25)

It is found that the second term is small compared with the first term after substituting all material properties into the equation. Thus,

\[ \frac{\partial R}{\partial \omega} \approx \left( \frac{100 d_{33} m_s}{G m_c} \right) \omega^2 \]  

(26)

\[ \frac{\partial R}{\partial V_{ba}} \approx \left( \frac{200 d_{33} m_s}{G m_c} \right) V_{ba} \omega \]  

(27)
Eq (26) shows that the isolation is a quadratic function of the frequency. As the frequency increases, the isolation will increase as frequency squared. Eq (27) shows that the isolation varies linearly with feedback voltage at constant frequency.

Experimental validation – prototype model

An experimental model was designed and tested in the laboratory to validate the theory. Fig 2 shows the experimental model with a layer of piezoelectric PVDF actuator. The model has a 6.35 mm (0.25 inch) thick steel base with a standard 10–32 stud which can be mounted on a shaker. A 1 mm thick PVDF polymer with an effective surface area of $4 \times 10^{-5}$ m$^2$ is sandwiched between two 6.35 mm Plexiglas layers which provide the same boundary conditions to the piezoelectric actuator. The bottom Plexiglas is epoxied to the steel base, and an interchangeable metal plate is screwed onto the top Plexiglas layer. A mini-accelerometer is attached above this metal plate. Thus, the seismic mass consists of all the items above the piezoelectric actuator – the Plexiglas, metal plates and the mini-accelerometer. The vibration of this seismic mass was monitored by the mini-accelerometer. The acceleration signal was phase shifted, amplified and stepped up using a transformer and then injected into the piezoelectric polymer. There were two cases to be verified: piezoelectric excitation induced by the converse effect, and active feedback position control.

Piezoelectric exciter

In this case, voltages of various amplitudes and frequencies were applied across the piezoelectric layer while no base excitation (mechanical) was applied. This causes the seismic mass to accelerate and the resultant acceleration induced by the converse piezoelectric effect can be measured. The spectra of the excitation and seismic mass acceleration signals were observed and the peaks recorded.

Active position control

The model was mounted on a shaker that could be excited at various frequencies using a function generator. The seismic mass acceleration was sensed by the mini-accelerometer. The acceleration signal was phase shifted, amplified and applied across the piezoelectric layer in such a way that the piezoelectric vibration was 180° out of phase with that of the base. The shaker was excited at various frequencies and amplitudes from 250 Hz to 2.5 kHz. The experimental set-up is shown in Fig 3.

The base excitation was kept constant in this set of experiments. Four excitation amplitudes to the shaker were chosen for each frequency. Since the base excitation is constant, the feedback gain is varied so that the feedback voltage injected into the piezoelectric polymer can be controlled. The spectrum of the undamped seismic mass acceleration signal was observed and the frequency and amplitude recorded. Then the feedback was applied and the spectra of the feedback and attenuated seismic mass acceleration signals were observed and the peaks recorded. In this way data for a three-dimensional surface plot was collected.

Results and discussion

As discussed previously, two cases were studied, i.e., piezoelectric excitation and active vibration isolation. Their analytical and experimental results are presented in this section.

Piezoelectric exciter

For the initial evaluation of the pure conversive piezoelectric effect, only voltages were injected into the piezoelectric actuator and response spectra observed as shown in Fig 4. The difference between the experimental result and the analytical prediction was found to be 8%.
**Active position control and isolation**

Base excitations $G_b$ (g/s), excitation frequencies $f$ (Hz), feedback voltages $V_{fe}$ (V) and resultant accelerations $G_r$ (experimental) can be acquired from the experiments. Substituting the first three values into Eq (18), we can calculate $G_r$ (theoretical). Using Eq (23), we can calculate $R(\exp)$ and $R(\th)$, which are then plotted. The independent variables in this situation were the base excitation frequency and the feedback voltage on the x and y axes respectively. The dependent variable is the isolation or error on the vertical z-axis. The active vibration isolation plots are presented in Fig 5.

Experimentally, the active vibration isolation is found to vary between 0.3% at 500 Hz and a feedback voltage of $12 V_{rms}$ to a maximum of 48% at 2500 Hz and a feedback voltage of $85 V_{rms}$. Analytically, the isolation ranges from 0.07% at 250 Hz and a feedback voltage of 11.4 $V_{rms}$ to a 47.5% at 2500 Hz and feedback voltage of 53.4 $V_{rms}$. The theory predicts that the isolation gradient should be a linear function of excitation voltage for a given frequency, Eq (27), and a quadratic function of frequency for a given excitation voltage, Eq (26). Both the experimental and analytical data plots show this tendency. Due to an equipment limitation and the system stability, it was only tested up to 25 kHz. The performance could be even better if higher feedback voltages were available.

The absolute errors between the analytical and experimental data of the resultant seismic mass acceleration can be observed in Fig 5. They are calculated to be within 7%. It is observed that the theoretical data are higher than those obtained experimentally at high frequency due to the energy dissipation at high feedback gains. The other possible sources of errors could be: interaction between the direct effect and the converse effect in the piezoelectric PVDF actuator; the feedback signal not being exactly 180° out of phase with the seismic mass acceleration; and non-linearity associated with the experimental model (e.g. epoxy). Note that the theory developed and verified is for the transverse direction only. The operation should also be kept within the linear range of the mechanical system with the piezoelectric actuator. Otherwise, non-linear control techniques should be used.

**Conclusions**

High-precision and high-accuracy operation is of importance in robotic manipulation and many other modern manufacturing systems. This paper presents an active micro-position feedback control technique using piezoelectric actuators. A general mathematical model of the piezoelectric actuator was first formulated and the theory associated with the model was also proposed. The model is rather general, and includes both mechanical and electrical boundary conditions. Analytical solutions for the actuation and isolation control at variable feedback gains were also derived. A physical model made of a $\beta$-phase piezoelectric polyvinylidene fluoride (PVDF) polymer was designed and evaluated in the laboratory. A mini-accelerometer was used to
monitor the system responses, and the output signals were processed, phase shifted, fed back, and then injected into the piezoelectric actuator to counteract the base excitations. The effectiveness of the actuation and isolation was evaluated.

Piezoelectric excitation solely introduced by a converse piezoelectric effect was first investigated to evaluate the resultant mass acceleration due to the electric voltage excitations. On the active position controls, studies were conducted by varying the feedback gains and keeping the constant base excitations. It shows that the active piezoelectric isolation is a linear function of feedback voltages at a given frequency and a quadratic function of frequency at a given voltage. The analytical solutions compared excellently with the experimental results.

Note that this technique is primarily for micro-position feedback controls of high-precision operations. For relatively large stroke movements, it would require a much thicker piezoelectric actuator. Some other factors, such as breakdown voltage, temperature effects, noise, phase shift, time delay, etc. could also influence the control effectiveness. The potential applications of the technique include positioning, grinding, laser mounts, polishing, machining etc.

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