NON-LINEAR JOINT DYNAMICS AND CONTROLS OF JOINTED FLEXIBLE STRUCTURES WITH ACTIVE AND VISCOELASTIC JOINT ACTUATORS

H. S. TZOU

Department of Mechanical Engineering, University of Kentucky, Lexington, Kentucky 40506-0046, U.S.A.

(Received 20 July 1989, and in revised form 12 February 1990)

Studies on joint-dominated flexible space structures have attracted much interest recently due to the rapid developments in large deployable space systems. This paper describes a study of the non-linear structural dynamics of jointed flexible structures with initial joint clearance and subjected to external excitations. Methods of using viscoelastic and active vibration control technologies, joint actuators, to reduce dynamic contact force and to stabilize the systems are proposed and evaluated. System dynamic equations of a discretized multi-degrees-of-freedom flexible system with initial joint clearances and joint actuators (active and viscoelastic passive) are derived. Dynamic contacts in an elastic joint are simulated by a non-linear joint model comprised of a non-linear spring and damper. A pseudo-force approximation method is used in numerical time-domain integration. Dynamic responses of a jointed flexible structure with and without viscoelastic and active joint actuators are presented and compared. Effectiveness of active/passive joint actuators is demonstrated.

1. INTRODUCTION

Joint-dominated structures have been being very popular in many civil and marine engineering applications; e.g., trusses, bridges, drilling platforms, etc. In these applications, the joints are usually fixed. However, joint mobility allowing fast relative motion of two jointed members is essential in many aerospace and mechanical systems: e.g., deployable space structures, robots and linkages. Although joint mobility is not important in static applications, joint characteristics can be very crucial in the dynamic performance, e.g., speed and accuracy, of the mechanical system. Moreover, internal joint dynamic contacts can introduce excessive noise and vibration, accelerate wear and fatigue, and result in premature system failures.

With the development of large flexible space systems, joint-dominated flexible structures have become even more important. The dynamics and control of large jointed flexible structures have attracted much attention from many researchers. Rhodes discussed some design considerations in the static behavior of joints for deployable space truss structures [1]. Coyner and Bachtell studied a multi-bay statistically determinate box truss antenna [2]. Crawley and O'Donnell investigated a joint's non-linearity using force-state mapping identification [3]. Gaul studied jointed structures using an equivalent linearized joint model [4]. Benack analyzed the dynamic responses of impact in high-speed systems with clearances [5]. Dubowsky et al. studied clearance impacts in spatial mechanisms [6]. Tzou investigated dynamic contact modeling using a pseudo-force approximation method (with Schiff) [7], the dynamic effect of design tolerance (with Hunter and Gadre) [8], joint dynamics [9], spatial joint dynamics theoretically (with Rong) [10], the application
of a viscoelastic link damper to structural vibration control [11, 12], and active vibration controls for distributed parameter systems [13, 14]. In these studies, a joint control mechanism to improve the system dynamics was not considered. Accordingly, a study of the structural dynamics of jointed flexible structural systems with viscoelastic or active joint actuators is relevant to designing more effective and accurate large flexible joint-dominated structural systems: e.g., deployable space structures.

This paper describes a study of the non-linear structural dynamics of jointed flexible structures with initial joint clearances and subjected to external excitations. Methods of using viscoelastic and active vibration control technologies, joint actuators, to reduce dynamic contact forces and to stabilize the systems are also proposed and evaluated. The system dynamic equations of a discretized multi-degree-of-freedom flexible system with initial joint clearances are derived. Dynamic contacts in an elastic joint are simulated by a non-linear joint model comprised of a non-linear spring and damper. A pseudo-force approximation method is used in a non-linear time-domain integration of the equations [7]. Dynamic responses of a jointed flexible structure with and without viscoelastic and active joint actuators are studied and their effectiveness evaluated.

2. MATHEMATICAL FORMULATION OF JOINTED STRUCTURES

A generalized jointed flexible structural system, in which each flexible members is elastically joined to its neighbors, is illustrated in Figure 1. It is assumed that there is a small clearance in each joint.

![Figure 1. Flexible members with elastic joints.](image)

A joint-dominated structural system can be discretized into \( n \) unit member-joint subsystems with equivalent lumped masses, equivalent springs, and equivalent dampers. Each unit system consists of a lumped mass \( m_i \), an equivalent linear spring \( k_i \), and an equivalent linear damper \( c_i \), derived from the flexible member (a detailed nomenclature is given in Appendix B). The mass motion \( z_i \) is assumed to be translational (however, this generalized approach can be easily extended to include rotation and/or spatial motion). In addition, a non-linear joint model with a non-linear joint spring \( k_i' \), and a non-linear joint damper \( c_i' \) links each two successive flexible members to model the dynamic action of each joint, as illustrated in Figure 2.

2.1. NON-LINEAR JOINT MODEL

The non-linear joint model consists of a set of non-linear joint spring \( k_i' \) and damper \( c_i' \) and is activated by a step function \( U(z_i - z_{i-1} - \mu_i') \): i.e., when the relative displacement \( z_i - z_{i-1} \) is greater than an initial joint gap \( \mu_i' \), the non-linear joint spring and damper are activated. The step function \( U(\ldots) \) is defined as

\[
\begin{align*}
U(y_i') &= 0, \quad \text{when } (z_i - z_{i-1}) < \mu_i' \\
U(y_i') &= 1, \quad \text{when } (z_i - z_{i-1}) \geq \mu_i'.
\end{align*}
\]  

(1)
The non-linear joint spring \( k'_j \) and damper \( c'_j \) can be generally approximated by polynomial functions, i.e.,

\[
c'_j(\delta z) = \sum_{p=1}^{\hat{p}} c'_p \delta z^{p-1}, \quad k'_j(\delta z) = \sum_{q=1}^{\hat{q}} k'_q \delta z^{q-1},
\]

where \( c'_p \) are the damping coefficients, \( k'_q \) are the non-linear stiffness coefficients,

\[
\{\delta \dot{z}\} = \{\dot{z}_i\} - \{\dot{z}_j\}, \quad \{\delta z\} = \{z_i\} - \{z_j\}.
\]

Here \( i \) and \( j \) are the two adjacent nodes connected by the joint element. \( \delta \dot{z} \) and \( \delta z \) are the relative velocities and the relative displacements respectively. It is intended to study the dynamic contacts of adjacent unit subsystems and their effects on entire system dynamics.

### 2.2. Joint Dynamics Controls

Control of structural dynamics of the jointed flexible structures can be achieved either passively or actively. In this study, a viscoelastic joint actuator, a passive joint controller of standard linear model, and an active joint actuator modeled by a non-linear spring and damper are incorporated with the joint to suppress the dynamic contacts and to stabilize the jointed structural system. The standard linear passive joint actuator model has a primary stiffness \( k'_i \), in parallel with a secondary spring \( k'_j \) and a viscous damper \( c'_i \). The active actuator consists of a non-linear control spring \( k'_i \) and a non-linear control damper \( c'_i \), as shown in Figure 2. To keep the mathematical model general, the viscoelastic and the active actuators are separated by gaps \( \mu'_1 \) and \( \mu'_2 \) and are activated by step functions \( U(z_i - z_{i-1} - \mu'_1) \) and \( U(z_i + z_{i-1} - \mu'_2) \) respectively. Should any controller be activated all the time, the gap \( \mu \) would be set to zero.

#### 2.2.1. Viscoelastic joint actuator: standard linear model

As illustrated in Figure 2, a standard linear model is used to approximate the control action of a viscoelastic joint actuator. When the viscoelastic actuator is subjected to a sinusoidal excitation \( F = F_0 \sin \omega t \), a complex stiffness \( F/z \) can be determined,

\[
F/z = (k'_i + k'_j(\omega c'_i)^2/R_i) + j\omega c'_i(k'_j)^2/R_i.
\]

where \( R_i = (k'_i)^2 + (\omega c'_i)^2 \) and \( j = \sqrt{-1} \). The three parameters in the model can be re-organized as two parameters, an equivalent spring \( k^*_i \) and an equivalent damper \( c^*_i \):

\[
k^*_i = k'_i + k'_j(\omega c'_i)^2/R_i, \quad c^*_i = c'_i(k'_j)^2/R_i.
\]

Only the equivalent stiffness and damping of the viscoelastic joint actuator will be used in the theoretical derivation of the system dynamic equations.
2.2.2. Active joint actuator

The non-linear active joint actuator is also modeled by a non-linear spring, $k^a_j$, and damper, $c^a_j$, both represented by polynomial functions,

$$c^a_j = \sum_{i=1}^{m} c^a_i \delta z^{i-1}, \quad k^a_j = \sum_{i=1}^{m} k^a_i \delta z^{i-1}. \quad (6a, b)$$

Here $c^a_j$ are the damping coefficients and $k^a_j$ are the stiffness coefficients of the active joint actuator, as in the definition of the non-linear joint model.

2.3. SYSTEM DYNAMIC EQUATIONS

Substituting all energy expressions related to each physical system into Lagrange's equations yields a generalized non-linear matrix equation of motion:

$$\begin{bmatrix}
    m_1 & 0 & 0 & 0 \\
    0 & m_2 & 0 & 0 \\
    0 & 0 & m_3 & 0 \\
    \vdots & \vdots & \vdots & \vdots \\
    0 & 0 & 0 & m_n
\end{bmatrix}
\begin{bmatrix}
    \ddot{z}_1 \\
    \ddot{z}_2 \\
    \ddot{z}_3 \\
    \vdots \\
    \ddot{z}_n
\end{bmatrix}
+ \begin{bmatrix}
    c_1 + c_2 + c^a_2 + c^a_1 & -c_2 - c_1 U(\gamma)_1 - c^a_2 U(\gamma)_2 - c^a_1 U(\gamma)_3 \\
    -c_2 - c_1 U(\gamma)_1 - c^a_2 U(\gamma)_2 - c^a_1 U(\gamma)_3 & c_2 + c_1 + c^a_2 + c^a_1 \\
    \ddots & \ddots & \ddots & \ddots \\
    0 & \ddots & \ddots & \ddots \\
    k_1 + k_2 + k^a_2 + k^a_1 & -k_2 - k_1 U(\gamma)_1 - k^a_2 U(\gamma)_2 - k^a_1 U(\gamma)_3 \\
    -k_2 - k_1 U(\gamma)_1 - k^a_2 U(\gamma)_2 - k^a_1 U(\gamma)_3 & k_2 + k_1 + k^a_2 + k^a_1 \\
    \ddots & \ddots & \ddots & \ddots \\
    0 & \ddots & \ddots & \ddots \\
\end{bmatrix}
\begin{bmatrix}
    \dddot{z}_1 \\
    \dddot{z}_2 \\
    \dddot{z}_3 \\
    \vdots \\
    \dddot{z}_n
\end{bmatrix}
= \begin{bmatrix}
    k(\mu_1 U(\gamma)_1) + k^a_1 \mu_1 U(\gamma)_1 \\
    -k(\mu_1 U(\gamma)_1) + k^a_1 \mu_1 U(\gamma)_1 + k^a_2 \mu_1 U(\gamma)_2 \\
    \ddots & \ddots & \ddots \\
    -k^a_{n-1} \mu_{n-1} U(\gamma)_{n-1} - k^a_{n-1} \mu_{n-1} U(\gamma)_{n-1} \\
\end{bmatrix}
\begin{bmatrix}
    F_1 \\
    F_2 \\
    \vdots \\
    F_n
\end{bmatrix}
+ \begin{bmatrix}
    \Gamma^{\prime}_1 + \Gamma^v_1 + \Gamma^c_1 \\
    \Gamma^{\prime}_2 + \Gamma^v_2 + \Gamma^c_2 \\
    \vdots \\
    \Gamma^{\prime}_n + \Gamma^v_n + \Gamma^c_n
\end{bmatrix}.$$  

Here $\gamma^j_i = (z_j - z_{j+1} - \mu^j_i)$, $U(\gamma^j_i)$ is a step function as defined earlier, and the superscript $x$ denotes either $j$, $v$ or $c$, representing joint, viscoelastic or active joint actuators respectively. Also

$$\Gamma^j_j = \frac{1}{2} \left( [k^j_{j-1}(z_j - z_{j-1} - \mu^j_{j-1})] \frac{\partial U(\gamma^j_{j-1})}{\partial z_j} - k^j_j(z_j - z_{j-1} - \mu^j_j)^2 \frac{\partial U(\gamma^j_j)}{\partial z_j} \right).$$  

(7)
\[ c_i^e = c_i^c U(y_i^c) + c_i^s U(y_i^s - 1), \quad c_i^c = c_i^c U(y_i^c - 1) \quad \text{and} \quad c_0^c = 0, \]
\[ k_i^e = k_i^c U(y_i^c) + k_i^s U(y_i^s - 1), \quad k_i^c = k_i^c U(y_i^c - 1) \quad \text{and} \quad k_0^c = 0. \]

Finally, \( \mathcal{X} \) denotes either \( J, \gamma \), or \( \mathcal{C} \), similar to \( j, v \), or \( c \) as discussed earlier.

The system equations can be simply expressed as
\[
[M][\dot{\mathcal{X}}] + [C][\ddot{\mathcal{X}}] + [K][\mathcal{X}] = \{F(t)\} + \{F^c(t)\} + \{F^v(t)\} + \{F^s(t)\},
\]
where \([M]\) is the system mass matrix, \([C]\) and \([K]\) are the non-linear system damping and stiffness matrices, \(\{\dot{\mathcal{X}}\}, \{\ddot{\mathcal{X}}\}, \{\mathcal{X}\}\) are the acceleration, velocity and displacement vectors, \(\{F(t)\}\) is the external excitation force vector, \(\{F^c(t)\}\) is the non-linear joint force vector, \(\{F^v(t)\}\) is the non-linear viscoelastic control force vector; and \(\{F^s(t)\}\) is the non-linear active control force vector. The non-linear system matrices can be divided into linear (associated with flexible members) and non-linear (associated with joints, viscoelastic and active joint actuators) components as
\[
[C] = [C]^d + [C]^v + [C]^e, \quad [K] = [K]^d + [K]^v + [K]^e;
\]
where \([C]^d\) and \([K]^d\) are the linear components and \([C]^v, [C]^e, [K]^v, [K]^e\) and \([K]^e\) are the non-linear components associated with each individual physical system, having different dynamic characteristics. The system equation then becomes
\[
[M][\ddot{\mathcal{X}}] + ([C]^d + [C]^v + [C]^e)[\dot{\mathcal{X}}] + ([K]^d + [K]^v + [K]^e)[\mathcal{X}] = \{F(t)\} + \{F^c(t)\} + \{F^v(t)\} + \{F^s(t)\},
\]
Moving all non-linear terms to the right side of the equation and summing non-linear terms to related force terms as a pseudo-joint contact force vector \(\{F_j(t)\}\), a pseudo-viscoelastic control force vector \(\{F_v(t)\}\), and an active control force vector \(\{F_a(t)\}\) give
\[
[M][\ddot{\mathcal{X}}] + [C]^d[\dot{\mathcal{X}}] + [K]^d[\mathcal{X}] = \{F(t)\} + \{F_j(t)\} + \{F_v(t)\} + \{F_a(t)\},
\]
where
\[
\{F(t)\} = \{F_1, F_2, F_3, \ldots, F_n\}^T
\]
\[
\{F_j(t)\} = -([C]^d[\dot{\mathcal{X}}] + [K]^d[\mathcal{X}]) + \{F^c(t)\}
\]
\[
\{F_v(t)\} = -([C]^v[\dot{\mathcal{X}}] + [K]^v[\mathcal{X}]) + \{F^v(t)\}
\]
\[
\{F_a(t)\} = -([C]^e[\dot{\mathcal{X}}] + [K]^e[\mathcal{X}]) + \{F^s(t)\}
\]
\[
\{F_v(t)\} = -([C]^{T}\{\dot{z}\} + [K]^{T}\{z\}) + \{F^{*}(t)\} \\
= -\begin{bmatrix}
  c_{0}^{y} & -c_{0}^{z}U(\gamma_{0}) & \cdots & 0 \\
  c_{1}^{y} & c_{1}^{z} & \cdots & 0 \\
  -c_{2}^{y}U(\gamma_{1}) & -c_{2}^{z}U(\gamma_{2}) & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  \vdots & \vdots & \ddots & -c_{n-1}^{y}U(\gamma_{n-1}) \\
  k_{0}^{y} & -k_{0}^{z}U(\gamma_{0}) & \cdots & 0 \\
  -k_{1}^{y}U(\gamma_{1}) & k_{1}^{z} & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  \vdots & \vdots & \ddots & -k_{n-1}^{y}U(\gamma_{n-1}) \\
  k_{n}^{y} & 0 & \cdots & k_{n}^{z}
\end{bmatrix} \begin{bmatrix}
  \dot{z}_{1} \\
  \dot{z}_{2} \\
  \vdots \\
  \vdots \\
  \dot{z}_{n}
\end{bmatrix} \\
+ \begin{bmatrix}
  k_{0}^{x} \mu_{0}^{x}U(\gamma_{0}) \\
  \vdots \\
  \vdots \\
  k_{n}^{x} \mu_{n}^{x}U(\gamma_{n})
\end{bmatrix}.
\]

(17)

\[
\{F_c(t)\} = -([C]^{T}\{\dot{z}\} + [K]^{T}\{z\}) + \{F^{*}(t)\} \\
= -\begin{bmatrix}
  c_{0}^{x} & -c_{0}^{y}U(\gamma_{0}) & \cdots & 0 \\
  c_{1}^{x} & c_{1}^{y} & \cdots & 0 \\
  -c_{2}^{x}U(\gamma_{1}) & -c_{2}^{y}U(\gamma_{2}) & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  \vdots & \vdots & \ddots & -c_{n-1}^{x}U(\gamma_{n-1}) \\
  k_{0}^{x} & -k_{0}^{y}U(\gamma_{0}) & \cdots & 0 \\
  -k_{1}^{x}U(\gamma_{1}) & k_{1}^{y} & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  \vdots & \vdots & \ddots & -k_{n-1}^{x}U(\gamma_{n-1}) \\
  k_{n}^{x} & 0 & \cdots & k_{n}^{y}
\end{bmatrix} \begin{bmatrix}
  \dot{z}_{1} \\
  \dot{z}_{2} \\
  \vdots \\
  \vdots \\
  \dot{z}_{n}
\end{bmatrix} \\
+ \begin{bmatrix}
  k_{0}^{x} \mu_{0}^{x}U(\gamma_{0}) \\
  \vdots \\
  \vdots \\
  k_{n}^{x} \mu_{n}^{x}U(\gamma_{n})
\end{bmatrix}.
\]

(18)

2.4. TIME-DOMAIN DIRECT INTEGRATION: A PSEUDO-FORCE METHOD

The assembled system equations can be directly integrated in the time domain by using a pseudo-force method [7] incorporated with the Wilson-θ method to accommodate the non-linear forces derived from the non-linearities related to joints and active and visco-elastic joint actuators. In the non-linear integration, the system equations at time \( t + \theta \Delta t \) can be expressed as

\[
[M][\ddot{z}(t + \theta \Delta t)] + [C]'(\dot{z}(t + \theta \Delta t)) + [K]'(z(t + \theta \Delta t)) = \{F^{*}(t + \theta \Delta t)\},
\]

(19)
where \( F^*(t + \theta \delta t) \) is the linear extrapolated load vector and can be written as
\[
\begin{align*}
F^*(t + \theta \delta t) &= \{F^*(t)\} + \theta\{F^*(t) + [F_2(t + \delta t)] + [F_\nu(t + \delta t)] + [F_C(t + \delta t)] - [F^*(t)]\}. \\
\end{align*}
\]  
(\ref{eq:20})

When this pseudo-force method is used, the homogeneous equations describing the system motion remain unchanged. The non-homogeneous equations differ by additions of the pseudo-non-linear joint force \( \{F_\nu(t)\} \) associated with the non-linear joints, the pseudo-non-linear viscoelastic force \( \{F_C(t)\} \) associated with the non-linear viscoelastic actuators, and the pseudo-non-linear control force term \( \{F_i(t)\} \) associated with the non-linear active actuators. Note that each joint, passive viscoelastic actuator and active actuator can have different dynamic characteristics, as indicated in each general expression.

3. CASE STUDY: A JOINTED FLEXIBLE SYSTEM WITH VISCOELASTIC OR ACTIVE JOINT ACTUATORS

In order to illustrate the joint contact dynamics and controls, a physical system consisting of a joint and four flexible members has been studied (see Figure 3). The joint is exaggerated to show its joint stiffness and damper, and to the joint will be added viscoelastic and active joint actuators later. Each member has cross-sectional area \( A = 3.23 \times 10^{-5} \text{ m}^2 \), density \( \rho = 7.84 \times 10^3 \text{ kg/m}^3 \) and flexural rigidity \( EI = 1.41 \times 10^5 \text{ N/m}^2 \). The first and the fourth members are fixed at the lower ends. The second and third members are elastically jointed together by the non-linear joint model discussed earlier. The initial joint gap between the second and the third links is 1 mil \( (2.54 \times 10^{-5} \text{ m}) \). The joint stiffness function is assumed to be \( k_i = 1.0 \times 10^{11} + 7.0 \times 10^9 \Omega(2.54 \times 10^{-5}) \text{ N/m} \), and the contact damping is defined as \( c_i = (1.23 \times 10^7) \Omega(2.54 \times 10^{-5}) \text{ N s/m} \), in which \( \Omega(2.54 \times 10^{-5}) \) is the step-function as discussed before. A 2 percent damping is assigned to all four flexible members. The system equation can be derived by setting the contact and actuator constants zero except for the second degree of freedom in equation (7).

For small-amplitude external excitations, the adjacent two elastic members will oscillate independently if the amplitude is smaller than the joint gap; i.e., \((z_{1} - z_{3}) < \mu_i \) (note that the connecting linear spring in Figure 3 is assumed to be very small, and thus it effect on members 2 and 3 is minimal). The joint gap has no effect on the individual member's oscillation. Joint contacts induced by external base excitation will be investigated and applications of a passive viscoelastic joint actuator and an active joint actuator will be studied.

![Non-linear joint model](image)

Figure 3. A jointed flexible structure system (not to scale).
3.1. NON-LINEAR JOINT CONTACTS

A sinusoidal base acceleration with an amplitude of 0.02 g and frequency of 30 Hz was applied to the jointed flexible structure. The excitation to the two base nodes was out of phase to emphasize the joint effect in system dynamics. It is recalled that the non-linear joint model is defined by a non-linear spring and damper activated by a step function. Thus, joint dynamic contact between the flexible members occurs when the relative displacement exceeds the gap. The time history responses of the two nodes located at either side of the non-linear joint (indicated as subscripts 1 and 2 in Figure 3) are presented in Figure 4, in which the bottom time history is the base excitation.

![Non-linear contact dynamic responses of the jointed structure.](image)

Since an out-of-phase base excitation of large amplitude has been applied to the two base nodes, the two nodes on either side of the joint contact every time at every peak. For extremely large amplitude excitation, it can be expected that the system responses will become more violent with time and eventually the system will breakdown. Penetration (numerical) of nodal points is also observed, which is used to estimate joint contact forces. This phenomenon is not likely to be possible in a real system. The penetration can be reduced by increasing the joint stiffness. However, a stiffer joint spring can introduce higher contact forces causing high-frequency oscillation of the nodal masses and resulting in numerical instability which is highly undesirable in the numerical analysis. Moreover, in reality, the joint stiffness needs to be identified according to its physical design, materials and construction. An equivalent contact force of the very first contact is calculated and used in future comparison with other cases: (1) with a viscoelastic joint actuator and (2) with an active joint actuator.

When the excitation frequency is decreased to 3.75 Hz, it is observed that the number of consecutive contacts within a half-cycle of the excitation frequency increases (see Figure 5). Because of the relatively long period of excitation, the member is forced to move back causing consecutive contacts. While they are not in contact, the flexible member oscillates at its natural frequency of 16.5 Hz. Again, penetration phenomena during contact oscillation are observed. This is only numerically possible as discussed earlier.
3.2. Viscoelastic Joint Actuator

To reduce joint contact force and to stabilize the flexible system, a passive viscoelastic joint actuator might be used. There are a variety of viscoelastic materials of different properties and geometries applicable to various circumstances. In this study, it was assumed that the joints between flexible members were filled with viscoelastic rubber, and also that the rubber actuator works both in tension and compression, as illustrated in Figure 6.

As discussed earlier, there are initially three parameters in a standard linear model. These three parameters can be simplified to two parameters, \( k_s \) and \( c_s \), if the excitation frequency is known. Furthermore, for a viscoelastic rubber actuator, the secondary stiffness \( k_s \) can be assumed to be three times that of the primary stiffness \( k_p \), and \( c_s \) can be estimated from experiments via damping ratio measurements. Dynamic responses of the flexible joint model with \( k_p = 3.5 \times 10^6 \text{ N/m} \) at damping ratios \( \zeta = 20 \) percent and 30 percent are shown in Figures 7 and 8 respectively.

From Figures 7 and 8, it is observed that when using a stiffer viscoelastic joint actuator, the peak amplitude decreases from the original \( 1.14 \times 10^{-4} \text{ m} \) (4.5 mils) (Figure 4) to \( 6.35 \times 10^{-5} \text{ m} \) (2.5 mils) for \( \zeta = 20 \) percent and \( 5.08 \times 10^{-5} \text{ m} \) (2.0 mils) for \( \zeta = 30 \) percent. The joint contacts are finally broken after a number of contact cycles, and the larger
Figure 7. Dynamic responses of the system with viscoelastic joint actuator ($\zeta = 0.2$).

Figure 8. Dynamic responses of the system with viscoelastic joint actuator ($\zeta = 0.3$).

damper ($\zeta = 30$ percent) contributes faster stable response and better dynamic performance. However, when the dynamic joint contact is broken, the responses of these two different viscoelastic actuators do not show much difference. A diagram showing the relationship between the first contact loads and different damping ratios of the viscoelastic joint damper is provided in Figure 9 (note that the maximum contact load can also be extrapolated by comparing the relative amplitude of mass penetration in any given figure).
In order to properly select a viscoelastic joint actuator for a given design, a number of design parameters need to be considered. A general design equation is derived as

$$
\mu^i = \frac{mZ\omega^2}{\sqrt{[(k^i_0 - m\omega^2)^2 + (c^i_0\omega)^2]^2}}.
$$

from which the required equivalent stiffness and damping of the viscoelastic joint actuator can be estimated by using the available parameters, such as mass, \(m\), vibration amplitude and frequency, \(Z\) and \(\omega\), and joint gap, \(\mu^i\). If there are \(n\) modal responses involved in the structural vibration, the modal participation factors, \(\Psi\), can be used to estimate the respective amplitudes of the modes (a detailed derivation is provided in Appendix A); one has

$$
\mu^i = \sum_{i=1}^{n} \frac{\Psi_i mZ_0 \omega_i^2}{\sqrt{[(k^i_0 - m\omega_i^2)^2 + (c^i_0\omega_i)^2]^2}}.
$$

The most conservative estimation would be the summation of each modal response participating in the motion. In practical applications, several iterations may be required to estimate appropriate viscoelastic joint actuators. Should the temperature fluctuate in the real environment, the temperature effects on the viscoelastic joint actuator should also be carefully considered.

### 3.3. Active Joint Actuator

The active joint actuator is activated by the step function in the same way as the viscoelastic joint actuator. However, the control force is determined by using the relative velocity of the joint nodes–velocity feedback. The feedback force is controlled by a feedback gain (i.e., it is equivalent to adjusting \(c^i_0\)) which directly changes the damping constant (control constant) \(c^i_0\) as discussed earlier. Control forces generated from the actuator can directly counteract the relative motion of two jointed flexible members. A block diagram showing the set-up is given in Figure 10.

In Figures 11 and 12, the actively controlled responses of the jointed flexible structure model with \(c^i_0 = 4.9 \times 10^8 \text{ N/m s} \) and \(9.8 \times 10^8 \text{ N/m} \) are shown respectively. Higher velocity feedback gain \((c^i_0)\) implies higher constants \(c^i_0\) in the active joint actuator. The controlled peak response is \(6.22 \times 10^{-5} \text{ m} \) (2.45 mils) at \(c^i_0 = 0.288U(\mu_0)\) and it continues to reduce with time (Figure 11). The peak response reduces to \(6.0 \times 10^{-5} \text{ m} \) (2.37 mils) when \(c^i_0 = 0.288U(\mu_1)\), and it reduces at a faster rate (Figure 12). Both figures show the effectiveness of the active joint actuator. It is shown in Figure 13 that the first contact loads decrease with an increase of \(c^i_0\).
4. SUMMARY AND CONCLUSIONS

A study of the non-linear structural dynamics of jointed flexible structures with initial joint clearances and subjected to external out-of-phase excitations has been presented in this paper. In order to reduce the dynamic contact force and to stabilize the jointed structural system, methods of using passive viscoelastic and active joint actuators were proposed and evaluated. System dynamic equations of a discretized multi-degree-of-freedom flexible system with initial joint clearances and joint actuators were derived. Dynamic contacts in an elastic joint were simulated by a non-linear joint model consisting of a non-linear spring and damper. A pseudo-force approximation method was used in the numerical time-domain integration. Dynamic responses of a jointed flexible structure with and without passive viscoelastic joint actuator or active joint actuator were presented and compared.

Although at low-amplitude oscillation ($\delta_1 < \gamma_1$) the joint gap has very little effect to the structural dynamics, joint contacts can introduce significant dynamic loads when the relative oscillation exceeds the joint gap ($\delta_1 > \gamma_1$), and the large contact loads result in poorer dynamic performance and even system failures. Low-frequency excitation can increase the number of consecutive contacts within a half-cycle of the excitation period.
at lower amplitudes. External excitation of large amplitude can introduce very significant joint contact forces and eventually result in system failure.

Applications of viscoelastic joint actuator were also studied, showing the effectiveness of the viscoelastic joint actuator, which can eliminate joint contacts. Stiffer viscoelastic actuators show better control of the joint dynamics. However, it should be noted that there is always a limit for optimal performance. A general selection guideline is that the viscoelastic actuator should be stiff enough to prevent the joint contact but not too stiff to constrain the relative motion of the flexible members. This can introduce additional external forces to the flexible members. A general design equation was proposed for viscoelastic joint actuator selection.

An active joint actuator was also proposed and its effectiveness on the system dynamics was studied. The active actuator generates a (countering) feedback force based on the relative velocity of the joint. The study showed that, as feedback gains increase, the dynamic contact load reduces at a faster rate, and the joint contact loads also decrease. The study presented was largely a simulation study to illustrate the control effectiveness
of passive viscoelastic and active joint actuators. The techniques need to be further implemented in physical systems for practical applications.

ACKNOWLEDGMENTS

This research was supported, in part, by a grant (NO. 5-AM2) from the Center for Robotics and Manufacturing Systems at the University of Kentucky, Lexington, Kentucky. Computer simulation was carried out by G. C. Wan using a Nonlinear Dynamic Analysis Finite Element Program (NEDAP) originally developed by Tzou.

REFERENCES


APPENDIX A: DESIGN EQUATION FOR VISCOELASTIC JOINT ACTUATORS

The original standard linear model (Figure A1(a)) can be simplified to an equivalent model (Figure A1(b)) with an equivalent stiffness $k'_2$ and an equivalent damping $c'_2$ as described in equations (5a, b). The relative displacement is defined by $z'_2 = z_{-1} - z_n$, and the excitation frequency is $\omega$. 
When estimating the required stiffness \( k_i^v \) and damping \( c_i^v \) for mass \( m_{i+1} \), the transfer function \( T(j\omega) \) can be derived as

\[
T(j\omega) = 1/[(-m_{i+1}\omega^2 + k_i^v) + j\omega c_i^v] = z_i^*/m_{i+1}Z\omega^2,
\]

(A1)

where the variables are defined as before, and \( j = \sqrt{-1} \).

It is desirable, by using the viscoelastic joint actuator, to enforce the condition \( z_i^* = \mu_i^* \) on the joint relative displacement, to minimize the contact load in the joint. The minimum required viscoelastic joint actuator is, at least, strong enough to prevent joint contact: i.e., \( z_i^* = \mu_i^* \). Thus, the design equation can be rewritten as

\[
\mu_i^* = \frac{m_{i+1}Z\omega^2}{(-m_{i+1}\omega^2 + k_i^v) + j\omega c_i^v} \quad \text{or} \quad \mu_i^* = \frac{m_{i+1}Z\omega^2}{[(-m_{i+1}\omega^2 + k_i^v)^2 + (\omega c_i^v)^2]^{1/2}}.
\]

(A2, A3)

The required stiffness and damping of the viscoelastic joint actuator can be estimated by using an iteration technique.

APPENDIX B: NOMENCLATURE

| \( [ ] \) | matrix |
| \( \{ \} \) | vector |
| \( A \) | cross-section area |
| \( c_i \) | equivalent link damping constant |
| \( c_i^v \) | damping function for the \( i \)th active joint actuator |
| \( c_i^j \) | non-linear joint damper for the \( i \)th joint |
| \( c_i^f \) | Equivalent damping function for the \( i \)th viscoelastic joint actuator |
| \( c_i^j^v \) | joint damping coefficient for the \( i \)th viscoelastic joint actuator |
| \( c_i^j^c \) | damping coefficient for the \( i \)th viscoelastic joint actuator |
| \( c_i^e \) | damping coefficient for the \( i \)th active joint actuator |
\[[C]\]^{\varepsilon} + [C]^{\varepsilon} + [C]^{\varepsilon} + [C]^{\varepsilon}

\[[C]^{\varepsilon}\]
damping matrix associated with active joint actuator

\[[C]^{\varepsilon}\]
joint damping matrix

\[[C]^{\varepsilon}\]
linear damping matrix

\[[C]^{\varepsilon}\]
damping matrix associated with viscoelastic joint actuator

\[E I\]
flexural rigidity

\[F_0\]
amplitude of excitation

\{F(t)\}
linear force vector

\{F_c(t)\}
pseudo-force vector derived from active joint actuators

\{F_j(t)\}
non-linear joint force vector

\{F_v(t)\}
pseudo-force vector derived from viscoelastic joint actuators

\[j = \sqrt{-1}\]

\[k_i\]
equivalent link stiffness constant

\[k_i^{\varepsilon}\]
stiffness function for the \(i\)th active joint actuator

\[k_i^{\varepsilon}\]
non-linear joint stiffness for the \(i\)th joint

\[k_i^{\varepsilon}\]
equivalent stiffness function for the \(i\)th viscoelastic joint actuator

\[k_i\]
joint stiffness coefficient for the \(i\)th viscoelastic joint actuator

\[k_i^{\varepsilon}\]
stiffness coefficient for the \(i\)th active joint actuator

\[k_i^{\varepsilon}\]
primary stiffness of viscoelastic joint actuator

\[k_i^{\varepsilon}\]
secondary stiffness of viscoelastic joint actuator

\[[K]\]^{\varepsilon} + [K]^{\varepsilon} + [K]^{\varepsilon} + [K]^{\varepsilon}

\[[K]^{\varepsilon}\]
stiffness matrix associated with active joint actuator

\[[K]^{\varepsilon}\]
joint stiffness matrix

\[[K]^{\varepsilon}\]
linear stiffness matrix

\[[K]^{\varepsilon}\]
stiffness matrix associated with viscoelastic joint actuator

\[m_i\]
equivalent link mass

\[\{U(\gamma)\}\]
unit step function vector

\[z_i\]
displacement of mass \(i\)

\{z\}
displacement vector

\{\dot{z}\}
velocity vector

\{\ddot{z}\}
acceleration vector

\{\delta z\}
relative displacement vector

\{\delta \dot{z}\}
relative velocity vector

\[\rho\]
mass density

\[\mu_i\]
initial joint clearance

\[\mu_i^{\varepsilon}\]
initial clearance for active joint actuator

\[\mu_i^{\varepsilon}\]
initial clearance for viscoelastic joint actuator

\[\Psi_i\]
modal participation factor for the \(i\)th mode

\[\omega\]
extcitation frequency

\[\theta\]
integration constant in the Wilson-\(\theta\) method, \(\theta = 1.4\)