DISTRIBUTED PIEZOELECTRIC SENSOR/ACTUATOR DESIGN
FOR DYNAMIC MEASUREMENT/CONTROL OF
DISTRIBUTED PARAMETER SYSTEMS:
A PIEZOELECTRIC FINITE ELEMENT APPROACH

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Advanced structures with integrated self-monitoring and control capabilities are becoming very important due to the rapid development of "intelligent" mechanical systems and space structures. Since the structures are distributed and flexible in nature, distributed dynamic measurement and active vibration suppression are of importance to their performance. In this paper, a new structure (shell or plate) containing an integrated distributed piezoelectric sensor and actuator is proposed. The distributed piezoelectric sensing layer monitors the structural oscillation due to the direct piezoelectric effect and the distributed actuator layer suppresses the oscillation via the converse piezoelectric effect. For modeling flexibility and versatility, a new piezoelectric finite element with internal degrees of freedom is derived. The performance of a plate model with distributed piezoelectric sensor/actuator is evaluated. Applications to distributed dynamic measurement and control of the advanced structures are also demonstrated.

1. INTRODUCTION

Advanced structures with integrated self-monitoring and control capabilities are increasingly becoming important due to the rapid development of "intelligent" space structures and mechanical systems [1-4]. Since these structures are, in general, distributed and flexible in nature, distributed dynamic measurement and active vibration suppression are essential to their performance. Vibration suppression and control of distributed parameter systems (e.g., plates and shells in this study) always represents a challenge, both in theory and practice. Theoretical development has been constantly advanced in the past 20 years [2, 3, 5-8]. However, due to the limitation of materials and actuator design, practical application of the theory to general distributed parameter systems still needs to be further explored. Besides, in order to control or suppress the undesirable structural oscillation of a distributed parameter system, an accurate measurement of the structural vibration is required. Conventional transducers (such as accelerometers, strain gages, and pressure transducers, etc.) are "discrete" in nature: i.e., measuring the responses at spatially "discrete" locations. Some natural frequencies and mode shapes could be missed if the transducers are placed at nodal modes or lines. Thus, the development of a "distributed" sensor can be essential for new-generation lightweight, high-performance structures. This paper is concerned with thin piezoelectric layers which are coupled with conventional materials and used as distributed sensors and distributed actuators in an intelligent advanced structure design.

The direct piezoelectric effect, a charge/voltage generated by an imposed force/pressure to a piezoelectric, has been widely applied to variety of transducer designs: e.g.,
accelerometers, pressure transducers, etc. [9]. However, the converse piezoelectric effect, and induced stress/strain due to an externally applied voltage/charge, is not that common as compared with the direct effect. Some applications of the converse effect have been made in flexible mirrors [10] and linear translators [11]. Also, Tzou and Gadre have designed a piezoelectric exciter and a vibration isolator [12, 13]. In this paper, the advanced structure (distributed parameter system) is a shell or plate configuration with one piezoelectric layer serving as a distributed sensor and the other layer serving as a distributed actuator. The direct effect is used in distributed sensing and the converse effect in distributed active vibration suppression and control of the advanced structure. Thus, the sensing layer detects the oscillation of the distributed systems and the actuator controls the vibration of the system. The piezoelectric material used in the finite element analysis of the advanced structures is a $\beta$-phase polymeric piezoelectric polyvinylidene fluoride (PVDF). However, the developed piezoelectric finite element can also be used for other types of piezoelectric materials, e.g., piezoceramics.

Polyvinylidene fluoride (PVDF) was initially discovered by Kawai in 1969 [14]. Raw polymeric PVDF ($\alpha$-phase) is an electrical insulator and it does not have any intrinsic piezoelectric properties. If the raw material is polarized during the manufacturing process, PVDF transforms to a $\beta$-phase—a tough and flexible semi-crystalline material and it can be made to strain either in one or two directions in the film plane. Since $\beta$-phase PVDF possesses a strong direct piezoelectric effect, it has been used in many transducer applications: e.g., sonar, medical ultrasonic equipment, robot tactile sensors, acoustic pick-ups, force and strains gages, etc. [9]. Due to its distinct characteristics, such as flexibility, durability, manufacturability, etc., PVDF is an ideal material for the distributed sensing and vibration suppression/control of distributed parameter systems (e.g., beams, plates, shells, etc.).

Application of the flexible piezoelectric PVDF to vibration control and active damping of beam structures have been studied in recent years [15, 16]. Crystalline piezoceramic materials also have been investigated in a longitudinal actuation [17] and a distributed sensor/actuator for beam structures [4, 18]. Tzou applied a PVDF film as an active damper in a flexible structure [19] and as an active vibration isolator and exciter (with Gadre) [12, 13]. A theory of multi-layered shells coupled with the piezoelectric shell actuators has been derived and evaluated by Tzou and Gadre [1]. Distributed active vibration control of a shell coupled with PVDF has been investigated [3, 20], and distributed sensing theory for a shell with PVDF also has been proposed and evaluated [21, 22]. Up to now, research in this area has been primarily focused on experimental and theoretical studies. General piezoelectric finite element development is relatively limited [20, 22]. In general, experimental models are limited by size, cost, noise and many other laboratory unknowns. Theoretical models can be more general, but analytical solutions are restricted to relatively simple geometries and boundary conditions. When the geometry and/or boundary conditions become relatively complicated, difficulties occur with both theoretical and experimental models. Thus, the finite element development becomes very important in modeling and analysis of advanced flexible structures with integrated distributed piezoelectric sensors and/or actuators.

Finite element techniques are very popular and important in many modern engineering designs and analyses. A piezoelectric finite element was developed and applied in piezoceramic transducer designs [23, 24]. However, the derived isoparametric hexahedron and tetrahedral elements are too thick for thin shell/plate applications. In general, the proposed advanced (intelligent) structure is composed of a master shell/plate with a coupled or embedded piezoelectric sensor/actuator. The thickness of the master structure is about two to three orders thicker than that of the piezoelectric layer. It would be very
inefficient and time consuming if the entire structure were to be modeled by hexahedral or tetrahedral solid elements. Thus, the development of a new thin piezoelectric solid finite element would be very important to the modeling and simulation of large flexible distributed systems—shells and plates with distributed piezoelectric sensor/actuator.

In this paper, the development of a “thin” piezoelectric solid element with internal degrees of freedom (DOFs) is presented, and its application to distributed dynamic measurement and active vibration suppression and control of an advanced “intelligent” plate is studied. The distributed sensing phenomena and effectiveness of the piezoelectric actuator are also evaluated. The dynamic equations of a piezoelectric element are first derived by using the piezoelectric constitutive equations [25]. This leads to a formulation of the entire system matrix equation of motion. In order to improve computation efficiency, a Guyan reduction scheme [26] is used to condense the DOFs associated with the electrical potential, which can be recovered if the distributed measurement is desired. The dynamic response of the distributed system is calculated by a direct integration algorithm—the Wilson—θ method and the pseudo-force method [27].

2. FORMULATION OF PIEZOELECTRIC ELEMENT AND SYSTEM EQUATION

In this section, a thin piezoelectric finite solid element with internal DOFs is derived by using a variational method and Hamilton’s principle. The system matrix equation is also formulated by assembling all of the element matrices.

2.1. DIRECT AND CONVERSE PIEZOELECTRIC EFFECTS

It is assumed that the mechanical and the electrical forces in an oscillating piezoelectric are balanced at any given time instant. Thus, the piezoelectric equations can be decoupled: i.e., a quasi-static approximation is used in the analysis. It is also assumed that the temperature variation is negligible in a fast oscillating piezoelectric: i.e., the pyroelectric effect is not considered in the analysis. The linear piezoelectric constitutive equations coupling the elastic field and the electric field can be respectively expressed as the direct and the converse piezoelectric equations [25] (a list of nomenclature is given in the Appendix):

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\
e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\
e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36}
\end{bmatrix}
\begin{bmatrix}
S_{11} \\
S_{22} \\
S_{33} \\
S_{12} \\
S_{23} \\
S_{31}
\end{bmatrix}
+ \begin{bmatrix}
e_{11} & e_{12} & e_{13} \\
e_{21} & e_{22} & e_{23} \\
e_{31} & e_{32} & e_{33}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]  \quad (1)

\[
\begin{bmatrix}
T_{11} \\
T_{22} \\
T_{33} \\
T_{12}
\end{bmatrix} = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
c_{33} & c_{32} & c_{34} & c_{35} & c_{36} \\
c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66}
\end{bmatrix}
\begin{bmatrix}
S_{11} \\
S_{22} \\
S_{33} \\
S_{12}
\end{bmatrix}
- \begin{bmatrix}
e_{11} & e_{21} & e_{31} \\
e_{12} & e_{22} & e_{32} \\
e_{13} & e_{23} & e_{33}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]  \quad (2)

Equation (1) describes the direct effect and equation (2) the converse effect. A piezoelectric element defining all mechanical and electrical variables is illustrated in Figure 1.

Equations (1) and (2) can be written simply as

\[
\{D\} = [e]^T[S] + [e^E]^T[E], \quad \{T\} = [e^E]^T[S] - [e]^T[E],
\]  \quad (3, 4)
where \( \{D\} \) is the electric displacement vector, \([e]\) is the dielectric permittivity matrix, \([e]\) is the transpose of \([e]\), \([S]\) is the strain vector, \([e^T]\) is the dielectric matrix at constant mechanical strain, \([E]\) is the electric field vector, \([T]\) is the stress vector, and \([c^T]\) is the elasticity matrix for a constant electric field.

2.2. A NEW PIEZOELECTRIC FINITE ELEMENT FORMULATION

The Lagrangian \( \mathcal{L} \) of a bounded piezoelectric body is defined by the summation of all kinetic energy \( \mathcal{F} \) and potential energy \( \mathcal{U} \) (including strain and electrical energies):

\[
\mathcal{L} = \int_{\mathcal{V}} (\mathcal{F} - \mathcal{U}) \, d\mathcal{V} = \int_{\mathcal{V}} \left( \frac{1}{2} \rho(\dot{q}) \{q\} - \frac{1}{2} \{S\}'\{T\}' - \{E\}'\{D\} \right) \, d\mathcal{V}. \tag{5}
\]

Here \( q \) is the velocity (time derivative of the displacement \( q \)), \( \mathcal{L} \) is the Lagrangian, and \( \mathcal{V} \) is the piezoelectric volume. The virtual work \( \delta W \) done by the external forces and the applied surface charge \( \sigma \) is

\[
\delta W = \int_{\mathcal{V}} \{q\}' \{P_b\} \, d\mathcal{V} + \int_{\mathcal{S}_1} \{q\}' \{P_1\} \, d\mathcal{S} + \{\delta q\}' \{P_1\} \, d\mathcal{S}_1 + \{\delta q\}' \{P_1\} \, d\mathcal{S}_2 - \int_{\mathcal{S}_2} \delta \phi \sigma \, d\mathcal{S}_2. \tag{6}
\]

where \( \{P_b\} \) is the body force, \( \mathcal{S}_1 \) the surface area, \( \{P_1\} \) the surface force, \( \{P_1\} \) the concentrated load, and \( \sigma \) the surface charge. Based on the Lagrangian and the virtual work defined above, the dynamic equations of a piezoelectric structure can be derived by using Hamilton’s principle,

\[
\int_{t_1}^{t_2} \delta (\mathcal{L} + W) \, dt = 0, \tag{7}
\]

where \( t_1 \) and \( t_2 \) define the time interval, and all variations must vanish at \( t = t_1 \) and \( t = t_2 \). Thus, substituting equations (5) and (6) into equation (7) yields the variational equation as

\[
\int_{\mathcal{V}} \left[ \rho \{\delta q\}' \{q\} - \{\delta S\}'\{c\}'\{S\} + \{\delta S\}'\{e\}'\{E\} - \{\delta E\}'\{e\}'\{S\} \right. \\
- \{\delta E\}'\{e\}'\{E\} + \{\delta q\}' \{P_b\} \right] \, d\mathcal{V} + \int_{\mathcal{S}_1} \{\delta q\}' \{P_1\} \, d\mathcal{S}_1 + \int_{\mathcal{S}_2} \delta \phi \sigma \, d\mathcal{S}_2 + \{\delta q\}' \{P_1\} = 0. \tag{8}
\]
potential vector is usually condensed in the time domain integration. However, a recovery scheme can be set up if the sensing information is required.

Consider the static case in equation (31): i.e.,

\[
\begin{bmatrix}
[K_{qq}] & [K_{q\phi}] \\
[K_{q\phi}] & [K_{\phi\phi}] \\
\end{bmatrix}
\begin{bmatrix}
\{q\} \\
\{\phi\} \\
\end{bmatrix}
=
\begin{bmatrix}
\{F\} \\
\{G\} \\
\end{bmatrix}
\]

(32)

A congruent transformation matrix \([T_c]\) can be calculated as

\[
[T_c] = \begin{bmatrix} [I], -[K_{\phi\phi}]^{-1} \end{bmatrix}.
\]

(33)

After performing the condensation of the \(\{\phi\}\) DOFs, the system dynamic equation is written as

\[
[M_{qq}]{\ddot{q}} + [C_{qq}]{\dot{q}} + [K^*]{q} = \{F\} - [K_{qq}][K_{\phi\phi}]^{-1}\{G\},
\]

(34)

where

\[
[K^*] = [K_{qq}] - [K_{qq}][K_{\phi\phi}]^{-1}[K_{q\phi}].
\]

(35)

In equation (34), there are two excitation forces associated with the piezoelectric structures: i.e., the mechanical forces and the electrical forces. The electrical potential vector can be recovered by

\[
\{\phi\} = [K_{\phi\phi}]^{-1}(-[K_{qq}]{q}).
\]

(36a)

The system dynamics is governed by equation (34) and the distributed dynamic measurement (voltage distribution) can be calculated by using equation (36). In the free vibration analysis, \(\{G\}\) is set to zero so that the voltage distribution associated with each mode can be estimated. Note that \(\{G\}\) is usually zero in the distributed sensor layer. Thus, the distributed sensor output is estimated by

\[
\{\phi\} = [K_{\phi\phi}]^{-1}([-K_{qq}]{q}).
\]

(36b)

4. DISTRIBUTED DYNAMIC MEASUREMENT AND ACTIVE VIBRATION CONTROL

The basic configuration of an “intelligent” structure is composed of a master structure sandwiched between two piezoelectric thin layers acting as the distributed sensor and the actuator, respectively. A plate with the above configuration, in which the bottom layer is acting as a distributed sensor and the top as a distributed actuator, is illustrated in Figure 2 (the specified dimension is for a case study presented later).

![Figure 2](image_url)

Figure 2. An integrated distributed piezoelectric sensor/actuator design. Plate dimensions for calculations: 10 cm x 10 cm x 0.31 cm.
The distributed sensor generates a voltage output when the structure is oscillating; and this signal is amplified and fed back into the distributed actuator. The micro-structural action of the piezoelectric actuator, in which a negative voltage introduces positive strains in the actuator resulting in a moment counteracting the upward motion of the structure, is shown in Figure 3. Thus, the control law is established such that the moment generated will oppose the motion in the transverse direction. Hence, a negative voltage should be applied when the transverse motion is upward and vice versa.

![Figure 3. Vibration suppression by the distributed piezoelectric actuator.](image)

As discussed earlier, there are two force terms in the system equations of motion: i.e., the mechanical force component and the electrical force component as indicated in equation (34). In the feedback control, the electrical force component, the second term in equation (34), can be regarded as a feedback force \( \{F_j\} \),

\[
\{F_j\} = [K_{ee}][K_{eo}]^{-1}\{G\}. \quad (37a)
\]

Note that \( \{G\} \) is a function of the feedback voltage in terms of the output signal from the distributed sensing layer as described in equation (36). Thus, the feedback force can be written as

\[
\{F_j\} = [K_{ee}][K_{eo}]^{-1}[C][K_{eo}]^{-1}(-[K_{eo}]]\{q\}), \quad (37b)
\]

where \([C]\) is a gain matrix. Substituting into the dynamic system equation yields

\[
[M_{qq}][\ddot{q}] + [C_{qq}][\dot{q}] + [K^*][q] = \{F\} + ([K_{eo}][K_{eo}]^{-1}[C][K_{eo}]^{-1}[K_{eo}]]\{q\}). \quad (38)
\]

Since only velocity feedback is considered in this study, an equivalent damping force can be defined as

\[
[C_j][\dot{q}] = (\partial/\partial t)(K_{eo})^{-1}[C][K_{eo}]^{-1}(-[K_{eo}]]\{q\})
\]

\[
= (-[K_{eo}][K_{eo}]^{-1}[C][K_{eo}]^{-1}[K_{eo}]]\{\dot{q}\}). \quad (39)
\]

Thus, the system equation of motion becomes \([M_{qq}][\ddot{q}] + ([C_{qq}]+[C_j])[\dot{q}] + [K^*][q] = \{F\} \), which implies that

\[
[M_{qq}][\ddot{q}] + ([C_{qq}]+[C_j])[-[K_{eo}][K_{eo}]^{-1}[C][K_{eo}]^{-1}[K_{eo}]]\{q\}) + [K^*][q] = \{F\}. \quad (40)
\]
The control force induced by the feedback surface voltage/charge can effectively enhance the system damping and therefore suppress the vibration of a distributed system. As discussed in the finite element formulation, the feedback control forces are assumed to be proportional to velocity (see equation (39)). Thus, two different feedback control algorithms are evaluated: (1) constant-gain feedback control, and (2) constant-amplitude feedback control. The feedback voltages for these two feedback control algorithms are shown in Figure 4.

![Figure 4. Feedback voltages of the constant-gain and constant-amplitude control algorithms.](image)

4.1. CONSTANT-GAIN FEEDBACK CONTROL

In the first case, the feedback gain is constant while the feedback amplitude varies with respect to the negative oscillating velocity (negative velocity, constant-gain feedback control): i.e.,

\[
\{G\} = -[C]^* \frac{\partial \{q\}}{\partial t} = -[C]^* \{q\},
\]

where \([C]^*([-K_{dd}][K_{dd}]^{-1}[C][K_{dd}]^{-1}[K_{dd}])\) is a modified feedback gain matrix.

4.2. CONSTANT-AMPLITUDE FEEDBACK CONTROL

In the second case—negative velocity, constant-amplitude feedback control—the feedback amplitude is constant and the sign is opposite to the velocity; see Figure 3. Only the sign changes when the vibration changes its direction: i.e.,

\[
\{G\} = -[C]^* \text{sign} \frac{\partial \{q\}}{\partial t} = -[C]^* \text{sign} \{\{q\}\},
\]

where \text{sign}\ is the sign function,

\[
\text{sign} \{x\} = \begin{cases} 
-1 & \text{if } x < 0 \\
0 & \text{if } x = 0 \\
+1 & \text{if } x > 0
\end{cases}.
\]

In the finite element analysis, the time history responses of the piezoelectric system are calculated by using a time-domain direct integration algorithm, the modified Wilson-θ method, and a pseudo-force method [27] to accommodate the control force derived from the applied surface changes.
DISTRIBUTED SENSOR/ACTUATOR DESIGN

5. DYNAMIC EVALUATION OF AN INTEGRATED DISTRIBUTED SENSOR/ACTUATOR DESIGN

In this section, the dynamic characteristics of a plate with surface coupled distributed piezoelectric sensor and actuator are evaluated. The distributed dynamic measurement of the first three modes is demonstrated by an eigenvalue analysis, in which the mode shapes and associated voltage distributions are obtained. Evidence of the effectiveness of the distributed active vibration suppression and control is also presented.

5.1. MODEL DEFINITION

A Plexiglas cantilever plate (10 cm x 10 cm x 0.31 cm) with a distributed piezoelectric PVDF layer (40 μm) serving as a distributed actuator on the top surface, and another PVDF on the bottom surface as a distributed sensor, was used as a case study. The plate with the distributed sensor/actuator was divided into 75 elements, 25 for each layer, modeled by the piezoelectric finite elements developed earlier (see Figure 5). The material properties of the piezoelectric PVDF polymer are summarized in Table 1. It should be noted that some assumptions were made in the finite element analysis: (1) the resistance of deposited metal electrode on each side of the PVDF was assumed negligible, so that the applied voltage could quickly reach a steady state; (2) the expanded or contracted

![Figure 5. Finite element modeling of a plate with distributed piezoelectric sensor/actuator.](image)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Material properties of the piezoelectric PVDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
<td>PVDF</td>
</tr>
<tr>
<td>Dielectric permittivity</td>
<td></td>
</tr>
<tr>
<td>(\varepsilon_{31})</td>
<td>0.0460</td>
</tr>
<tr>
<td>(\varepsilon_{32})</td>
<td>0.0460</td>
</tr>
<tr>
<td>(\varepsilon_{33})</td>
<td>0.0000</td>
</tr>
<tr>
<td>Dielectricity</td>
<td></td>
</tr>
<tr>
<td>(\varepsilon_{31})</td>
<td>0.1062 x 10^{-9}</td>
</tr>
<tr>
<td>(\varepsilon_{22})</td>
<td>0.1062 x 10^{-9}</td>
</tr>
<tr>
<td>(\varepsilon_{33})</td>
<td>0.1062 x 10^{-9}</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.2900</td>
</tr>
<tr>
<td>Mass density</td>
<td>0.1800 x 10^4</td>
</tr>
<tr>
<td>Modulus</td>
<td>0.2000 x 10^{10}</td>
</tr>
</tbody>
</table>
strains settle very fast without time delay; (3) the velocity information can be obtained
and instantaneously used as a feedback signal.

As discussed earlier, the distributed piezoelectric sensing layer should respond to the
plate oscillation and generate an electric voltage representing the distributed dynamic
response of the plate. This distributed sensing phenomena are demonstrated in an
eigenvalue analysis. The active distributed vibration suppression and control of the plate
is studied and evaluated in a snap-back analysis in which an initial displacement (first
mode) was imposed.

5.2. MODAL VOLTAGE DISTRIBUTION: DISTRIBUTED SENSING PHENOMENA

In equation (36), the output signals of each node on the distributed piezoelectric sensor
layer can be calculated as a function of the displacements. (Note that the \( \{ G \} \) vector is
zero in an eigenvalue analysis.) After the nodal voltage is calculated, the overall voltage
distribution of the plate can be plotted by connecting all nodal voltage amplitudes. Thus,
for a given mode, the modal voltage distribution (distributed sensing phenomena) can
be observed. The first three plate mode shapes and modal voltage distributions are
illustrated in Figures 6–8.

![Figure 6. First mode shape and modal voltage distribution.](image)

It is observed that the first mode is a bending mode, the second mode a torsion mode,
and the third mode a warping mode. According to equation (36), the output amplitude
is inversely proportional to the displacement; i.e., the maximum positive voltage occurs
at the maximum negative displacement. Voltage drops at two corners, \( V(0, 0) \) and
\( V(0, 100) \), are observed in all three figures. This is because that the strains are (numerically)
smaller at these two boundary nodes in the finite element calculation. The sensitivity of
each mode could also change because the tension and compression vary in different
modes. The modal voltage distribution could also be refined if more elements and a finer
mesh were used in the finite element modeling.

5.3. ACTIVE DISTRIBUTED VIBRATION SUPPRESSION AND CONTROL

The distributed piezoelectric actuator on the top surface of the plate contracts or
expands depending on negative or positive feedback voltages (the converse piezoelectric
In general, a positive feedback voltage is needed for an upward (positive) displacement. An initial displacement (first mode) is imposed and then the plate is set free. The vibration amplitude decays depending on the modal damping and the feedbacks. An initial modal damping was assumed to be 0.9% (based on a laboratory experiment), and the damping ratio change was evaluated when comparing the control effectiveness of the distributed piezoelectric actuator with different control algorithms. It should be noted that the tip velocity (node $P$) was used in the feedback controls (since in practical applications this is the simplest approach to implement).
5.3.1. Negative velocity, constant gain feedback control

In this case, the feedback voltage amplitude varies with respect to the negative velocity as shown in equation (41). Since the node P's velocity was used in the feedback, equation (41) is modified as

\[ \{G\} = -[C]^*\{\dot{q}_p\}. \]  \hspace{1cm} (44)

The gain matrix \([C]^*\) is also changed in order to evaluate its control effectiveness. The vibration amplitudes were suppressed by the distributed piezoelectric actuator, and the damping ratio change was calculated and plotted versus feedback gains as shown in Figure 9. This shows that the damping ratio increases when the feedback gain increases.

5.3.2. Negative velocity, constant amplitude feedback control

In the second case, although the feedback amplitude is constant, the feedback voltage changes sign when the nodal velocity \(\dot{q}_p\) changes its direction: i.e.,

\[ \{G\} = -[C]^* \text{ sign } [ \{\dot{q}\}]. \]  \hspace{1cm} (45)

The damping ratio was studied at different feedback voltages as plotted in Figure 10. This shows that the damping ratio also increases as the feedback voltage increases.

On the basis of equations (43), (44) and (40), increasing \([C]\) (or \([C]^*\)) results in a higher damping matrix in the system equation. Thus, the plate oscillation should be

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Figure 9. Damping calculations at negative velocity, constant-gain feedback control.

Figure 10. Damping calculation at constant-amplitude feedback control.
damped out much faster at higher feedback gain, as shown in Figures 9 and 10. Note that a single node velocity was used in the feedback controls, even though the distributed voltage was discussed in the previous section. This single signal was amplified and fed back to all nodes of the actuator.

6. DISCUSSION AND CONCLUSIONS

Advanced "intelligent" structures with integrated sensors, actuators and control electronics are becoming increasingly important in high-performance space structures and mechanical systems. In this paper, an integrated distributed piezoelectric sensor/actuator design for large distributed system (shells and plates) has been proposed and the dynamic characteristics of the system studied and evaluated.

In order to evaluate the dynamic performance of the structure, a finite element technique was used. Because the electrical charge is distributed on both the top and bottom surfaces of a piezoelectric layer, the conventional thin plate/shell elements are difficult, if not impossible, to model these surface characteristics. Besides, conventional isoparametric hexahedron elements are too thick for "thin" plate modeling and analysis. Thus, a new "thin" piezoelectric solid element with internal DOFs was developed by using a variational principle and the dynamic system equation was formulated by using Hamilton's principle. Guyan's reduction scheme was employed to condense the internal DOFs and the "slave" DOFs (with minor physical significance) in order to improve computation efficiency. Electrical potential DOFs were also condensed when performing time-domain integration, and these DOFs could be recovered if the distributed sensing interested. The dynamic responses were calculated by using a modified Wilson-θ method and a pseudo-force method. Note that the developed finite element methodology is general, and can be used to model piezoelectric structures made of PVDF, piezoceramics, etc.

Performance of a thin plate model coupled with the distributed piezoelectric sensor/actuator was studied and evaluated in a free-vibration analysis and a snap-back analysis by using the finite elements discussed above. Distributed dynamic measurement of the plate was demonstrated and the voltage distributions of the first three modes were illustrated. It was observed that the voltage of any node is determined by the local strain: i.e., a higher local strain generates a higher output signal. In reality, however, there would be problems in measuring these voltages in a distributed manner: e.g., the measurements would be limited by data acquisition systems and computers.

Active distributed vibration suppression and control of the plate was also studied by using two control algorithms: (1) constant-gain negative velocity feedback and (2) constant-amplitude negative velocity feedback. The effectiveness of each feedback algorithm was studied via damping evaluation. It was observed that the damping ratio, in general, increases when the feedback voltage/gain increases. Both feedback control algorithms showed positive effects on the control of the cantilever plate. Note that a single point signal was used in the feedback control of the plate—single-input and single-output (SISO). Since the voltage distribution of the distributed piezoelectric sensor can be calculated in the time-domain, the SISO control can be extended to a multi-input and multi-output (MIMO) control with a variable gain matrix. The effectiveness of the MIMO control needs to be investigated further.

It should also be noted that the piezoelectric PVDF used in the distributed sensor/actuator of the advanced structure has a "breakdown" voltage of around 2000 volts. When the feedback voltage exceeds this breakdown voltage, the dipolar molecular structure of the PVDF will be destroyed. Additionally, the temperature variation in the piezoelectric crystal could also affect the overall performance, which is not considered
in this analysis. The performance of other piezoelectric materials, such as PZTs, and the effect of piezoelectric orientation need to be explored further.

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DISTRIBUTED SENSOR/ACTUATOR DESIGN


APPENDIX: NOMENCLATURE

[ ] matrix
{ } vector
\( \ldots \) transpose of vector or matrix
\( [c_{ee}] \) the elasticity matrix evaluated at constant electric field
\( [c_{dc}] \) element damping matrix
\( \mathcal{C} \) feedback gain
\( [\mathcal{C}] \) feedback gain matrix
\( [C] \) system damping matrix
\( [C]_e \) equivalent non-linear damping matrix
\( [D] \) the electric displacement vector
\( [E] \) the electric field vector
\( [e] \) dielectric permittivity matrix
\( [F] \) equivalent non-linear force
\( [g] \) element charge vector
\( [G] \) external applied charge vector
\( H \) electric enthalpy
\( [k_{ee}] \) element stiffness matrix
\( [k_{ee}] \) element piezoelectric stiffness matrix
\( [k_{ee}] \) element dielectric stiffness matrix
\( [k^*] \) condensed system stiffness matrix
\( \mathcal{Z} \) Lagrangian
\( [L] \) elasticity operator
\( [m] \) element consistent mass matrix
\( [M] \) system mass matrix
\( [N]_e \) shape function matrix related to nodal displacement
\( [N]_e \) shape function matrix related to nodal potential energy
\( [P] \) body force vector
\( [P] \) surface force vector
\( [P] \) concentrated load vector
\( [q] \) nodal displacement vector
\( [q] \) displacement vector
\{q\} velocity vector
\{\dot{q}\} acceleration vector
\{S\} the strain tensor
\mathcal{F}_1 surface where surface forces applied
\mathcal{F}_2 surface where surface charges applied
\{T\} the stress tensor
\{T_r\} transformation matrix
\dot{\mathcal{U}} internal energy
\gamma piezoelectric volume
\delta\mathcal{W} virtual work
\alpha,\beta Rayleigh's coefficients
\{s^*\} dielectric matrix evaluated at constant strain
\{\phi_n\} nodal electrical potential vector
\{\phi\} electrical potential vector
\sigma surface charge
\nabla gradient operator