Integrated Distributed Sensing and Active Vibration Suppression of Flexible Manipulators Using Distributed Piezoelectrics

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Structural oscillation of flexible robot manipulators would severely hamper their operation accuracy and precision. This article presents an integrated distributed sensor and active distributed vibration actuator design for elastic or flexible robot structures. The proposed distributed sensor and actuator is a layer, or multilayer, of piezoelectric material directly attached on the flexible component needed to be monitored and controlled. The integrated piezoelectric sensor/actuator can monitor the oscillation as well as actively and directly constrain the undesirable oscillation of the flexible robot manipulators by direct/converse piezoelectric effects, respectively. A general theory on the distributed sensing and active vibration control using the piezoelectric elements is first proposed. An equivalent finite element formulation is also developed. A physical model with distributed sensor/actuator is tested in laboratory; and a finite element model with the piezoelectric actuator is simulated. The distributed sensing and control effectiveness are studied.

INTRODUCTION

Because of the rapid development of industrial automation, high performance robots and manufacturing systems are increasingly in demand today. In ad-
dition, these are required to be lightweight, strong, and capable of high precision and efficiency. Under such conditions, high forces acting on the flexible robot or machine components result in high amplitude oscillations. This leads to uncontrolled vibrations of large amplitudes that are highly destructive. Besides, these problems lead to a drastic reduction in the accuracy and precision of robot and machine operations. One method is to strengthen the structures to withstand these vibrations but this leads to strength and weight requirements far in excess of those demanded by the loading alone. Thus, effective vibration control is crucial under such circumstances. The techniques have to be efficient and should not increase the size and weight of the components substantially.

Vibration control techniques are generally classified into two major categories: (1) the "passive" method in which the impressed force does work in a damper (e.g., automotive shock absorbers) and (2) the "active" method (actuator/controller) in which the effects of the undesirable force are counteracted by an auxiliary mechanism. In an earlier study, it was observed that although the passive damping using rubber materials provides a sharp increase in damping initially, the effects of using over strong rubber damper could reduce its effectiveness and even further increase its dynamic loads. Passive vibration controls of flexible robot manipulators using viscoelastic damping materials were studied. In this article, an "active" distributed control technique counteracting the undesirable oscillations of flexible manipulators using polymeric piezoelectric material is proposed and evaluated.

Effective control always depends on accurate real-time monitoring of the robot's operation. Accurate acceleration and force measurements are required for high precision manipulation and control. In addition, the sensors should be resistant to the environment. In various sensor designs for dynamic measurements and control, piezoelectricity is probably the most widely used electromechanical characteristic. In general, a piezoelectric material responds to mechanical forces and/or pressures and generates an electrical charge. This is called the direct piezoelectric effect. Conversely, application of an electric field to the material can produce mechanical stress or strain, which is called the reciprocal, or converse piezoelectric effect. In this article the direct effect is used for distributed sensing and the converse effect is for the active distributed control of flexible robot manipulators.

Today there are many natural and synthetic piezoelectric materials available. In general, most of them are crystalline solids that are dense, brittle and difficult to fabricate into complex shapes. In the distributed sensing and control applications, a flexible piezoelectric material is required to be closely coupled with the robot or machine components. Polyvinylidene fluoride (PVDF) is a tough and pliant piezoelectric polymer. It can be produced in large sheets and complex geometries. In addition, PVDF can operate at temperatures up to 100°C, has a dynamic sensitivity that runs from about 10^-8 to 10^6 N/m, (a 286 dB range) and its maximum response frequency is in the GHz range. Its dielectric constant is about 100th that of ceramics, making it possible for a film sensor to be much smaller and lighter than a similar ceramic device for a given voltage. All these qualities, along with low production cost have made PVDF
very attractive for distributed sensing and control applications.\textsuperscript{2-4,10,11} Active control effectiveness of piezoceramics were also studied.\textsuperscript{12-14} (However, because the piezoceramics are rigid and brittle, there are limitations for “flexible” robot applications.) An integrated distributed sensing and active vibration controls of flexible robot manipulators is proposed and evaluated in this study.

In this article, a brief review of piezoelectricity theory is presented and a new distributed sensing and control theory is proposed. The utilization of the direct and converse piezoelectric effect for distributed sensing and active vibration control is respectively discussed. A finite element formulation of a flexible manipulator and an equivalent finite element formulation for the distributed actuator are derived. An experimental model and a finite element model are studied.

A DISTRIBUTED SENSING THEORY

In this section, the piezoelectric theory is briefly reviewed. This leads to a development of a distributed sensing theory. Applications of the generic theory to flexible manipulators are also demonstrated. (Note that a piezoelectric polymer is used as an example in the formulation. However, the derived theory is general, which can be applied to other piezoelectric materials.)

Piezoelectricity

In general, the piezoelectricity is an electromechanical phenomenon that couples the electric field with the elasticity. There are two fundamental equations representing the direct and converse piezoelectric effects respectively. The linear piezoelectric constitutive equations can be written as,\textsuperscript{15,16}

\begin{equation}
\{S\} = \{c^{p}\} \cdot \{T\} - \{h\}' \cdot \{D\},
\end{equation}

\begin{equation}
\{E\} = \{\beta^{s}\} \cdot \{D\} - \{h\} \cdot \{S\};
\end{equation}

and

\begin{equation}
\{S\} = \{s^{e}\} \cdot \{T\} + [d]' \cdot \{E\},
\end{equation}

\begin{equation}
\{D\} = \{e^{r}\} \cdot \{E\} + [d] \cdot \{T\};
\end{equation}

where \(\{T\}\) is the stress vector; \(\{c^{p}\}\) is the elasticity matrix evaluated at constant dielectric displacement; \(\{S\}\) is the strain vector; \(\{h\}\) is the piezoelectric constant matrix; \(\{D\}\) is the electric displacement vector; \(\cdot \)' indicates the matrix transpose; \(\{E\}\) is the electric field vector; \(\{\beta^{s}\}\) is the dielectric impermeability matrix evaluated at constant strain; \(\{s^{e}\}\) is the elastic compliance matrix measured at constant electric field; \(\{e^{r}\}\) is the dielectric matrix evaluated at constant strain:

\(\{T\} = [T_{11}, T_{12}, T_{13}, T_{21}, T_{22}, T_{23}, T_{31}, T_{32}]'.\)
Then, the amplified high voltage is injected into the distributed piezoelectric actuator layer generating a feedback force to control the oscillation of distributed systems. This theory will be presented in the next section.

A DISTRIBUTED VIBRATION CONTROL THEORY

In this section, a distributed active vibration control theory is first developed. Application of the theory to distributed vibration control of flexible manipulators is also demonstrated in this section.

An Active Distributed Vibration Control Theory

According to the converse piezoelectric effect, a voltage applied to the distributed piezoelectric actuator layer results in two in-plane strains ($\alpha_1$ and $\alpha_2$ planes) in the layer (Fig. 3). Since the piezoelectric actuator layer is not constrained and free from external in-plane forces, the stress effects are neglected in the analysis. Besides, the applied control voltage $\phi_a$ in the distributed actuator is very large compared with the voltage $\phi$ induced by the direct effect.
Finite Element Modeling of a Flexible Manipulator

In the finite element modeling, a flexible robot manipulator is modeled by flexible beam elements with translation and rotation. For a rotating flexible manipulator, a geometrically nonlinear beam theory is required to include the influence of centrifugal forces on the bending stiffness. The higher-order strains for a flexible rotating beam can be calculated by a successive approximation from the linear beam theory. A single-link flexible manipulator (modeled by a beam element) translating and rotating on a two-dimensional plane and a free-body diagram of a beam segment are illustrated in Figure 4. (Note that \( XZ \) denotes the global coordinate system with origin \( O \) and \( z \) denotes the local coordinate system originated at \( O' \).)

Note that the effects of the shear deformation and the rotary inertia are included in deriving the equation of motion. That is, Timoshenko beam theory is chosen for the study of the elastic deformation. The bending moment and shear force are defined by,\(^{19}\)

\[
V_z = K A G^* \left\{ \theta_s - \frac{du_z}{dx} \right\},
\]

(18)
Figure 6. Distributed sensor output of an impulse response of the flexible beam.

observed that all three modes are monitored and controlled simultaneously via the distributed PVDF actuator.

It is observed that the system oscillation is effectively suppressed by the distributed piezoelectric actuator (Fig. 7). The oscillation amplitudes of the first three modes are all attenuated by the PVDF distributed actuator. Sixty hertz noise and its harmonics are also observed. The responses below 10 Hz were

Figure 7. Frequency responses of the flexible beam measured by the distributed sensor
significantly distorted due to the low frequency limitation of the PVDF polymer itself. As the control gain increases, the control effect also increases. (However, there is breakdown voltage (= 2000 volts), i.e., the PVDF dipole structure will be destroyed.) This control effect is used to estimate the coefficients of the polynomial functions of the control element in the finite element analysis.

Case 2: A Flexible Robot Arm with Distributed Piezoelectric Actuator

In this case, dynamics of the flexible manipulator arm with distributed active piezoelectric actuator was studied and the effectiveness of the piezoelectric actuator was evaluated using the finite element technique. A single-link flexible robot arm (60.96 cm × 2.54 cm × 0.508 cm) made of steel was discretized into eight elements with the first node hinged at the origin for finite element analysis (Fig. 8). The active piezoelectric actuator was modeled by the active control element connecting every two adjacent nodes; and its effectiveness in controlling the transient and steady-state responses was evaluated. The sensing action can be modeled by the internal calculation of displacement, velocity, and/or acceleration in the time-domain integration. The flexible arm was input an angular acceleration $\alpha(t)$ for a duration of 0.3 s followed by a deceleration of the same period. The excitation profile is defined as follows.

**Acceleration phase** ($0 \leq t \leq 0.3\ s$)

\[
\theta(t) = (1/0.3)\left[(t^2/2) + (0.3/2\pi)^2(\cos(2\pi t/0.3) - 1)\right] \text{ rad},
\]

\[
\omega(t) = (1/0.3)\left[1 - (0.3/2\pi)\cdot\sin(2\pi t/0.03)\right] \text{ rad/s},
\]

\[
\alpha(t) = (1/0.3)[1 - \cos(2\pi t/0.3)] \text{ rad/s}^2;
\]

**Deceleration phase** ($0.3 < t \leq 0.6\ s$)

\[
\dot{\theta}(t) = 0.3 - (1/0.3)\left[(0.6 - t)^2/2 + (0.3/2\pi)^2\cdot[\cos(2\pi(0.6 - t)/0.3) - 1]\right] \text{ rad}.
\]

![Figure 8. A flexible arm with distributed piezoelectric actuator.](image-url)
\[
\omega(t) = (1/0.3)((0.6 - t) - (0.3/2\pi) \cdot \sin[2\pi(0.6 - t)/0.03]) \text{ rad/s},
\]
(42)
\[
\alpha(t) = -(1/0.3)[1 - \cos[2\pi(0.6 - t)/0.3]] \text{ rad/s}^2;
\]
(43)

Stop (t \geq 0.6 \text{ s})

\[
\theta(t) = 0.3 \text{ rad},
\]
(44)
\[
\omega(t) = 0.0 \text{ rad/s},
\]
(45)
\[
\alpha(t) = 0.0 \text{ rad/s}^2.
\]
(46)

In the finite element model, the distributed actuator (control) element was assumed to have a damping function \( \Lambda_n = C^* \cdot G \cdot S(0) \text{ lb-s/in.} \), where \( C^* \) is an electromechanical constant experimentally determined for the distributed actuator. \( G \) is the feedback gain constant and \( S(0) \) is the unit step function as discussed before. \( S(0) \) indicates that the damping function activates all the time. \( C^* \) is the moment about the neutral axis of the two layered beam per unit feedback volt, \( V_{\text{feedback}} \). The feedback voltage induces a longitudinal strain in the piezoelectric actuator layer. This in turn produces a moment about the resultant neutral axis of the flexible arm that counteracts the arm motion. This effect manifests itself as an increase in the measured damping ratio.

The feedback voltage is directly proportional to the feedback gain constant, \( G \). In the experimental setup, the following relation applies: \( V_{\text{feedback}} = G \times \text{velocity (velocity feedback)} \). This same concept was applied in implementing the finite element analysis. In the NDAFEPA, we have the nonlinear control element terms as polynomials of \( \{SU\} \). The control element is assumed to have a damping function \( \Lambda_n = C^* \cdot G \cdot S(0) \text{ lb-s/in.} \), where \( C^* \) is a constant for a specific distributed actuator design. These parameters were estimated from the laboratory experiments.

In this case, two different values of the active damping coefficients, \( \Lambda_n = 0.03 \) and \( 0.12 \) were investigated, respectively. Higher order damping coefficients were all set zero. All stiffness coefficients were also set zero, i.e., \( \Gamma_n = 0 \). Figure 9 shows two cases: (a) \( \Lambda_n = 0.03 \) and (b) \( \Lambda_n = 0.12 \).

Figure 9 clearly shows the steady state vibration was controlled efficiently. However, the maximum transient response did not change much. The damping ratios of the system were also estimated and plotted with respect to the damping coefficient \( \Lambda_n \) as shown in Figure 10. It is observed that increasing the feedback gain of the distributed piezoelectric actuator can increase the damping ratio of the flexible system. The steady-state response can be effectively controlled.

Higher order terms of the damping coefficients \( \{\Lambda_n\} \) were also evaluated. It did not show much change in the responses. This could be the relative velocity of two adjacent nodes is relatively small so that they did not contribute much to the vibration control of the system.
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**NOMENCLATURE**

- $\{\}$: Vector
- $[\ ]$: Matrix
- $a_x, a_z$: Accelerations
- $A$: Cross-section area
- $A_1, A_2$: Lame's parameters
- $A_s$: Sensor surface area
- $C^*$: Electromechanical constant
- $[C]$: Linear damping matrix
- $[C]_r$: Coriolis matrix
- $d_m^a$: Distance measured from the neutral surface to the actuator mid-surface
- $d_s^i$: Distance measured from the neutral surface to the sensor mid-surface
- $d_{ij}$: Piezoelectric constant
- $[d]$: Piezoelectric matrix
- $D_i$: Dielectric displacement
- $\{D\}$: Dielectric displacement vector
- $E_i$: Electric field
- $\{E\}$: Electric field vector
- $F_x, F_z$: Force resultants
- $f_n$: Natural frequency
- $\{F(i)\}$: Linear force vector
- $\{F_d(i)\}$: Pseudo force vector derived from the actuator
- $\{F_r(i)\}$: Pseudo force vector derived from the viscoelastic damper
- $G$: Feedback gain of active damper
- $G^*$: Shear modulus
- $h_r$: Piezoelectric sensor thickness
- $h^a$: Piezoelectric actuator thickness
- $h_{ii}$: Piezoelectric constant
- $[h]$: Piezoelectric matrix
- $I$: Area moment of inertia
- $K$: Timoshenko shear coefficient
- $[K]$: Linear stiffness matrix
- $M_i$: Moment resultant
- $[M]$: Mass matrix
- $p_1$: Damping coefficient
- $P_x$: Longitudinal force
- $P_z$: Transversal force
- $q_i$: Stiffness coefficient
- $R_i$: Radius of curvature
- $S(\ )$: Unit step function vector
[\mathbf{s}] \quad \text{Elastic compliance matrix}
\mathbf{S}_{ij} \quad \text{Strains}
\{\mathbf{s}\} \quad \text{Strain vector}
\{\mathbf{T}\} \quad \text{Stresses}
\{\mathbf{T}\} \quad \text{Stress vector}
\mathbf{u}_i \quad \text{Displacement in } i\text{th direction } (i = 1, 2, 3)
\{\mathbf{U}\} \quad \text{Displacement vector}
\{\mathbf{U}\} \quad \text{Velocity vector}
\{\mathbf{U}\} \quad \text{Acceleration vector}
\{\delta \mathbf{U}\} \quad \text{Relative displacement vector}
\{\delta \mathbf{U}\} \quad \text{Relative velocity vector}
\mathbf{V}_i \quad \text{Shear force}
Y_\sigma \text{ or } Y^* \quad \text{Young's modulus}
y_i \quad \text{Amplitude of } i\text{th peak}

\alpha \quad \text{Angular acceleration}
\alpha_1, \alpha_2, \alpha_3 \quad \text{Curvilinear coordinates}
\beta_{ij} \quad \text{Dielectric permeability constants}
\{\delta\} \quad \text{Dielectric permeability matrix}
\epsilon_{ij} \quad \text{Dielectric constants}
\{\mathbf{e}\} \quad \text{Dielectric matrix}
\varphi \quad \text{Electric potential}
\varphi_s \quad \text{Sensor output}
\varphi_v \quad \text{Feedback voltage to actuator}
\Lambda_{ij} \quad \text{Damping function of the actuator element}
[\Lambda_{ij}] \quad \text{Damping matrix of the actuator element}
\rho \quad \text{Mass density}
\Gamma_{ij} \quad \text{Stiffness function of the actuator element}
[\Gamma_{ij}] \quad \text{Stiffness matrix of the actuator element}
\lambda, \xi \quad \text{Initial velocity constraint}
\varphi, \psi \quad \text{Initial displacement constraint}
\omega \quad \text{Excitation frequency}
\omega_n \quad \text{Natural frequency}
\omega_i \quad \text{Natural frequency of the } i\text{th mode}
\theta \quad \text{Angular displacement}
\theta_0 \quad \text{Bending slope}
\zeta \quad \text{Damping ratio}

References