DISTRIBUTED SENSING AND FEEDBACK CONTROLS OF DISTRIBUTED PARAMETER SYSTEMS

Horn S. TZOU

Department of Mechanical Engineering
University of Kentucky
Lexington, Kentucky 40506-0046 U.S.A.

Distributed sensing and control of large flexible structures have drawn much attention in recent years. This paper presents a development of a thin piezoelectric finite element applied to sensing and control of distributed parameter systems, e.g., large flexible shells, plates, etc. The piezoelectric element coupled with a distributed system can act like a distributed sensor monitoring the system via the direct piezoelectric effect; and like a distributed actuator controlling the system oscillation by the converse piezoelectric effect. System equations of a piezoelectric structure are formulated using Hamilton's principle. Feedback control algorithms are also implemented to control the vibration of distributed parameter systems. A finite element program is developed and a physical system is studied. Distributed sensing phenomena and effectiveness of the distributed actuator are also presented.

1. INTRODUCTION

In recent years, sensing and control of distributed parameter systems have steadily increased their importance due to the rapid development of large flexible space structures and flexible mechanical systems. Conventional sensors, such as accelerometer and pressure transducer, are "discrete" in nature. They usually measure the responses of "discrete" locations. Some natural frequencies and modeshapes could be missed if the transducers are placed at modal nodes or lines. Thus, development of a "distributed" sensor can be essential for new light-weight high-performance structures. Control of distributed parameter systems also represents a challenge, both in theory and practice. Theories on the control of distributed parameter systems have been advanced and received an increasing attention in recent years (Butkovskii, 1962; Wang, 1966; Sakawa, 1966; Lions, 1968; Robinson, 1971; Brichinin, 1973; Tzafestas, 1970; and Burke & Hubbard, 1987). However, the difficulties still exist especially in practical applications.

Piezoelectric materials have long been used in many transducer designs and applications. Usually, the "direct" piezoelectric effect, a voltage/charge generated by an applied force/pressure, is used in transducer applications. Conversely, applying a voltage to the material can also induce stress/strain. This is referred to as the "converse" piezoelectric effect. In this paper, the direct effect is used in distributed sensing and the converse effect is used for distributed active vibration control. The

† Also Research Faculty, Center for Robotics and Manufacturing Systems, U.K.
generic distributed parameter system is a shell or plate with one layer of piezoelectric material as a distributed sensor and the other layer serving as a distributed actuator. The sensing layer detects the oscillation of the distributed system; and the actuator controls the vibration of the system. The piezoelectric material used in this research is a polymeric piezoelectric polyvinylidene fluoride (PVDF) $(\beta$-phase).

The PVDF was initially discovered by Kawai in 1969. The original raw polymeric PVDF $(\alpha$-phase), like all other piezoelectric materials, is an electrical insulator. If polarized during the manufacturing process, the PVDF becomes a tough and flexible piezoelectric semi-crystal $(\beta$-phase), and it can be made to strain either in one or two directions in the film plane. The strong direct piezoelectric effect has been applied to transducer designs for sonar, medical ultrasonic equipment, robotic tactile sensors, acoustic pickups, force and strain gauges, etc. (Tzou and Pandita, 1987). Due to its distinct characteristics, such as flexibility, durability, manufacturability, etc., the PVDF also becomes a potential ideal material for distributed sensing and distributed vibration control applications (Tzou, 1987, 1988, Tzou & Gadre, 1988; Bally, 1987, Tzou & Zhong, 1988; Tzou & Tseng, 1987–88, b,c). Tzou applied a PVDF film as an active damper in a flexible structure (1987) and an active vibration isolator and exciter (1987 ) and Tzou et al. (1988) developed active vibration control of a shell coupled with the PVDF was studied (Tzou and Tseng, 1987–88, b,c). Distributed sensing theory for a shell element using the PVDF polymer was also proposed and evaluated (Tzou & Zhong, 1988; Tzou & Tzou, 1988–88). Bally also used a piezoelectric film to enhance the damping ratio of a cantilever beam. In general, theoretical solutions are only available for relatively simple geometries and boundary conditions. When the geometry and/or boundary conditions become a little bit more complicated, the analytical solutions are very difficult to find. Thus, a generalized finite element development is definitely needed to model the distributed parameter systems coupled with the distributed piezoelectric PVDF sensors and/or actuators.

A finite element technique was used to study the sensing effect of piezoceramic transducers (Allik & Hughes, 1979; Nallon et al., 1983). However, the derived elements are too thick for thin shell/plate applications. Because the master structure is about two to three order thicker than the PVDF layer, it would be very inefficient if the tetrahedral element is used. Thus, a thin piezoelectric finite element is developed for distributed sensing and control of large flexible distributed systems. The dynamic equations of a piezoelectric element are formulated according to the piezoelectric constitutive equations (Tiensten, 1969); then the whole composite shell structure is assembled to form the dynamic system equation. Finally, the degrees of freedom associated with the electrical potential are condensed in order to save memory storage and computing time. The electrical potential energy can be recovered later through the system equation derived.

The dynamic response of the distributed system is calculated by a direct integration algorithm — the Wilson-$\theta$ method and the pseudo-force method (Tzou and Schiiff, 1987) to accommodate the equivalent forces derived from the external stress and applied voltage. A finite element program is developed based on the methodology stated above. The program is capable of solving the electromechanics (multi-field) problems and the distributed sensing and active vibration control analysis of shells and plates. An example on the distributed sensing and active vibration controls of a plate is presented in this paper. Distributed sensing phenomena and control effectiveness are also studied.

2. FINITE ELEMENT FORMULATION

The linear constitutive relationship coupling the elastic field and the electric field can be written as (IEEE, 1978).

\[
\begin{align}
\{T\} &= \{c_E\}^{T}\{S\} + \{e\}\{E\}, \\
\{D\} &= \{e\}^{T}\{S\} + \{c_E\}\{E\},
\end{align}
\]

where $(T)$ is the stress tensor; $(D)$ is the electric displacement vector; $(S)$ is the strain tensor; $(E)$ is the electric field vector; $(c_E)$ is the elasticity tensor at constant electric field; $(e)$ is the dielectric permittivity tensor; $(T)$ is a matrix transpose; $(D)$ is the dielectric tensor at constant mechanical strain. The piezoelectric equations are formulated provided that both mechanical and electrical forces are instantaneously balanced and can be decoupled. Namely, a quasi-static approximation is assumed. Here, the pyroelectric effect is not considered because for a fast vibrating crystal the temperature variation is assumed to be negligible.

For an electromechanical medium, the electric energy $W$ takes the place of the internal energy $U$ in a Lagrange equation. Hence, the Lagrangian $L$ for this bounded piezoelectric medium is defined by

\[
L = \int_{V} \left[ -\frac{1}{2}\rho\{\dot{u}\}^T\{\ddot{u}\} - \frac{1}{2}\{\dot{S}\}^T\{\ddot{S}\} - \{\dot{E}\}^T\{\ddot{E}\} \right] \, dV,
\]

where $V$ is the PVDF body and $\rho$ is the density. The virtual work $\delta W$ done by the external forces $F_x$ and the prescribed surface charge $q$ is

\[
\delta W = \int_{V} \{\dot{u}\}^T\{\ddot{u}\}dV + \int_{S_1} \{\dot{u}\}^T\{\ddot{u}\}dS_1 + \{\dot{u}\}^T\{\ddot{u}\}dS_2 - \int_{S_2} \delta q \cdot dS_2,
\]

where $(P_x)$ is the body force, $(P_f)$ the surface force, $(P_c)$ the concentrated load, and $\sigma$ the surface charge. The equations which define the behavior of a piezoelectric structure can be derived using Hamilton's principle

\[
\int_{t_1}^{t_2} (L + W) dt_1 = 0,
\]

where $t_1$ and $t_2$ are integral time interval; and all variations must vanish at $t = t_1$ and $t = t_2$. Hence, from Eqs. (2) and (3), the resultant variation principle can be rewritten as

\[
\int_{V} \left[ \{\dot{u}\}^T\{\ddot{u}\} - \{\dot{S}\}^T\{\ddot{S}\} + \{\dot{E}\}^T\{\ddot{E}\} \right] dV + \int_{S_1} \{\dot{u}\}^T\{\dot{u}\}dS_1 - \int_{S_2} \delta \sigma \cdot dS_2 + \{\dot{u}\}^T\{\dot{P_f}\} = 0.
\]

To generate the electromechanical relationship for a finite element, the displacement vector $\{u\}$ and electric potential $\phi$ are expressed in terms of nodal
5. DISCUSSION AND CONCLUSIONS

In this paper, a thin piezoelectric finite element was developed for the distributed sensing and active control of distributed parameter systems. The variational principle and finite element discretization techniques were used to derive the dynamic system equations. Then, Guyan's reduction scheme was employed to condense the degrees of freedom associated with the electrical potential. This can be recovered in a secondary computation if distributed sensing phenomena interested. In order to apply the piezoelectric PVDF film (3-phases) as a distributed actuator, two control algorithm were implemented. A finite element program capable of simulating the distributed sensing and control was developed according to the theory discussed above.

A thin square plate with a distributed PVDF sensor on the bottom surface and a distributed actuator on the top was studied in the finite element analysis. Distributed sensing phenomena for the first three modes were evaluated. In general, the voltage amplitude depends on the local strains; and connecting the nodal voltages gives the overall voltage distribution of the square plate.

Distributed active vibration control of the plate was also studied using the two control algorithms developed in this paper, namely, the constant amplitude feedback and the constant gain feedback. Both feedback controls showed positive effects and increased the system damping ratios. In general, a higher feedback voltage, regardless the control algorithm, can introduce higher control force suppressing the vibrations. However, there is a "breakdown" voltage for the piezoelectric PVDF, i.e., 2000 volts. Beyond this point, the dipolar molecular structure will be destroyed. That is the piezoelectricity associated with the PVDF will vanish and the material becomes useless in both the distributed sensing and control applications.

ACKNOWLEDGEMENT

The open-access policy on an IBM-3090-309E supercomputer, where the analysis was carried out, at the University of Kentucky is greatly acknowledged. This research was also supported, in part, by a grant from the National Science Foundation (No. EEE-8610671) and the Kentucky EPSCOR program: and a seed grant (No. 6-AG1) from the Center for Robotics and Manufacturing Systems at the University of Kentucky.

REFERENCES


