Active Vibration Isolation by Polymeric Piezoelectret with Variable Feedback Gains

Horn S. Tzou* and Malind Gadre†
University of Kentucky, Lexington, Kentucky

ACTIVE vibration involves the cancellation of unwanted oscillation with an equal and opposite excitation that is artificially generated by an arrangement of active vibration control devices. Numerous active control systems using electromagnetic, pneumatic, hydraulic, and viscoelastic force generators have been investigated. In recent years, piezoelectrets were also used as active actuators for active vibration control of distributed parameter systems because of their potential applications in large flexible and precision space structures.

This paper presents a theoretical and experimental study of an active piezoelectric vibration isolation technique using polymeric piezoelectric polyvinylidene fluoride (PVDF) with variable feedback gains. Injecting feedback voltage into a PVDF isolator results in a thickness change of the isolator due to its converse piezoelectric effect. If this thickness change is adjusted 180 deg out of phase with the base excitation, the PVDF isolator (with appropriate feedback gain) can effectively cancel any disturbance from the base.

Received Nov. 30, 1987; revision received April 21, 1988. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1988. All rights reserved.
*Assistant Professor, Department of Mechanical Engineering, Member AIAA.
†Research Assistant; currently, graduate student, University of California, Berkeley.
electric boundary conditions is developed. Analytical solutions are compared with experimental results.

**Theory Development**

In this section, a piezoelectric isolator with general electrical (simulating feedback voltage) and mechanical (simulating base excitation) boundary conditions is investigated. A general solution is to be developed so that active vibration isolation resulting from the converse piezoelectric effect can be evaluated.

The converse piezoelectricity can be expressed as:

\[ e_{33} = d_{33} \cdot E + \kappa \cdot \varepsilon_{33} \] \hspace{1cm} (1a)  

\[ p_{33} = \frac{1}{\varepsilon} \left[ e_{33} - d_{33} \cdot E \right] \] \hspace{1cm} (1b)

where \( \varepsilon \) is the strain, \( \sigma \) the stress, \( E \) the electric field, \( d \) the piezoelectric constant, and \( \kappa \) the elastic compliance. For a PVDF isolator with an effective area \( A \), the equation of motion in transverse vibration can be written as:

\[
\frac{d^2q_3}{dt^2} = \frac{1}{\rho A} \left( \frac{\partial^2 \varepsilon_{33}}{\partial \alpha_3^2} - \frac{\partial^2 p_{33}}{\partial \alpha_3^2} \right)
\]

where \( q_3 \) is the transverse displacement in the \( \alpha_3 \) direction and \( \rho \) the mass density. Assume that the electric field is constant over the thickness, i.e., \( \left( \frac{\partial E}{\partial \alpha_3} \right) = 0 \), and the transverse strain is defined as:

\[ \varepsilon_{33} = \frac{\partial q_3}{\partial \alpha_3} \] \hspace{1cm} (3)

Thus, a one-dimensional wave equation for the piezoelectric PVDF isolator can be derived:

\[ \ddot{q}_3 = \lambda^2 \frac{\partial^2 q_3}{\partial \alpha_3^2} \] \hspace{1cm} (4)

where \( \lambda \) is the velocity of wave propagation in the PVDF medium, \( \lambda = \sqrt{1/\rho} \). Applying the separation of variables method and using Eq. (3) yield a general solution of \( e_{33} \):

\[ e_{33}(\alpha_3, t) = (\omega/\lambda)[c_0 \cos(\omega \alpha_3/\lambda) - c_1 \sin(\omega \alpha_3/\lambda)] \cdot (c_2 \sin \omega t + c_3 \cos \omega t) \] \hspace{1cm} (5)

where \( c_0, c_1, c_2, \) and \( c_3 \) are constants to be determined by boundary conditions. Consider two general electrical boundary conditions and two general mechanical boundary conditions at \( \alpha_3 = 0 \) and \( \ell \) (\( \ell \) is the thickness of the PVDF isolator):

\[ \sigma_{33} = \sigma_{330} \sin \omega t, \hspace{1cm} E = E_0 \sin \omega t \] \hspace{1cm} (6a)  

\[ \sigma_{33} = \sigma_{330}, \hspace{1cm} E = E_0 \sin \omega t \] \hspace{1cm} (6b)

Note that \( \sigma_{330} \) and \( E_0 \) are induced by base excitation and feedback voltage, respectively, \( (E_0 = V_0/f \) where \( V_0 \) is the feedback voltage) in later analysis. Substituting Eqs. (6) into Eq. (5) and using Eq. (3) yield a steady-state solution of \( q_3 \):

\[
\dot{q}_3(t) = \int_0^{\Omega} (d_{33}E_0 + k \sigma_{33}) \cos \left( \frac{\omega}{\lambda} \alpha_3 \right) \cdot \frac{1 - \cos(\Omega \ell)}{\sin(\Omega \ell)} \cdot \sin \left( \frac{\omega}{\lambda} \alpha_3 \right) \cdot \sin \omega t \cdot \partial \alpha_3
\]

\[ = \ell (d_{33}E_0 + k \sigma_{33}) \tan(\Omega/2) \sin(\Omega/2) \] \hspace{1cm} (7)

where \( \Omega = (\omega \ell / \lambda) \). The induced acceleration \( G_{2b}(t) \) expressed in g (gravity) is generated by the PVDF converse piezoelectric effect, and it can be expressed as:

\[ G_{2b}(t) = \frac{1}{\beta} \dot{q}_3(t) = -\frac{\omega^2}{\beta} (d_{33}E_0 + k \sigma_{33}) \cdot \sin \left( \frac{\tan(\Omega/2)}{(\Omega/2)} \right) \] \hspace{1cm} (8)

It is intended that this converse effect be used to actively isolate a seismic mass \( m_t \) from the base excitation. Substituting \( E_0 = V_0/t \) into Eq. (8) yields a converse piezoelectric-induce force \( F_{2b}(t) \):

\[ F_{2b}(t) = m_t \cdot g \cdot G_{2b}(t) \]

\[ = -m_t \cdot \omega^2 \cdot (d_{33}E_0 + k \sigma_{33}) \cdot \sin \left( \frac{\tan(\Omega/2)}{(\Omega/2)} \right) \] \hspace{1cm} (9)

Similarly, the force \( F_b(t) \) introduced by the base excitation is given by:

\[ F_b(t) = m_t \cdot g \cdot G_b(t) \] \hspace{1cm} (10)

where \( m_t \) is the total mass (including \( m_t \)) on a shaker. The resultant acceleration \( G \), due to the combining effects of excitations and feedbacks can be obtained by balancing the forces,

\[ G = \frac{1}{m_t} \left( F_b + F_{2b} \right) \]

\[ = \left[ \frac{G_{b0}}{g} \cdot \omega^2 \cdot \left( \frac{d_{33}}{g} \cdot V_{b0} + k \cdot \frac{m_t}{A} \right) \right] \cdot \frac{\tan(\Omega/2)}{\Omega/2} \cdot \frac{1}{m_t} \] \hspace{1cm} (11)

Substituting all material properties gives \( \left( \tan(\Omega/2)/(\Omega/2) \right) = 1 \).

Thus,

\[ G = \frac{G_{b0}}{m_t} \cdot \omega^2 \cdot \left( \frac{d_{33}}{g} \cdot V_{b0} + k \cdot \frac{m_t}{A} \right) \cdot \frac{m_t}{m_t} \times 100 \] \hspace{1cm} (12)

The vibration isolation resulting from the feedback-induced converse effect can be defined as the difference between the resultant acceleration and the base excitation. The isolation percentage \( R \) can be written as:

\[ R = \frac{G_{b}(t) - G_{2b}(t)}{G_{b}(t)} \times 100 \]

\[ = \frac{\omega^2}{G_{b0}} \cdot \left( \frac{d_{33}}{g} \cdot V_{b0} + k \cdot \frac{m_t}{A} \right) \cdot \frac{m_t}{m_t} \times 100 \] \hspace{1cm} (13)

The gradient of the isolation surface with respect to the excitation frequency and the feedback voltage is evaluated when the base excitation is constant \( (G_{b0} = G) \) and feedback gain varies \( (V_{b0} = C \times V_b) \), where \( C \) is the feedback gain and \( V_b \) is the accelerometer output. Thus,

\[ R = \frac{\omega^2}{G} \cdot \left( \frac{d_{33}}{g} \cdot C \cdot V_b + G \cdot \frac{m_t}{A} \right) \cdot \frac{m_t}{m_t} \times 100 \] \hspace{1cm} (14)

Substituting all material properties into the equation, it is observed that the second term is small compared with the first term. Thus, at constant feedback voltage, the active piezoelectric isolation is a quadratic function of frequency, and at constant frequency it is a linear function of feedback voltage.

**Experimentation**

A physical model made of PVDF polymer, Fig. 1, was designed and tested to validate the theory discussed earlier. The model base is made of two 1/4-in.-thick layers. The bottom layer is steel and is provided with a 10-32 stud so that the model can be mounted on a shaker. The second layer is made of plexiglas. Then, a 1-mm-thick layer of PVDF polymer with an effective surface area of \( 4 \times 10^{-4} \text{ m}^2 \) (2 x 2 cm) is epoxied to the plexiglas layer below. A second layer of plexiglas is glued onto the top surface of the PVDF to provide identical boundary conditions on either side of the PVDF isolator. An interchangeable metal plate is screwed onto this top plexiglas. A miniaaccelerometer is attached above this metal plate.
References


SOVIET/JAPANESE ABSTRACTS: HOW DO YOU USE THEM?

Please do us the favor of giving your responses to the following questions and mailing them to me at AIAA headquarters. Thanks very much.

Dr. George W. Sutton
Editor-in-Chief, AIAA Journal
370 L'Enfant Promenade SW
Washington, DC 20024-2518

☐ I read abstracts regularly.  ☐ Occasionally

☐ I follow up citations by __________________________

__________________________________________________________

☐ I want abstracts continued.

☐ Comments: ________________________________

__________________________________________________________

Date of issue: August 1988