STRUCTURAL DYNAMICS OF ELEVATOR COUNTERWEIGHT SYSTEMS AND EVALUATION OF PASSIVE CONSTRAINT

By H. S. Trott and A. J. Schiff

ABSTRACT: Structural dynamic responses imposed by large base excitation and the effectiveness of an intermediate constraint in elevator counterweight systems are studied. A nonlinear contact finite element with nonlinear stiffness and damping approximated by polynomial functions is developed to simulate the dynamic contacts in elevator counterweight systems. A passive vibration control device—intermediate constraint—is proposed and evaluated. Analysis results show that the number of consecutive contacts increases as the excitation frequency decreases, large contact force can occur in current design practice, and constraining the motions of two rails by intermediate ties can reduce the contact load by 40%.

INTRODUCTION

Elevators are essential for transportation and impact life safety in modern high-rise buildings and hospitals. In normal operation, a passenger car is, in general, guided between two side rails located on either side of the car. Several steel cables connected to the top assembly of the car go up to a gear-driven traction machine located at the top of the hoistway, then extend down to a counterweight system traveling adjacent to the passenger car in the opposite direction. The counterweight system consists of counterweight masses secured inside a pair of deep steel channels and is guided between two steel guide rails by four spring-loaded roller guides located at each of the four corners of the counterweight frame.

The vulnerability of elevator systems has been demonstrated in a number of earthquakes. In the 1971 San Fernando earthquake, for instance, elevator systems in the Los Angeles area were seriously damaged even though there was almost no structural damage to the buildings in which they were housed (Ayres and Sun 1973). It was estimated that at least 25% of the elevator systems were damaged in the off-Miyagi Prefecture earthquake in Japan in 1978 (Schiff et al. 1980). While there were other causes of elevator failure, such as deformed car and guide rails, damaged roller guides, derailed car and counterweight systems, fallen counterweights, crashed car roofs, loosed motors, damaged electrical equipment and control cabinets, etc., counterweight-related failures were observed to be the most significant cause of destruction in elevator systems (Ayres and Sun 1973).

From elevator damage reports, it appears that the motion most damaging to counterweight systems is the motion in the plane of counterweight frame due to building vibration. As a result, the counterweight frame hammers
against the guide rails and results in breaking roller guides and deforming the rails (Ayres and Sun 1973; Schiff et al. 1980; Yang et al. 1983).

Dynamic responses of an elevator car-roller-ride and a counterweight-roller-ride system were investigated by Muto et al. (1973), Yang et al. (1983) and Schiff et al. (1980) also studied the nonlinear dynamic response of a small experimental model using the finite element method. While some design improvements can also be suggested from the observation of elevator failures, a thorough understanding of the dynamic behavior of the elevator counterweight-guide rail systems subjected to imposed motion would enable implementation of well-founded design improvements. This paper presents a study on the structural dynamic responses of elevator counterweight systems and evaluates the effectiveness of a passive control device—an intermediate tie. Derivation of generalized system equations will be presented first. Finite element analysis of elevator counterweight systems, both original and modified, will follow and be compared to each other.

Due to the complexity of analytical equations, analytical solutions are not available for this study. Thus, a numerical technique, i.e., the finite element method, is used to model the counterweight systems. Since the dynamic contact within the systems is generally nonlinear, characteristics change with respect to time and state; thus, nonlinear finite element techniques need to be incorporated in the modeling and analysis. In general, there are two fundamental techniques in nonlinear finite element analysis: (1) Reformation of the system matrix method, in which the system matrices need to be updated according to the change of the state and iteration procedure that is usually exercised to balance the residual force (Brebbia and Corner 1969; Wilson et al. 1973); and (2) a pseudo force approximation method, in which an equivalent load derived from nonlinearity is calculated as a pseudo nonlinear force and added to a linear force vector (Haisler et al. 1972; Stricklin et al. 1971). In either case, computation is very time consuming, so that numerical schemes to improve computational efficiency are often applied.

In this study, a nonlinear contact element is implemented in a nonlinear dynamic analysis finite element program (NDAFEP) (Tzou 1983) in which the element is modeled by nonlinear stiffness and damping polynomial functions. In the analysis, an equivalent force derived from the contact element is transferred to the right side of the system equations as a pseudo force component and is, therefore, added to a linear excitation force vector in the time-domain integration. Thus, reforming system matrices due to a change in nonlinearity is not required in the analysis. The dynamic responses of the systems are determined by a direct integration algorithm, i.e., the Wilson-θ method (Wilson et al. 1973), with minor modifications to accommodate the pseudo forces derived from the nonlinearity (Tzou and Schiff 1987). The Wilson-θ method applies a linear acceleration algorithm. Thus, the pseudo force can only be determined by extrapolation. Stricklin et al. (1971) found that the linear extrapolation is sufficient to define the load. Due to the nature of the nonlinear problem, relatively small integration time steps are required to ensure numerical convergence. Guyan's (1965) reduction scheme is also implemented in the NDAFEP so that the computational efficiency can be improved.

**Formulation**

It is assumed that there is a complex dynamic system with multiple independent spring-mass-damper subsystems, as shown in Fig. 1. Each spring-mass-damper system is considered as a unit system that moves only in translational direction, $x_i$. A unit system has mass $m_i$, linear spring $k_i$, linear damper $c_i$, and nonlinear damper $c'_i$. Dynamic contact is defined in a way such that the relative displacement, $x_{i+1} - x_{i-1}$, is greater than an initial clearance, $q_i$, from which the nonlinear spring and damper start activating. Dynamic contact of adjacent unit systems and its effect on overall system response are of interest in this study, which also leads to implementing passive dynamic control of the complex system.

The equation of motion of the complex system can be derived from Lagrange's equation:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial U}{\partial x_i} = F_i \quad (1)$$

where $T$ is the kinetic energy; $D$ is the energy dissipation; $U$ is the potential energy; $F_i$ is the excitation; and $x_i$ is the generalized coordinate. The energy expressions of Eq. 1 can be written as:

$$T = \frac{1}{2} \sum_{i=1}^{n} m_i \dot{x}_i^2 \quad (2)$$

$$D = \sum_{i=1}^{n} \left( c_i \dot{x}_i + \frac{1}{2} \sum_{j=2}^{n} c'_{i,j} (x_{i-1} - x_{j-1})^2 U(q_j - q_{j-1}) \right) \quad (3)$$

$$U = \sum_{i=1}^{n} \left( k_i x_i^2 + \frac{1}{2} \sum_{j=2}^{n} k'_{i,j} (x_{i-1} - x_{j-1})^2 U(q_j - q_{j-1}) \right) \quad (4)$$

where $q_{j-1} = (x_{j-1} - x_{j-2})$, $U(q_j - q_{j-1})$ is a step function, i.e., $U(q_j) = 0$ when $(x_{j} - x_{j-1}) < q_j$, and $U(q_j) = 1$ when $(x_{j} - x_{j-1}) \geq q_j$. Substituting the

![FIG. 1. Complex Dynamic System with Contact Potential](image-url)
or, in brief:

\[ [M] \ddot{\mathbf{x}} + [C] \dot{\mathbf{x}} + [K] \mathbf{x} = \{ F(t) \} + \{ F^N(t) \} \]  

(6)

where \([M]\) is the system mass matrix; \([C]\) and \([K]\) are the nonlinear system damping and stiffness matrices, respectively; \(\mathbf{x}\), \(\dot{\mathbf{x}}\), and \(\ddot{\mathbf{x}}\) are the acceleration, velocity, and displacement vectors, respectively; \(\{ F(t) \}\) is the external excitation force vector; and \(\{ F^N(t) \}\) is the nonlinear force vector.

It should be noted that the system equation of motion (Eq. 5) is decoupled and linearized when \(\phi_n\) goes to infinity. For a three-mass model of elevator counterweight systems, i.e., rail weights-frame-rail, \(\phi_n\) goes to infinity; and for a five-mass model, i.e., rail-frame-weights-frame-rail, \(\phi_n = \infty\) where \(n = 0\). The nonlinear system matrices can be divided into linear and nonlinear components as

\[ [C] = [C]^L + [C]^N \]  

(7)

\[ [K] = [K]^L + [K]^N \]  

(8)

in which \([C]^L\) and \([K]^L\) are the linear components; and \([C]^N\) and \([K]^N\) are the nonlinear components. Substituting Eqs. 7 and 8 into Eq. 6 yields

\[ [M] \ddot{\mathbf{x}} + ([C]^L + [C]^N) \dot{\mathbf{x}} + ([K]^L + [K]^N) \mathbf{x} = \{ F(t) \} + \{ F^N(t) \} \]  

(9)

and moving all nonlinear terms to the right side of the equation yields

\[ [M] \ddot{\mathbf{x}} + ([C]^L) \dot{\mathbf{x}} + ([K]^L) \mathbf{x} = -[C]^N \dot{\mathbf{x}} + -[K]^N \mathbf{x} + \{ F(t) \} + \{ F^N(t) \} \]  

(10)

Thus, an equivalent nonlinear force, \(\{ F_e(t) \}\), can be calculated from the current dynamic conditions.

\[ \{ F_e(t) \} = -[C]^N \dot{\mathbf{x}} + -[K]^N \mathbf{x} + \{ F^N(t) \} \]  

(11)

With this approach, the homogeneous equations describing the system motion remain unchanged, i.e., the system mass, stiffness, and damping matrices are kept constant. The nonhomogeneous equations differ by an addition of the pseudo nonlinear force term, \(\{ F_e(t) \}\), associated with the nonlinear stiffness and damping of the nonlinear element, as shown in Eq. 12

\[ [M] \ddot{\mathbf{x}} + [C]^L \dot{\mathbf{x}} + [K]^L \mathbf{x} = \{ F(t) \} + \{ F_e(t) \} \]  

(12)

**MODELING OF NONLINEAR DYNAMIC CONTACT**

To model the dynamic contacts between two subsystems, it is assumed that the dynamic contact can be simulated and represented by a highly nonlinear spring and damper combined as a nonlinear contact element. As discussed earlier, the spring and damper are activated by a unit step function in which \(U(t) = 1\) when \(x_i - x_{i+1} \geq \phi_n\). The nonlinear force \(\{ F_e(t) \}\) generated from the contact element can be approximated by summing the stiffness and damping forces.

\[ \{ F_e(t) \} = \{ x_i(x_{i+1} - \phi_n) \} \cdot \{ U(t) \} + \{ x_i(x_{i+1} - \phi_n) \} \cdot \{ U(t) \} \]  

(13)
where $[\kappa_0(\delta \xi)]$ and $[\kappa_1(\delta \xi)]$ are the damping and stiffness matrices, respectively, of the nonlinear contact element; $\lambda_0(\delta \xi)$ and $\kappa_1(\delta \xi)$ are the damping and stiffness functions, respectively; $[\psi]$ is the initial clearance vector; $[L(\gamma)]$ is the step function vector; and $[\delta \xi]$ and $[\delta \xi]$ are the relative velocity and relative displacement vectors, respectively. The nonlinear damping function, $\lambda_1(\delta \xi)$, can be formulated as a polynomial function of the relative velocity, $\delta \xi$.

$$\lambda_1(\delta \xi) = \sum_{p=1}^{m} \lambda_p \delta \xi^{p-1} \quad p = 1, 2, \ldots, m \quad (14)$$

where $\lambda_p$ are the damping coefficients of the contact element. A viscous damping can also be approximated by letting higher-order terms equal zero, i.e., $\lambda_p(\delta \xi) = \lambda_1$ = constant. The polynomial function of the nonlinear stiffness, $\kappa_1(\delta \xi)$, is formulated as

$$\kappa_1(\delta \xi) = \sum_{q=1}^{n} \kappa_q \delta \xi^{q-1} \quad q = 1, 2, \ldots, n \quad (15)$$

where $\kappa_q$ are the nonlinear stiffness coefficients of the contact element. Similar to the damping case, a linear spring can be approximated by retaining the first term, $\kappa_1$, in the equation, i.e., $\kappa_1(\delta \xi) = \kappa_1$ and $\kappa_2 = \kappa_3 = \ldots = \kappa_n = 0$. The relative displacement and relative velocity vectors can be defined as

$$[\delta \xi] = [x_i] - [x_j] \quad (16a)$$

$$[\delta \xi] = [x_i] = [x_j] \quad (16b)$$

where $i$ and $j$ are the two adjacent nodes that may come in contact and that are connected by the contact element.

Substituting Eqs. 14-16 into Eq. 12, the system's equation of motion becomes

$$[M] \cdot [\ddot{x}] + [C] \cdot [\dot{x}] + [K] \cdot [x] = [F(t)] + \theta \cdot ([F(t + \delta t)] - [F(t)]) \quad (20)$$

where $[F(t + \theta \cdot \delta t)]$ is the linear extrapolated load vector and can be written as

$$[F(t + \theta \cdot \delta t)] = [F(t)] + \theta \cdot ([F(t + \delta t)] - [F(t)]) \quad (21)$$

If there is an additional nonlinear contact forces, the extrapolated force vector at the time $t + \theta \cdot \delta t$ should be

$$[F(t + \theta \cdot \delta t)] = [F(t)] + \theta \cdot ([F(t + \delta t)] + [F(t + \delta t)] - [F(t)]) \quad (22)$$

or $[M] \cdot [\ddot{x}(t + \theta \cdot \delta t)] + [C] \cdot [\dot{x}(t + \theta \cdot \delta t)] + [K] \cdot [x(t + \theta \cdot \delta t)]$

$$= [F(t)] + \theta \cdot ([F(t + \delta t)] + \sum_{p=1}^{m} \lambda_p \delta \xi(t + \delta t)^{p-1} \cdot [\dot{x}(t + \delta t)] + \sum_{q=1}^{n} \kappa_q \delta \xi(t + \delta t)^{q-1} \cdot [\dot{x}(t + \delta t)] - [F(t)]) \quad (23)$$

A problem associated with this formulation is that the nonlinear contact forces are determined by extrapolation, which may cause numerical instability when suddenly changing dynamic contact status. Thus, relatively small integration steps and an artificial damping are incorporated in the time-domain integration process to stabilize the numerical problem.

**STRUCTURAL DYNAMIC ANALYSIS OF ELEVATOR COUNTERWEIGHT SYSTEMS**

The earthquake excitation of building structures is complex and random in nature. However, the structural response that directly excites the elevator system is a sinusoidal-like oscillation at the frequency of the building structure. Thus, sinusoidal inputs are assumed to be the external base excitation in finite element models of counterweight systems in the forced-vibration analysis.
Since the counterweights tend to move to one side of the counterweight frame as a result of the imposed motion, the counterweights would stay in contact with one side of the frame for the duration of the half-period of input excitation. For this reason, counterweights are assumed to be attached to one side of the frame, and only the stiffness contributed by one side of the counterweight frame is used in forming the center beam in the finite element model. A simplified spring-mass-damping system representing the finite element model is presented in Fig. 2.

An elevator counterweight system consists of counterweight masses, counterweight frame, spring-loaded roller guides, and guide rails. The counterweight masses are usually a stack of casting iron or cut steel blanks secured inside a pair of steel channels that form the sides of the counterweight frame. The counterweight masses are located at the lower two-thirds of the counterweight frame. In this study, the total weight of the counterweights is assumed to be 7,400 lbs (3,290 kg). Two C8-18.75 standard steel channels are used for the sides of the frame, and two 10-in.-wide (25.4-cm) and 0.5-in.- thick (1.27-cm) steel plates are used for both the top and bottom cross members of the frame. The unit weight of the channel is 18.75 lbs/ft (32.8 N/cm) (ANSI A17.1 1978; AISC 1975). Besides, one 12-lb/ft (21-N/cm) rail is positioned on either side of the counterweight frame and is assumed to be separated from the frame by a 0.5-in. (1.27-cm) gap. The total weight of the counterweights used in the finite element model is 5,400 lbs (2,400 kg), which is divided into five lumped masses attached to a center beam. The weight of the top and bottom steel plates are lumped into two concentrated masses located at either end of the beam, which is supported by four linear springs with spring constant of 54 lbs/in. (94.6 N/cm) to represent the spring-loaded roller guides at the four corners of the frame. Thus, the contact between the counterweights and the frame is not considered. A 12-lb/ft (21-N/cm) guide rail is positioned on either side of the center beam separated by a 0.5-in. (1.27-cm) initial gap, as discussed earlier.

The contact element is composed of a bilinear spring and a viscous damper, which is used to connect adjacent nodes on the frame and rails. Before contact occurs, the first section of the bilinear spring activates. Since the stiffness of the first section is only $1.7 \times 10^{-11}$ of the rail stiffness, the effect on the structural dynamics is not significant. When the relative displacement of the corresponding coordinates exceeds the gap, the second section of the bilinear spring, i.e., the contact spring, activates; it generates a large contact force between the pair of coordinates. Also, a viscous damper in parallel with the contact spring is used to reduce high-frequency oscillation of the nodal mass. The counterweight system is assumed to be centered between two floors. Thus, only a single-span guide rail with simply supported ends is used in the finite element model, as shown in Fig. 3.

The differences between the real system and the model should be noted. In the real system, the guide rail is continuous and spans several floors and the ends of the side members of the counterweight frame tend to have rigid boundary conditions, while the model has spring-supported ends.

**Eigenvalue Analysis**

The natural frequencies and modeshapes of the single-span model have been studied using the available analytical equation and the NDAFEP program, which are summarized in Table I and Fig. 4. The first natural

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical solution (Hz)</th>
<th>Consistent [m]</th>
<th>Lumped [m]</th>
</tr>
</thead>
<tbody>
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<td>39.8377</td>
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<td>46.4576</td>
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<td>9-10</td>
<td>80.8786</td>
<td>82.0329</td>
<td>82.0499</td>
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</table>

*No analytical solution was found.
frequency of the center beam with five masses attached is calculated by converting all the masses to a single mass and the spring constants of the roller-guide springs to an equivalent spring constant, and then using the equation $f = \sqrt{(K/M)/2\pi}$. The equation for solving beam natural frequencies is $f_0 = \lambda \cdot (E \cdot h^2/12L^3)^{1/2} \cdot \pi$ with $\lambda = 9.87$ for the first beam mode and $39.5$ for the second beam mode, where $\lambda$ is the mass per unit length in lb-sec²/ft, $L$ is the beam length in ft, and $M$ is the concentrated mass in lb-sec²/ft. (Harris and Crede 1976).

By examining Table 1, there is no significant difference between the analytical and numerical solutions. Because the analytical solution of the first mode is obtained by neglecting the flexibility of the counterweight frame, an error of 3.72% is observed for the first natural frequency. Usually the estimation of the first mode is the most accurate. The third and fourth and ninth and tenth frequencies, respectively, are the first and second modes of all rails. Also, a difference of 0.03% and 1.45% error can be found for the third and fourth and ninth and tenth frequencies, respectively, between the analytical and numerical solutions. A difference of 2.22% and 3.52% can also be observed for the third and fourth and the ninth and tenth frequencies between the consistent and lumped mass formulations. Thus, the finite element solutions compare favorably with the available closed-form solutions.

The first and second frequencies are basically the first two modes of the frame weights on the roller-guide springs. Because the weights are located at the lower two-thirds of the frame, the relative amplitude at the bottom is much greater than at the top, as shown in Fig. 4. The third and fourth frequencies are the first modes of the rails, which are essentially the third to sixth modes of the center beam with masses. The ninth and tenth frequencies correspond to the in- and out-of-phase second modes of the rails.

**Nonlinear Forced-Vibration Analysis**

Sinusoidal accelerations with an amplitude of 0.4 g (154.6 in/s²) (392.68 cm/s²) and frequencies of 1.2, 0.5, and 0.3 Hz are applied to the base of the counterweight model. The noncontact stiffness of the bilinear spring is set at 5.0 x 10⁴ lb/in. (8.76 x 10⁴ N/cm) and the contact stiffness is set at 1.3787 x 10⁸ lb/in. (2.42 x 10⁴ N/cm). The nonlinear damper is adjusted to be viscous damping, which has a damping constant of 8.5 x 10⁶ lb-sec/in. (1.49 x 10⁵ N-s/cm). A 2% rail damping is used, which is implemented by assigning $\alpha = 0.0$ and $\beta = 3.15 x 10^{-6}$ to Rayleigh’s coefficients in the system damping matrix. A time step size of 0.001 sec is used in the time-domain integration.

The nonlinear dynamic responses of the original systems with lumped mass formulation are presented in Figs. 5-7. The plots are arranged in such a way that only the displacements of the center mass with two adjacent nodes on the side rails are plotted. The center time history in the top plot is the transverse displacement of the center mass; the top time history is that of the node on the left rail; the bottom time history is that of the node on the right rail. The bottom plot is the input base acceleration.

Each plot shows the penetration of the rail by the mass, which would not be possible in reality. Rail oscillations with 2% damping can be observed after breaking the contacts in Figs. 5 and 7. The damped response of the rail in Fig. 7 is distorted because the rail is in contact with other counterweight masses. The contact of the adjacent counterweight mass lifts the rail, which can be observed in the first contact of the top time history in all three figures. By decreasing the base excitation frequency from 1.2 (Fig. 5), to 0.5 (Fig. 6), to 0.3 Hz (Fig. 7), the number of consecutive contacts between the counterweight frame and the rails has increased from one, to two, to four contacts for each half-excitation cycle. The phenomena have also been observed from the experimental results of a physical model and the numerical analysis of its finite element model (Yang et al. 1983).

The total rail load is calculated by summing the contribution of each individual mass load, referred to as an equivalent load, to the side rail. The equivalent load is calculated by multiplying the contact stiffness and maximum mass penetration in which the velocity equals zero. In the case of 0.4-g and 0.5-Hz sinusoidal base excitation, the total rail load is found to be 9,700 lbs (4.31 x 10⁶ N), which is greater than the rail design load (1/2 g) applied to the counterweights, by a factor of 2.6. Because the counterweights are located at the lower 2/3 of the counterweight frame, the lower support load is observed to be 5,800 lbs (2.58 x 10⁶ N), which is also greater than the design load (ANCI A 17.1 1978).

The maximum deflection, relative to the base, of the rails is observed to be 0.9 in. (2.29 cm) at 0.5-Hz base excitation. For the purpose of comparing the response of the various models, the load associated with the third mass will be calculated when the system is subjected to 0.4-g, 0.5-Hz sinusoidal base excitation. In this case, the equivalent load, i.e., mass 3 applied to the rail, is 2,800 lbs (11.25 x 10⁵ N). In the load calculation, no correction was made due to the difference of the boundary conditions between the real system and the finite element model.
FIG. 5. Dynamic Response of Counterweight Systems Subjected to 0.4-g, 1.2-Hz Base Excitation

FIG. 6. Dynamic Response of Counterweight Systems Subjected to 0.4-g, 0.5-Hz Base Excitation
**Effect of Intermediate Constraint**

In appendix F of the revised 1980 elevator codes (ANSI A 17.1 1978), it is recommended that an intermediate tie be used to connect two guide rails. The purpose of the tie is to constrain the relative displacement of the two rails, thus preventing the roller guides from disengaging the rails, which would prevent the counterweight frame swinging from its guide rails. The action of the tie also causes both rails to support the counterweight for motions in the plane of the rails, while the counterweight load was only supported by a single rail in the previous design. The modification is implemented by adding a U-shaped tie 48 in. (122 cm) long, 6 in. (15.2 cm) deep, made of 4-in. by 1/2-in. (10.2 cm × 1.27 cm) steel stock rigidly connecting two side rails at the midpoints. The equivalent stiffness of the tie is $4.0 \times 10^8$ lbs/in. $(7.0 \times 10^7$ N/m) in the finite element model, which is exhibited by the original U-shaped ties when a unit load is applied. The geometrical and material properties of the other elements are exactly the same as in the previous counterweight systems.

**Eigenvalue Analysis**

The natural frequencies are presented in Table 2, in which the analytical solutions of the eighth and ninth natural frequencies are calculated from the formula of a simple support beam without an intermediate tie; and the extra mass due to the addition of the tie is added to the total mass when calculating the third frequency of the system.

By comparing Tables 1 and 2, the same natural frequencies and modes of the center frame with attached masses can be observed. The third frequency of the system (18.4 Hz) is the first in-phase mode of the two rails, in which the effect of the tie mass is considered. There is only 0.035% difference between the analytical and numerical solutions. The out-of-phase mode of the two rails in the first model has been eliminated due to the constraint of the intermediate tie. The eighth and ninth natural frequencies of the system are the second modes of the two rails. Because there is a modal node of the second mode located in the middle of each rail, the effect of the intermediate tie on the in- and out-of-phase second modes of the rails is not significant.

**Table 2. Natural Frequencies of Counterweight Systems with Intermediate Tie (Hz)**

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<th>Mode</th>
<th>Analytical solution</th>
<th>Consistent [Hz]</th>
<th>Lumped [Hz]</th>
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*No analytical solution was found,*
Nonlinear Forced-Vibration Analysis

Other than the tie element, this finite element model is identical to that of the previous systems. Thus, the same input data are used in the dynamic analysis. Because the tie stiffens the structure, one input frequency is changed to show the same single contact phenomena between the mass and the rails. Sinusoidal accelerations with an amplitude of 0.4 g and frequencies of 1.0, 0.5, and 0.3 Hz are applied to the base of the model to study the dynamic response of the systems; the dynamic responses with lumped mass formulation are presented in Figs. 8–10. The plots are arranged in the same manner as discussed in the previous model. Only the deflections of the center mass with two nodes adjacent to the side rails are presented.

The dynamic responses are similar to those studied in the original systems, except the motions of both rails will always follow each other; this is imposed by the action of the intermediate tie. Also, the rail ringings only exist when the mass is moving between two rails. At this moment, two rails are ringing simultaneously at the rail’s natural frequency of 18 Hz. Lifting of rail-1 can be observed before the first contact in Figs. 9 and 10, and after the first contact in Fig. 8. Multiple contacts by other counterweight masses cause the complex waveform during the second excitation cycle in Fig. 8.

By decreasing the base excitation frequency, the number of consecutive contacts between the counterweight frame and the rails increases for each half excitation cycle. Because of the constraint of the tie, the maximum deflection of the rails, relative to the base, is observed to be about 0.53 in. (1.35 cm) at a 0.5-Hz base excitation. This deflection will not allow the counterweight to disengage from the guides. An equivalent rail load due to mass-3 is also calculated to be 1,650 lbs (7.34 × 10^3 N). This load is about 38.8% of that obtained in the analysis of previous systems. This reduction is the result of both rails supporting the counterweight load imposed by the intermediate tie. This indicates that the stiffness of the other side rail contributes a positive effect in improving the dynamic response.

Summary and Conclusion

Structural dynamics of elevator counterweight systems and the effectiveness of a passive vibration control device, i.e., the intermediate constraint, were studied and evaluated using the finite element method. A contact finite element using equivalent force approximation was developed and evaluated. The contact element had a polynomial representation for its stiffness and damping. The forces derived from the contact were directly added to the external force vector without reforming the system matrices. The dynamic response was obtained by direct integration, using the Wilson–B method with minor modifications. These techniques are implemented in a general-purpose finite element program, NADFEPA.

Dynamic contact can introduce destructive forces into the systems and damage them. The number of consecutive contacts within a half cycle of structural excitation increases when the excitation frequency decreases if the amplitude is kept the same. The dynamic contact load obtained from the analysis is larger than the design load in current design practice. This indicates that the design of elevator counterweight systems needs to be improved.
FIG. 9. Dynamic Response of Counterweight Systems with Intermediate Tie Subjected to 0.4-g, 0.5-Hz Base Excitation

FIG. 10. Dynamic Response of Counterweight Systems with Intermediate Tie Subjected to 0.4-g, 0.3-Hz Base Excitation
Implementing an intermediate constraint between two guide rails provides advantages over the previous design. The advantages result in the motions of the two rails always in phase. The dynamic contact load is reduced by about 40% compared with that of the original system. The tie can also prevent disengagement of the counterweight frame from the rails. Thus, even if a permanent deformation of the rails occurs, the counterweight should be retained in its guide rails so that there is a minimum effect on lift safety.

ACKNOWLEDGMENT

This research was supported by the National Science Foundation and the Purdue Research Foundation.

APPENDIX I. REFERENCES


APPENDIX II. NOTATION

The following symbols are used in this paper:

\[ A \] = cross-sectional area;
\[ [C]_s \] = system damping, \( [C] = [C]^t + [C]^n \);
\[ [C]^t \] = linear damping matrix;
\[ [C]^n \] = nonlinear damping matrix;
\[ c^r \] = nonlinear damper for ith unit system;
\[ D \] = energy dissipation;
\[ EI \] = flexural rigidity;
\[ F(t) \] = linear force vector;
\[ F^r(t) \] = pseudo force vector derived from nonlinear element;
\[ F^k(t) \] = nonlinear force vector;
\[ f_n \] = natural frequency, in Hz;
\[ [K] \] = \( [K]^t + [K]^n \);
\[ [K]^t \] = linear stiffness matrix;
\[ [K]^n \] = nonlinear stiffness matrix;
\[ k^r \] = nonlinear stiffness for ith unit system;
\[ [M] \] = mass matrix;
\[ T \] = kinetic energy;
\[ U \] = potential energy;
\[ \{U(t)\} \] = unit step function vector;
\[ \{x\} \] = displacement vector;
\[ \{v\} \] = velocity vector;
\[ \{a\} \] = acceleration vector;
\[ \alpha, \beta \] = Rayleigh's coefficients;
\[ \delta_{ij} \] = relative displacement vector;
\[ \omega \] = relative velocity vector;
\[ \varphi \] = initial displacement constraint;
\[ k_p \] = stiffness function of nonlinear element;
\[ k_p^r \] = stiffness matrix of nonlinear element;
\[ \lambda_p \] = damping coefficient;
\[ \lambda_p^r \] = damping function of nonlinear element;
\[ \lambda_p^m \] = damping matrix of nonlinear element;
\[ \mu \] = mass per unit length;
\[ \theta \] = integration constant;
\[ \omega_n \] = natural frequency of ith mode; and
\[ \zeta \] = damping ratio.