DEVELOPMENT AND EVALUATION OF A PSEUDO-FORCE APPROXIMATION APPLIED TO NONLINEAR DYNAMIC CONTACTS AND VISCOELASTIC DAMPING

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(Received 16 July 1986)

Abstract—A pseudo-force approximation method is used to study the nonlinear dynamic contacts and the effect of viscoelastic damping of beam-mass-beam systems. A pseudo-contact element with nonlinear stiffness and damping approximated by polynomial functions is defined and evaluated. To improve the system dynamic responses, viscoelastic damping is introduced into the systems using the standard linear model. The nonlinear forces derived from surface elasticity, penetration, instant contact velocity and viscoelastic damper are added to an excitation force vector to avoid reformation of system matrices. The system’s equations of motion are directly integrated using the Wilson-θ method with minor modifications to accommodate the dynamic contacts and the viscoelastic damping. A design equation is also proposed to estimate the required viscoelastic dampers.

1. INTRODUCTION

The finite element analysis dealing with geometrical and material nonlinearities has been studied for over two decades [1]. There are two basic approaches for solving nonlinear problems: one is reformation of system matrices [2-6] and the other is pseudo-force approximation [7-9]. The first method requires reformation of system matrices at certain load or time steps and an iteration procedure is usually required to balance the residual force. The second scheme needs to calculate an equivalent load derived from the nonlinearity and to apply the load as a pseudo-nonlinear force to the linear force vector. The difficulty arises in that the nonlinear force depends on the unknown displacement which can only be determined by extrapolation or iteration techniques. Stricklin found that the linear extrapolation is sufficient to define the load.

In beam-mass-beam systems, an initial gap which separates the mass and the beam is assumed. Under the effect of external excitation, the beam and the mass start to vibrate at a certain frequency. If the amplitude of vibration is smaller than the gap, they should oscillate independently. However, with the change of excitation amplitude and/or frequency, the vibration magnitudes of these systems also change accordingly. If the relative displacement of the two systems exceeds the separation, the mass comes into contact with the beams. A large contact force can be introduced into the systems and its interaction will further influence the dynamics of both systems. The contact force is composed of a repulsive force and an impact force. The first component is determined by the surface rigidity and the surface penetration and the second component is determined by the system mass and the contact velocity. In this study, a pseudo-contact element with nonlinear stiffness and damping represented by polynomial functions is developed and evaluated. To simulate different stages of contact, different combinations of stiffness and damping polynomials can be defined. In the finite element analysis, an equivalent contact force derived from the element stiffness, damping, surface penetration and instant velocity is transferred to the right side of the system equations as a pseudo-force component and is added to the external excitation force component in the time-domain integration process.

In order to improve the systems’ dynamics, viscoelastic damping is introduced into the systems between the mass and the beams. The viscoelastic damper alleviates the contact, provides extra damping and thus improves the overall system dynamics. The viscoelastic damping is modeled by the standard linear model [10, 11] and implemented in a similar way as that of the pseudo-contact element, i.e. the pseudo-viscoelastic element, in the finite element program.

The dynamic responses of the systems are determined by a direct integration technique—the Wilson-θ method [2-4]—with minor modifications to accommodate the pseudo-forces derived from system contact and viscoelastic damping. Owing to the nature of the nonlinear contact problem, a relatively small integration time step is required to obtain convergent results. Guyan’s reduction scheme [12]
and an artificial damping parallel to the contact element are implemented to improve the computation efficiency.

2. FINITE ELEMENT ANALYSIS

To demonstrate the concept of pseudo-force approximation applied to nonlinear dynamic contacts and viscoelastic damping, a general purpose nonlinear dynamic analysis finite element program (NDAFEP) is developed [13]. The NDAFEP has been tested and compared with the analytical technique and the SAP IV [14] in linear vibration analysis. A summary of the comparison is presented in a separate report [13].

2.1 Equation of motion of the system

The generalized equation of motion for a system with $N$ degrees of freedom can be formulated in a matrix form:

$$[M] \cdot \ddot{u} + [C] \cdot \dot{u} + [K] \cdot u = F(t),$$

where $[M]$, $[C]$ and $[K]$ are the system mass, damping and stiffness matrices; $\ddot{u}$, $\dot{u}$ and $u$ are the acceleration, velocity and displacement vectors; and $F(t)$ is the external excitation force vector. If the system is excited by a base excitation, the equation of motion can be rewritten as

$$[M] \cdot \ddot{u} + [C] \cdot \dot{u} + [K] \cdot u = -[M] \cdot \ddot{u}_b(t),$$

where $\ddot{u}_b$, $\dot{u}_b$ and $u_b$ are the relative acceleration, velocity and displacement vectors; and $\ddot{u}_b(t)$ is the base acceleration vector.

The system damping matrix is assumed to be proportional to the stiffness and mass matrices by Rayleigh's coefficients of $\alpha$ and $\beta$ [4], which can be written as

$$[C] = \alpha \cdot [M] + \beta \cdot [K].$$

The damping ratio of the $ith$ mode of the system is

$$\zeta_i = \frac{\alpha + \beta \omega_i^2}{2\omega_i},$$

where $\omega_i$ is the natural frequency of the $ith$ mode. The desirable damping ratio $\zeta_i$ for the system can be determined by adjusting $\alpha$ and $\beta$.

2.2 Pseudo-force approximation of dynamic contacts

In this study, two adjacent nodes located on two separate systems with the potential of coming into contact when the relative displacement exceeds an initial gap are connected by a pseudo-contact element. The contact element consists of nonlinear stiffness and damping represented by polynomial functions. The stiffness and damping properties of the element change with respect to the contact status. Instead of reforming the systems' matrices due to a change of the contact status, an equivalent contact force is calculated and directly applied to these two nodes which come into contact. With this approach, the form of the homogeneous equations describing the system motion is unchanged, i.e. the system mass, stiffness and damping matrices remain constant. The nonhomogeneous system equations differ by an addition of the contact force term, $F_{ct}(t)$, associated with the nonlinear stiffness and damping of the contact element as shown in eqn (5). The addition of the physical element is only done conceptually so that the element is referred to as the "pseudo"-contact element.

$$[M] \cdot \ddot{u} + [C] \cdot \dot{u} + [K] \cdot u = [F(t)] + F_{ct}(t) = \{F(t)\} + \{F_{ct}(t)\}. \quad (5)$$

The contact force generated from the pseudo-contact element can be approximated by the summation of the stiffness and the damping forces.

$$F_{ct}(t) = [C_{cm}(\delta u)] \cdot \delta u \cdot U_i(\varphi) + \{K_{cm}(\delta u)\} \cdot \{\delta u\} - \{\varphi\} \cdot U_i(\varphi) \quad (6)$$

where $F_{ct}(t)$ is the contact force vector; $C_{cm}(\delta u)$ and $K_{cm}(\delta u)$ are the contact damping and stiffness matrices; $C_{cm}(\delta u)$ and $K_{cm}(\delta u)$ are the damping and stiffness functions; $\varphi$ is the initial gap vector; $U_i(\varphi)$ is the step function vector; $\delta u$ and $\{\delta u\}$ are the relative velocity and the relative displacement vectors, respectively, which can be defined as

$$\{\delta u\} = \{u\} - \{u_b\} \quad \{\delta u\} = \{u\} - \{u_b\},$$

where $i$ and $j$ are the two adjacent nodes connected by the pseudo-contact element.

The approximation of the nonlinear damping function, $C_{cm}(\delta u)$, can be formulated as a polynomial function of the relative velocity, $\delta u$.

$$C_{cm}(\delta u) = \sum_{r=1}^{m} c_r \delta u^{r-1}, \quad s = 1, 2, \ldots, m, \quad (8)$$

where $c_r$s are the contact damping coefficients. A viscous damping only has the first term, in which $C_{cm}(\delta u) = c_1 = \text{constant}$. The polynomial function of the nonlinear stiffness, $K_{cm}(\delta u)$, is formulated as

$$K_{cm}(\delta u) = \sum_{r=1}^{n} k_r \delta u^{r-1}, \quad r = 1, 2, \ldots, n, \quad (9)$$

where $k_r$s are the contact stiffness coefficients. In a linear spring approximation, only the first term, $k_1$, is retained in the equation, i.e. $K_{cm}(\delta u) = k_1$ and $k_2 = k_3 = \cdots = k_n = 0$.

Substituting eqns (6)-(9) into (5), the system’s equation of motion becomes

$$[M] \cdot \ddot{u} + [C] \cdot \dot{u} + [K] \cdot u = \{F(t)\} + \{\delta u\} \cdot U_i(\varphi) + \{\{\delta u\} - \{\varphi\}\} \cdot U_i(\varphi) \quad (10)$$
2.3 Pseudo-force approximation of viscoelastic damping

The viscoelastic damping introduced between the mass and the beams dissipates the vibration energy and alleviates the dynamic contact. A standard linear model (Fig. 1) is used to model the viscoelastic damping.

The standard linear model consists of three parameters: a primary spring, \( K_{v1} \), in parallel with a serial of a secondary spring, \( K_{v2} \), and a viscous damper, \( C_v \) [10]. When the viscoelastic damper is subjected to a sinusoidal excitation at a frequency of \( \omega \), a complex stiffness, \( F/X \) [11], is defined as

\[
F = \left( K_v + \frac{K_{v2}(\omega C_v)^2}{K_{v2}^2 + (\omega C_v)^2} \right) \frac{\omega C_v K_{v2}^2}{K_{v2}^2 + (\omega C_v)^2}
\]

(11)

The real part is treated as an equivalent stiffness, \( K_{eq} \), and the imaginary part is considered to be an equivalent damping, \( C_{eq} \), multiplied by \( \omega \). The equivalent viscoelastic force, \( F_v \), derived from the viscoelastic damper, can be approximated by

\[
\{F_v(t)\} = [C_v(\delta u') + [K_v(\delta u') + [\delta u'] = \{F(t + \theta \cdot \delta t)\}
\]

(12)

where \( [C_v(\delta u')] \) and \( [K_v(\delta u')] \) are the equivalent viscoelastic damping and stiffness matrices, respectively; and \( \{\delta u'\} \) and \( \{\delta u\} \) are the relative velocity and relative displacement vectors as defined in eqn (7).

A pseudo-viscoelastic element is created in a similar way as the pseudo-contact element. The force, \( \{F_v(t)\} \), resulting from the viscoelastic damper is also transferred to the right side of the system equation as given in the following equation:

\[
[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} + \{F_v(t)\} + \{F_u(t)\}
\]

(13)

Substituting eqns (6) and (12) into eqn (13), the system’s equation becomes

\[
[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\}
\]

\[
+ \sum_{i=1}^{n} c_i \delta u^{u-1} \left( \{\dot{u}_i\} - \{u_i\} \right) \{u_i\} \\{\dot{\varphi} \}
\]

\[
+ \sum_{i=1}^{n} k_i \delta u^{u-1} \left( \{u_i\} - \{\varphi\} \right) \{\dot{u}_i\} \{\varphi\}
\]

\[
+ \frac{C_v K_{v2}^2}{K_{v2}^2 + (\omega C_v)^2} \left( \{\dot{u}_i\} - \{u_i\} \right)
\]

\[
+ \left( k_v + \frac{K_{v2}(\omega C_v)^2}{K_{v2}^2 + (\omega C_v)^2} \right) \left( \{u_i\} - \{\varphi\} \right).
\]

(14)

2.4 Pseudo-force method in direct time-domain integration

The system’s responses are directly integrated in the time-domain using the Wilson-θ method with minor modifications to accommodate the dynamic contact and the viscoelastic damping. In the linear integration, the system equation at time “\( t + \theta \cdot \delta t \)” can be formulated as

\[
[M]\{\ddot{u}(t + \theta \cdot \delta t)\} + [C]\{\dot{u}(t + \theta \cdot \delta t)\}
\]

\[
+ [K]\{u(t + \theta \cdot \delta t)\} = \{F(t + \theta \cdot \delta t)\},
\]

where \( \theta \) is a constant and \( \{F(t + \theta \cdot \delta t)\} \) is a linearly extrapolated load vector and it can be written as

\[
\{F(t + \theta \cdot \delta t)\} = \{F(t)\}
\]

\[
+ \theta \cdot (\{F(t + \delta t)\} - \{F(t)\})).
\]

(15)

The term inside the parentheses on the right side of eqn (16) is considered to be a load increment in one time step. If nonlinear forces, contact and/or viscoelasticity, exist in addition to the linear forces, the force vector at the time “\( t + \theta \cdot \delta t \)” should be rewritten as

\[
\{F(t + \theta \cdot \delta t)\} = \{F(t)\}
\]

\[
+ \sum_{i=1}^{n} \left( \{\ddot{u}_i(t + \theta \cdot \delta t)\} \{\dot{u}_i\} \{\varphi\}
\]

\[
+ \sum_{i=1}^{n} \left( \{u_i(t + \theta \cdot \delta t)\} \{\dot{u}_i\} \{\varphi\}
\]

\[
+ \left( \frac{C_v K_{v2}^2}{K_{v2}^2 + (\omega C_v)^2} \right) \left( \{\dot{u}_i(t + \theta \cdot \delta t)\} \{\varphi\}
\]

\[
+ \left( k_v + \frac{K_{v2}(\omega C_v)^2}{K_{v2}^2 + (\omega C_v)^2} \right) \left( \{u_i(t + \theta \cdot \delta t)\} \{\varphi\}
\]

\[
- \left( \{u_i(t + \delta t)\} \{\varphi\} \right).
\]

(16)

or

\[
\{F(t + \theta \cdot \delta t)\} = \{F(t)\}
\]

\[
+ \sum_{i=1}^{n} \left( \{\ddot{u}_i(t + \theta \cdot \delta t)\} \{\dot{u}_i\} \{\varphi\}
\]

\[
+ \sum_{i=1}^{n} \left( \{u_i(t + \theta \cdot \delta t)\} \{\dot{u}_i\} \{\varphi\}
\]

\[
+ \left( \frac{C_v K_{v2}^2}{K_{v2}^2 + (\omega C_v)^2} \right) \left( \{\dot{u}_i(t + \theta \cdot \delta t)\} \{\varphi\}
\]

\[
+ \left( k_v + \frac{K_{v2}(\omega C_v)^2}{K_{v2}^2 + (\omega C_v)^2} \right) \left( \{u_i(t + \theta \cdot \delta t)\} \{\varphi\}
\]

\[
- \left( \{u_i(t + \delta t)\} \{\varphi\} \right).
\]

(17a)

The problem involved in this equation is that the nonlinear forces are determined by an extrapolation method which causes numerical instability when the state in which large forces are added to the system equations within a very short time period is suddenly changed. In order to avoid an iteration process and a very fine integration time step, an artificial damping
is implemented to compensate the numerical problem and to improve the computation efficiency.

3. CASE STUDY

3.1 Case 1: a concentrated mass between two fixed beams

The first model studied here is an H structure—a concentrated mass supported by two contact elements between two fixed beams as shown in Fig. 2. The beam has a cross-section area $A = 0.015$ in.$^2$, mass density $\rho_m = 7.34 \times 10^{-4}$ lb-sec/in.$^4$ and flexural rigidity $EI = (1.03 \times 10^{5})(1.125 \times 10^{-6})$ lb-in.$^2$. Each fixed beam is divided into eight beam elements. The concentrated mass is equal to $1.82 \times 10^{-3}$ lb-sec$^2$/in. The initial gap between the mass and the beam is 0.02 in. The contact stiffness function is assumed to be $K_{cm} = 1.758 + 175.8 \delta u \cdot U_i(0.02)$ lb/in. and the contact damping is defined as $C_{cm} = 7.0 \times 10^{-2} \cdot U_i(0.02)$ lb-sec/in., in which $U_i(0.02)$ is the step function as discussed before. By assigning Rayleigh's coefficients $\alpha = 0.0$ and $\beta = 3.183 \times 10^{-3}$, a 2% damping is provided to the fixed beams.

3.1.1 Eigenvalue analysis. The first five natural frequencies are evaluated using the NDAFEPE, SAP IV and the available analytical techniques. The results are summarized in Table 1.

Referring to Table 1, the first natural frequency corresponds to the oscillation of the concentrated mass on the linear springs; the second and third natural frequencies correspond to the first mode of the two fixed beams with out-of-phase and in-phase motions; and the fourth and the fifth natural frequencies represent the out-of-phase and in-phase motions of the second mode of the two fixed beams. The results compare very favorably.

3.1.2 Linear vibration analysis. A sinusoidal base excitation with 0.001 g and 20 Hz is applied to the fixed base of the H structure to evaluate the linear dynamic response and to compare it with the SAP IV analysis. A detailed comparison can be found in Ref. [13]. The linear dynamic responses of the center mass and two adjacent nodes are presented in Fig. 3 in which the bottom history is the base excitation.

3.1.3 Contact vibration analysis. It is recalled that the concentrated mass is suspended by two contact elements and each element is connected to a fixed-end beam. The gap between the mass and the beam is assumed to be 0.02 in. Thus, dynamic contact between the mass and the side beams will occur when the relative displacement exceeds the 0.02-in. separation.

Sinusoidal accelerations with an amplitude of 0.1553 g and frequencies of 4 and 3 Hz are applied to the base of the H structure. The responses of the center mass and two adjacent nodes are presented in Figs 4 and 5. There are two sets of the time histories shown in one plot. The bottom time history represents the input base excitation and the top time histories are the responses of the center mass and the two adjacent nodes.

It is observed that the mass moves opposite to the base excitation and single contact within the half-cycle of 4 Hz excitation as shown in Fig. 4. When the excitation frequency is decreased to 3 Hz, keeping the same input amplitude, double contacts can be found within half the excitation cycle. Because of the relatively long period of excitation, the beam is forced to move back and cause the second contact. The effect of the soft spring pulling the other side beam and the ringing of the side beams after breaking the contact can also be observed in both figures. The damping

| Table 1. Natural frequencies of the H structure (Hz) |
|---------------------------------|-----------------|-----------------|-----------------|
| Method mode | Analytical solution | SAP IV† | NDAFEPE† |
| 1 | 6.9996 | 6.8275 | 6.8289 |
| 2 and 3 | $2.402 \times 10^2$ | $2.333 \times 10^2$ | $2.339 \times 10^2$ |
| 4 and 5 | $6.297 \times 10^2$ | $6.288 \times 10^2$ | $6.298 \times 10^2$ |

† Consistent mass formulation. ‡Apply Guyan's reduction scheme.
ratio of the beams calculated from the ringing is 2%, which matches the assumption. The mass penetrating the beams can also be observed in both figures. The penetration can be reduced by increasing the contact stiffness. However, the stiffer contact element introduces high frequency oscillation of the nodal masses and result in numerical instability, which is highly undesirable in the integration process.

3.2 Case 2: Five masses between two simply supported beams: an elevator counterweight model

3.2.1 Background. In modern high-rise buildings and hospitals, elevators are essential to transportation and life safety. In normal operation, a passenger car is, in general, guided between two side rails. Several steel cables connected to the top assembly
Fig. 5. Dynamic responses of the H structure at 3 Hz base excitation.

ably of the car go to a gear-driven traction machine located at the top of the hoistway, then extend to a counterweight system traveling adjacent to the passenger car in the opposite direction. The counterweight system consists of counterweight masses secured inside a pair of deep steel channels and is guided between two steel guide rails by four spring-loaded roller guides located at each of the four corners of the counterweight frame.

The vulnerability of the elevator system has been demonstrated in a number of earthquakes [15–17]. It was noticed that the damage to elevator systems was much more severe than that to building structures. While there were other causes of elevator failure, such as deformed rails, loosened motor and electrical control cabinets, etc., counterweight-related failures were the most significant destruction in the elevator damage reports [15]. Thus, the elevator system, especially the counterweight system, is highly vulnerable and the need to study its dynamics and to improve its design is clearly identified.

From the damage reports, it appears that the motion most damaging to the counterweight system is the motion in the plane of the counterweight frame and guide rails due to the building vibration at the structural natural frequency. As a result, the counterweight frame hammers against the guide rails, breaking roller guides and deforming rails [17, 18].

One of the proposed design modifications on the current elevator system is that a large gap is reserved on either side of the counterweights between the weights and the frame. For relatively small motion of the counterweights, this gap would provide a contact-free status [13, 19]. A new problem introduced in this configuration is that the inertia force of the counterweights can increase due to the acceleration provided by the extra clearance and it would even aggravate the contact when the external excitation increases. The contact dynamic response and the effectiveness of the viscoelastic damping are thereafter studied and discussed.

3.2.2 Finite element modeling. The earthquake excitation to a building structure is complex and random in nature. However, the structural response...
which excites the elevator system is a sinusoidal-like oscillation at the fundamental frequency of the building structure. Thus, the excitation to elevator counterweight systems is assumed to be a sinusoidal base excitation. Also, the counterweight system is assumed to be centered between two floors; therefore, only a single span of guide rails with simply supported ends is used in the finite element model. Because the gap between the counterweight frame and the guide rails is 0.5 in., which is 1/12 of that between the frame and the counterweights in the proposed design modification, it is assumed that the frame and the rail would come into contact first, and stay in contact for a duration of a half-cycle of the structural frequency. A simplified model representing elevator counterweight systems is shown in Fig. 6. The right side of the figure is considered to be the base of the system.

The elevator counterweight systems are modeled by beam elements with five concentrated masses representing the lumped counterweights. Each mass weighs 1480 lb in the model. Based on the assumption that the frame contacts the rails first, there is only a combined beam used on either side of the counterweight masses. The combined beam used in the model has a cross-sectional area $A = 9.26$ in.$^2$, flexural rigidity $EI = (2.9 \times 10^3)(6.47)$ lb-in.$^3$, and the damping ratio $\zeta = 0.05$. The initial gap between the weights and the combined beam is 6 in. The pseudo-contact element has the stiffness function $K_\infty = 5.4 \times 10^{-7} + 1.99 \times 10^{-7} \Delta u - U_i$ (6.0) lb/in. and the damping function $C_\infty = 85 U_i (6.0)$ lb-scc/in. The $k_d$ represents the contact status and it is used to support the concentrated masses in the finite element model to avoid numerical problems. The soft spring is only $7.36 \times 10^{-4}$ of the equivalent beam stiffness and $2.72 \times 10^{-7}$ of the second contact stiffness, $k_d$. The effect to the system dynamics is insignificant.

3.2.3 Eigenvalue analysis. Free vibration analysis is first conducted to evaluate the system natural frequencies and mode shapes and to determine the time step in the direct integration. The natural frequencies are summarized in Table 2.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical solution</th>
<th>NDAFE</th>
<th>Lumped [M]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>$8.451 \times 10^{-3}$</td>
<td>$8.451 \times 10^{-3}$</td>
<td>$8.451 \times 10^{-3}$</td>
</tr>
<tr>
<td>6-7</td>
<td>15.2040</td>
<td>15.2040</td>
<td>14.8711</td>
</tr>
<tr>
<td>8-9</td>
<td>60.8469</td>
<td>61.7302</td>
<td>63.9849</td>
</tr>
</tbody>
</table>

By examining Table 2, it can be seen that there is no significant difference between the analytical solutions and the numerical solutions. The analytical equation for solving the first five natural frequencies is $f_\alpha = (\sqrt{2\pi}/M)(2\pi)$ and for the beam frequencies it is $f_\beta = A(\mu/L)^{1/2}/(2\pi)$ with $A = 9.87$ for the first beam mode and $A = 39.5$ for the second beam mode, where $\mu$ is the mass per unit length in lb-sec$^2$/in.$^2$, $L$ is the beam length in in., $M$ is the concentrated mass in lb-sec$^2$/in. and $k_d$ is the soft contact stiffness in lb/in.

The first five natural frequencies are associated with the concentrated mass and the contact soft spring. In reality, these five frequencies do not exist because of the gap between the weights and the combined beam. The sixth and seventh natural frequencies are the first mode of the combined beam, and the eighth and ninth natural frequencies are the second modes of the combined beam, which are the out-of-phase and in-phase modes.

3.2.4 Contact vibration analysis. A sinusoidal base excitation with an amplitude of 0.4 g and a frequency of 0.75 Hz is applied to the elevator counterweight model. The time step used in the direct integration is 0.01 sec, which is 64% of the period of the ninth natural frequency of the system. The ratio is relatively large for a nonlinear dynamic analysis without an iteration process. The dynamic responses of five individual masses and their two adjacent nodes are presented in Figs 7–11. Figure 12 shows composed plots of all five figures. The plots are arranged in the same way as before. The bottom time history is the input base excitation. The center response curve in the top plot corresponds to the motion of the center mass, and the top and bottom response curves represent the motion of two corresponding nodes on the combined beams.

The contacts between the individual mass and the combined beam are clearly shown in these response curves. It can be observed that these five masses first contact beam 2 about the same time. After that, the contact sequence is relatively unpredictable. While the base excitation changes the direction, all masses move to the other side and contact beam 1. The motion of these masses is usually opposite to the direction of the base excitation with a time delay. The contact sequence to beam 1 can be observed, from Fig. 12 and Figs 7–11: the fourth mass contacts beam 1 first, the third is slightly after, then the sequence is fifth, second and first masses. The earlier mass contact lifts the beam prior to the arrival of the later contacts. The ringing of the beam can also be observed after the contact is broken. The beam oscillates at the beam natural frequency of 15 Hz and the amplitude decreases at 2% damping.

3.2.5 Viscoelastically damped forced-vibration analysis. To improve the contact dynamic response of the systems, viscoelastic damping is introduced between the mass and the combined beam. The viscoelastic damper softens the contact and provides extra damping which benefits the overall system responses. The difference between the finite element model and the real physical system, both equipped with a viscoelastic damper, should be noted as follows. (1) In the real system, as the relative displacement between the weights and the frame decreases, the viscoelastic damper pushes on the frame which has a relatively fixed end condition; then the frame pushes on the rail which is a continuous member.
Fig. 7. Dynamic response of the first mass with two adjacent nodes in the counterweight model with a large gap.

Fig. 8. Dynamic response of the second mass with two adjacent nodes in the counterweight model with a large gap.
Fig. 9. Dynamic response of the third mass with two adjacent nodes in the counterweight model with a large gap.

Fig. 10. Dynamic response of the fourth mass with two adjacent nodes in the counterweight model with a large gap.
Fig. 11. Dynamic response of the fifth mass with two adjacent nodes in the counterweight model with a large gap.

Fig. 12. Dynamic responses of all five masses with their adjacent nodes in the counterweight model with a large gap.
However, in the finite element model, the viscoelastic damper pushes directly on the combined beam, which is simply supported on either end. (2) When the relative displacement between the weights and the frame increases, the viscoelastic damper pulls only on the frame and its does not affect the guide rail in the real system; however, the damper pulls on the combined beam representing both the frame and the rail in the finite element model.

A series of tests with different primary stiffness $K_1$, secondary stiffness $K_2$, and damping $C$, was conducted using the five-mass model shown in Fig. 6. The secondary stiffness of the viscoelastic damper is assumed to be three times that of the primary stiffness for a damper made of rubber. The damping, $C$, is determined from $C = 2 \zeta \sqrt{(K_1 M)}$, where $M$ is the applied concentrated mass and $\zeta$ is the damping ratio in which $\zeta = 20\%$ for the viscoelastic rubber damper [20].

The dynamic responses of the center mass with two adjacent beam nodes in the finite element model equipped with different viscoelastic damping, $K_n = 0.0, 50.0, 100.0, 150.0$ and 200.0 lb/in., and subjected to 0.5 Hz and 0.4 g base excitation are presented in Fig. 13. In this plot, only the center mass, the third mass, motion and two adjacent nodes on the side beams are presented.

The responses of the original system in which $K_n = 0.0$ show some contact phenomena similar to those shown in Figs 7–12. When the primary stiffness increases to 50 lb/in., the contact is significantly reduced. As $K_n$ keeps increasing, the contact is eventually prevented. The equivalent loads on the combined beam applied by either the direct contact and/or the indirect compression of the viscoelastic damper are also estimated.

The equivalent beam load due to mass contact can be calculated by the required force to produce its absolute motion. There are primarily two components involved in the time history of the mass motion. The first component is associated with the imposed motion and the second component is related to the oscillation of the mass in conjunction with the combined beam. The amplitude and frequency of the structural vibration determine the first component, and those of the contact oscillation determine the second component. While the beam stiffness becomes stiffer, the contact frequency increases, which results in the increase of the contact force. It is also noticed that when the gap between the weights and the frame increases, the force associated with the imposed motion decreases, and that associated with the contact oscillation increases. The load when the mass does not contact the combined beam is calculated by multiplying the damper stiffness by the damper deformation. The maximum deformation of the damper occurs when the velocity is zero. Thus, no damping

<table>
<thead>
<tr>
<th>Primary spr. $K_n$ (lb/in.)</th>
<th>Mass max. disp. (in.)</th>
<th>Contact†</th>
<th>Beam load (lb)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>200.00</td>
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</tbody>
</table>

† 6-in. gap.
force is involved. The analysis results are summarized in Table 3, in which the primary stiffness of the viscoelastic damper, the maximum displacement of the third mass, the contact status and the contact beam load are listed.

Referring to Table 3, the minimum beam load is observed when \( K_p \) is equal to 100 lb/in., in which the mass contact is prevented. When the primary stiffness of the viscoelastic damper increases, the constraint applied to the beam and the mass results in an increase in the absolute motion of the mass and a further increase in the beam load. Thus, the appropriate viscoelastic damper should be rigid enough to prevent the contact and soft enough not to constrain the mass motion.

3.3 Viscoelastic damper design equation

Based on the study presented above, appropriate selection of the viscoelastic damper can effectively prevent the dynamic contacts and also significantly reduce the dynamic load applied to the beams. Equation (18) can be used to estimate the required stiffness and damping using the available parameters, such as mass \( M \), vibration amplitude and frequency \( Y \) and \( \omega \), and design gap \( Z \) between the two systems.

\[
Z = \frac{MYw^2}{[(K_m - M\omega^2)^2 + (C_m\omega^2)]^2}
\]  

(18)

In practical applications, several iterations may be required. If there is more than one modal response involved in the vibration, the modal participation factors can be used to estimate the amplitude of each mode. The worst combination would be the summation of each modal response participating in the motion.

4. SUMMARY AND CONCLUSIONS

In this paper, a pseudo-contact element and a pseudo-viscoelastic element using a pseudo-force approximation method have been developed and evaluated. The contact element has polynomial representations for its nonlinear stiffness and damping. The viscoelastic damper utilizes the standard linear model approximation in which a complex stiffness equation provides an equivalent stiffness and an equivalent damping function. The forces derived from the contact and the viscoelastic damper were directly applied to the external force vector without reforming the system matrices. The dynamic response was obtained by direct integration using the Wilson-\( \theta \) method with minor modifications.

Two cases—one, an H structure and the other, an elevator counterweight system—were studied using the finite element program NDAFEP. The studies showed contact dynamic behaviors in which the mass moves, in general, opposite to the direction of the base excitation and the number of consecutive contacts within half the excitation cycle increases as the excitation frequency decreases. The dynamic contacts introduce a large contact load into the systems.

To improve the system dynamics, a technique of incorporating viscoelastic dampers into the systems was proposed and evaluated. The analyses demonstrated the effectiveness of the viscoelastic damping in preventing the system contact and reducing the dynamic loads. A general design equation for selecting viscoelastic dampers was thus proposed. The suggested design equation is relatively simple and only requires information about the mass, the gap, the vibration amplitude and frequency.

Acknowledgements—This research was supported by the National Science Foundation and the Purdue Research Foundation.

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