



```

for jL ∈ 0.. n
    tmpi+iL, i+jL ← tmpi+iL, i+jL + Slocal(i)iL, jL
tmpN-1, N-1 ← tmpN-1, N-1 + Slocal(N)0, 0
tmpN, N ← 1
tmp

```

Superimpose the element matrices

Initialize for the Dirichlet boundary at x = L

Only S00(N) contributes to the global system matrix

S =

|    | 0 | 1   | 2   | 3   | 4   | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   | 13   |
|----|---|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|
| 0  | 1 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 1  | 0 | 160 | -80 | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 2  | 0 | -80 | 160 | -80 | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 3  | 0 | 0   | -80 | 160 | -80 | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 4  | 0 | 0   | 0   | -80 | 160 | -80  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 5  | 0 | 0   | 0   | 0   | -80 | 240  | -160 | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 6  | 0 | 0   | 0   | 0   | 0   | -160 | 320  | -160 | 0    | 0    | 0    | 0    | 0    | 0    |
| 7  | 0 | 0   | 0   | 0   | 0   | 0    | -160 | 320  | -160 | 0    | 0    | 0    | 0    | 0    |
| 8  | 0 | 0   | 0   | 0   | 0   | 0    | 0    | -160 | 320  | -160 | 0    | 0    | 0    | 0    |
| 9  | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | -160 | 320  | -160 | 0    | 0    | 0    |
| 10 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | -160 | 320  | -160 | 0    | 0    |
| 11 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | -160 | 320  | -160 | 0    |
| 12 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | -160 | 320  | -160 |
| 13 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | -160 | 320  |
| 14 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | -160 |
| 15 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 16 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 17 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 18 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| 19 | 0 | 0   | 0   | 0   | 0   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |

Initialize the Right-hand-Side

```

b :=
for i ∈ 0.. N
    tmpi ← 0
    tmp0 ← V1
    tmp1 ← - Slocal(0)1, 0 · V1
    tmpN-1 ← Slocal(N)0, 1 · V2
    tmpN ← V2
tmp

```

Compute the Solution:

$$c := S^{-1} \cdot b$$

|    |       |
|----|-------|
|    | 0     |
| 0  | 5     |
| 1  | 4.75  |
| 2  | 4.5   |
| 3  | 4.25  |
| 4  | 4     |
| 5  | 3.75  |
| 6  | 3.625 |
| 7  | 3.5   |
| 8  | 3.375 |
| 9  | 3.25  |
| 10 | 3.125 |
| 11 | 3     |
| 12 | 2.875 |
| 13 | 2.75  |
| 14 | 2.625 |
| 15 | 2.5   |
| 16 | 2     |
| 17 | 1.5   |
| 18 | 1     |
| 19 | 0.5   |

c =

Plot the Solution versus the Exact:

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \text{er1} & 0 & -\text{er2} & 0 & 0 & 0 \\ 0 & 0 & \text{er2} & 0 & -\text{er3} & 0 \\ \text{d1} & 1 & -\text{d1} & -1 & 0 & 0 \\ 0 & 0 & \text{d2} & 1 & -\text{d2} & -1 \\ 0 & 0 & 0 & 0 & L & 1 \end{bmatrix} \quad y := \begin{bmatrix} V1 \\ 0 \\ 0 \\ 0 \\ 0 \\ V2 \end{bmatrix}$$

$$z := A^{-1} \cdot y$$

$$z = \begin{bmatrix} -5 \\ 5 \\ -2.5 \\ 4.375 \\ -10 \\ 10 \end{bmatrix}$$

$$A1 := z_0 \quad B1 := z_1 \quad A2 := z_2 \quad B2 := z_3 \quad A3 := z_4 \quad B3 := z_5$$

$$\text{Vexact}(x) := \begin{cases} (A1 \cdot x + B1) & \text{if } x < d1 \\ (A2 \cdot x + B2) & \text{if } d1 \leq x < d2 \\ (A3 \cdot x + B3) & \text{if } d2 \leq x < L \\ 0 & \text{otherwise} \end{cases}$$

$$d1 = 0.25$$

$$\text{Vexact}(d1) = 3.75$$

$$xx := \begin{cases} \text{tmpx} \leftarrow 0 \\ \text{tmp}_0 \leftarrow 0 \\ \text{for } i \in 0..N-1 \\ \quad \begin{cases} \text{tmpx} \leftarrow \text{tmpx} + \text{sqrtg1} & \text{if } i < N1 \\ \text{tmpx} \leftarrow \text{tmpx} + \text{sqrtg2} & \text{if } N1 \leq i < N1 + N2 \\ \text{tmpx} \leftarrow \text{tmpx} + \text{sqrtg3} & \text{otherwise} \end{cases} \\ \text{tmp}_{i+1} \leftarrow \text{tmpx} \\ \text{tmp} \end{cases}$$

$$\text{Vex} := \begin{cases} \text{for } i \in 0..N \\ \text{tmp}_i \leftarrow \text{Vexact}(xx_i) \\ \text{tmp} \end{cases}$$

ii := 0.. N

