HW#5
EE699 – Fall 2007
Due 11/6/07

1. A parallel plate waveguide is excited by a TEz polarized source. The source is in the x = 0 plane. The waveguide is bound by PEC planes in the y = 0, and y = 1 m planes. The waveguide is also filled with three dielectric regions: Region 1 is defined by the region $0 < x < 0.25$ m, and has a dielectric constant ε_{r1}. Region 2 is defined by the region $0.25 < x < 0.75$ m, and has a dielectric constant ε_{r2}. Region 3 is defined by the region $x > 0.75$ m, and has a dielectric constant ε_{r3}. The TEz polarized source is excited by a Dirichlet boundary condition: $E_y(x,y)\big|_{x=0} = 1$ V/m.

   a. Determine a closed form expression for an analytical solution for $\vec{E}(x,y)$.

   b. You describe the theory for a finite element solution for the problem based on a Galerkin discretization of the vector wave equation. The transverse electric field $\vec{E}(x,y)$ is to be expanded via H(0)-curl vector basis functions distributed over triangular finite elements. The test functions are also H(0)-curl vector functions. The source boundary condition is to be constrained as a Dirichlet boundary condition in the solution domain. The PEC walls of the parallel plate waveguide are constrained via Dirichlet boundary conditions. The mesh is to be terminated in the x = 1 m plane via a Sommerfeld Radiation condition.

   c. You are to write a code to perform the finite element solution for $\vec{E}(x,y)$. This can be done using MathCad, MatLab, Mathematica, or FORTRAN90. You will then compute the mean error in your solution versus the analytical solution. The mean error is to be computed as:

$$\text{Mean Error} = \frac{1}{N_c}\sum_{i_c=1}^{N_c}\frac{\left|\vec{E}_{i_c}^{Exact} - \vec{E}_{i_c}^{FEM}\right|}{\left|\vec{E}_{i_c}^{Exact}\right|}$$

   where, N_c = the number of cells in the mesh, and $\vec{E}_{i_c}^{FEM}$ is the field evaluated at the center of each element. You will then plot the error versus the average cell size to study error convergence. Four meshes are provided to you in the zip file: *TEzEdgeMeshRectDomain.zip,* which can be downloaded from the homework web page. You can also compare surface plots (shaded contour type) of the FEM fields with the exact fields as part of your validation.

   To validate your solution, you will consider the following cases:

   i) f = 75 MHz, $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1$

   ii) f = 75 MHz, $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 2$, $\varepsilon_{r3} = 3$.