HW#4
EE699 – Fall 2007
Due 10/25/06

1. A parallel plate waveguide is excited by a TEz polarized source. The source is in the x = 0 plane. The waveguide is bound by PEC planes in the y = 0, and y = 1 m planes. The waveguide is also filled with three dielectric regions: Region 1 is defined by the region $0 < x < 0.25$ m, and has a dielectric constant ε_{r1}. Region 2 is defined by the region $0.25 < x < 0.75$ m, and has a dielectric constant ε_{r2}. Region 3 is defined by the region $x > 0.75$ m, and has a dielectric constant ε_{r3}. The TEz polarized source is excited by a Dirichlet boundary condition: $H_z(x,y)\big|_{x=0} = 1$ A/m. That is, $H_z(x,y)$ is a fixed constant in the x = 0 plane.

   a. Determine a closed form expression for an analytical solution for $H_z(x,y)$.

   b. You are to write a finite element solution for the problem based on a Galerkin discretization of the scalar wave equation. The magnetic field $H_z(x,y)$ is to be expanded via nodal basis functions distributed over triangular finite elements. The test functions are also Nodal functions. The source boundary condition is to be constrained as a Dirichlet boundary condition in the solution domain. The PEC walls of the parallel plate waveguide are constrained via Neumann boundary conditions. The mesh is to be terminated in the x = 1 m plane via a Sommerfeld Radiation condition. You are to write a code to perform the finite element solution for $H_z(x,y)$. This can be done using MathCad, MatLab, Mathematica, or FORTRAN90. You will then compute the mean error in your solution versus the analytical solution. The mean error is to be computed as:

$$\text{Mean Error} = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|H_{z_i}^{Exact} - H_{z_i}^{FEM}\right|}{\left|H_{z_i}^{Exact}\right|}$$

   where, N = the number of nodes, and H_{z_i} is the field evaluated at the node of the mesh. You will then plot the error versus the average cell size to study error convergence. Four meshes are provided to you in the zip file: *TEzNodalMeshRectDomain.zip*, which can be downloaded from the homework web page. You can also compare surface plots (shaded contour type) of the FEM fields with the exact fields as part of your validation.

   To validate your solution, you will consider the following cases:

   i) f = 75 MHz, $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1$

   ii) f = 75 MHz, $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 2$, $\varepsilon_{r3} = 3$. Note that for this case, mesh1 is very coarse, and the error will be quite large.