HW#3
1D Finite Element Method
EE699 – Fall 2007
Due 10/11/07

1. You are given a one-dimensional domain defined by $x \in (0, L)$, where for simplicity, let L = 1 m. The domain is inhomogeneously filled with a dielectric material, such that over the range $0 < x < 0.4$, $\varepsilon_r = 5$, and over the range $0.4 < x < 1$, $\varepsilon_r = 2.0$. The electrostatic potential is defined over the one-dimensional domain as $V(x)$ and has the boundary conditions $V(0) = 5$ V, and $V(1)$ = -5 V.
   a. Derive the analytical solution for $V(x)$ over the range $x \in (0, L)$ given the Dirichlet boundary conditions.
   b. You are to solve this problem via the finite element method. You can assume a uniform discretization within each homogeneous material media region. Also, assume linear interpolation functions over each discrete segment. The finite element solution can be developed within Matlab, MathCad, or Mathematica. Or, if you prefer, you can write a program in FORTRAN or C++ (with permission from the instructor). For the solution, you can assume a dense linear system of equations. In the process, you will need to: *i*) validate your system matrix values (use a fairly coarse discretization to do so). *ii*) validate your forcing vector. *iii*) validate your solution versus the known analytical solution.

2. You are given a one-dimensional domain defined by $x \in (0, L)$, where for simplicity, let L = 1 m. The domain is inhomogeneously filled with a dielectric material, such that over the range $0 < x < 0.4$, $\varepsilon_r = 5$, and over the range $0.4 < x < 1$, $\varepsilon_r = 2.0$. A time-haromonic electromagnetic field has been excited in the 1-dimensional domain at a frequency of f = 300 MHz. The electric field $E_z(x)$ is defined over the range $x \in (0, L)$ and has the boundary conditions $E_z(0) =$ 1 V/m, and $\partial E_z(x)/\partial x\big|_{x=L} = -jkE_z(x)\big|_{x=L}$, where k is the wave number in the material region at $x = L$ (i.e., $k = \sqrt{2}k_0$, and $k_0 = \omega / c = 2\pi / \lambda_0$ is the free space wave number.)
   a. Derive the analytical solution for $E_z(x)$ over the range $x \in (0, L)$ given the Dirichlet and Nuemann boundary conditions.
   b. You are to solve this problem using the finite element method. To do this, basically, follow the instructions of problem 1 part b. Again, present the validations that are specified by items *i), ii),* and *iii).* Note that for this problem, the finite element solution will *not* be exact, but will be a linear approximation to the problem. Thus, you should also do the following: *iv*) Plot the error in the solution versus the number of unknowns in the solution vector. The error can be estimated as:

   Mean Error $= \dfrac{1}{N}\sum_{i=1}^{N}\left|E_{z_i}^{FEM} - E_{z_i}^{Exact}\right| / \left|E_{z_i}^{Exact}\right|$, where N is the number of unknowns.