Example of Signal Flow Analysis

- Given a 4-port network: $$\begin{pmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{pmatrix} = \begin{pmatrix} S_{11} & 0 & 0 & S_{14} \\ 0 & S_{22} & S_{23} & 0 \\ 0 & S_{23} & S_{33} & 0 \\ S_{14} & 0 & 0 & S_{44} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{pmatrix}$$
- Port 3 is connected to Port 4 with a matched transmission line with electrical length of 60°.
- Determine the resultant insertion loss between ports 1 and 2.
- Solution by direct analysis:
  - Our goal is to find V_2^- / V_1^+ with port 2 matched, and port 1 driven by a matched source

$$\begin{pmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{pmatrix} = \begin{pmatrix} S_{11} & 0 & 0 & S_{14} \\ 0 & S_{22} & S_{23} & 0 \\ 0 & S_{23} & S_{33} & 0 \\ S_{14} & 0 & 0 & S_{44} \end{pmatrix} \begin{pmatrix} V_1^+ \\ 0 \\ V_4^- e^{-j\frac{\pi}{3}} \\ V_3^- e^{-j\frac{\pi}{3}} \end{pmatrix}$$

- From the 2nd, 3rd and 4th rows:

$$V_2^- = S_{23} V_4^- e^{-j\frac{\pi}{3}}, \quad V_3^- = S_{33} V_4^- e^{-j\frac{\pi}{3}}, \quad V_4^- = S_{14} V_1^+ + S_{44} V_3^- e^{-j\frac{\pi}{3}}$$

- Plugging the 2nd equation into the 3rd:

$$V_4^- = S_{14} V_1^+ + S_{44} S_{33} V_4^- e^{-j\frac{\pi}{3}} e^{-j\frac{\pi}{3}}$$

- Solving for V_4^-:

$$V_4^- = \frac{S_{14}}{1 - S_{44} S_{33} e^{-j\frac{2\pi}{3}}} V_1^+$$

- Plugging this into the expression for V_2^-, leads to:

$$V_2^- = \frac{S_{23} S_{14} e^{-j\frac{\pi}{3}}}{1 - S_{44} S_{33} e^{-j\frac{2\pi}{3}}} V_1^+$$

- Thus, the new $S_{1,2}$ of the effective 2 port is:

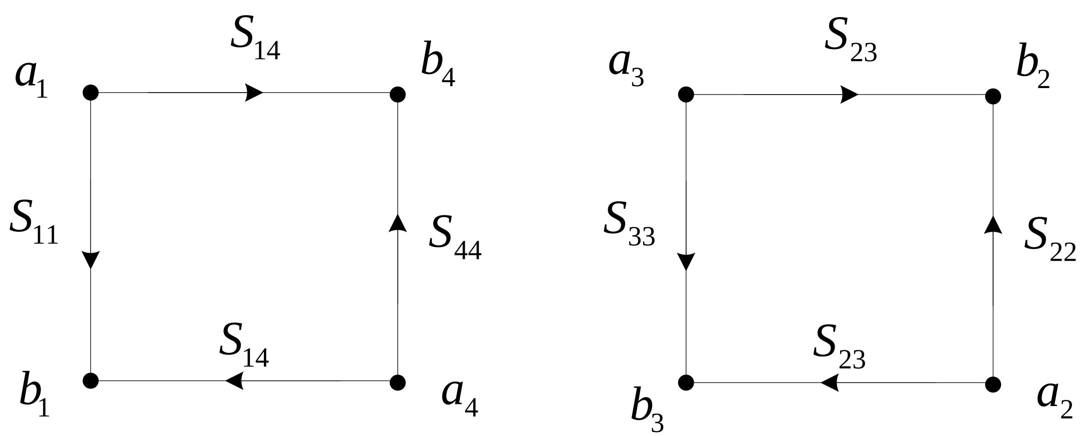
$$\tilde{S}_{1,2} = \frac{S_{23} S_{14} e^{-j\frac{\pi}{3}}}{1 - S_{44} S_{33} e^{-j\frac{2\pi}{3}}}$$

Solution via Signal Flow Analysis

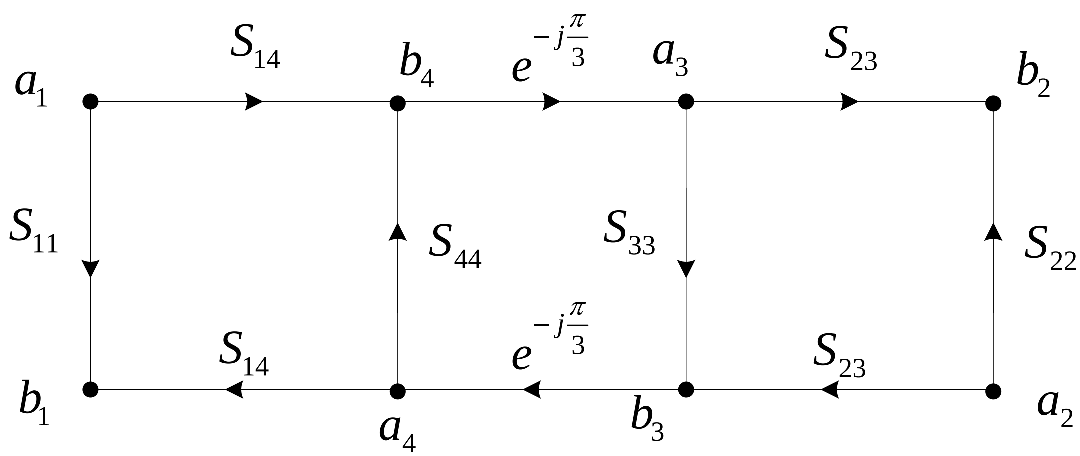
- This problem can be solved via a signal flow graph:

$$\begin{pmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{pmatrix} = \begin{pmatrix} S_{11} & 0 & 0 & S_{14} \\ 0 & S_{22} & S_{23} & 0 \\ 0 & S_{23} & S_{33} & 0 \\ S_{14} & 0 & 0 & S_{44} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{pmatrix}$$

Signal Flow Graph:

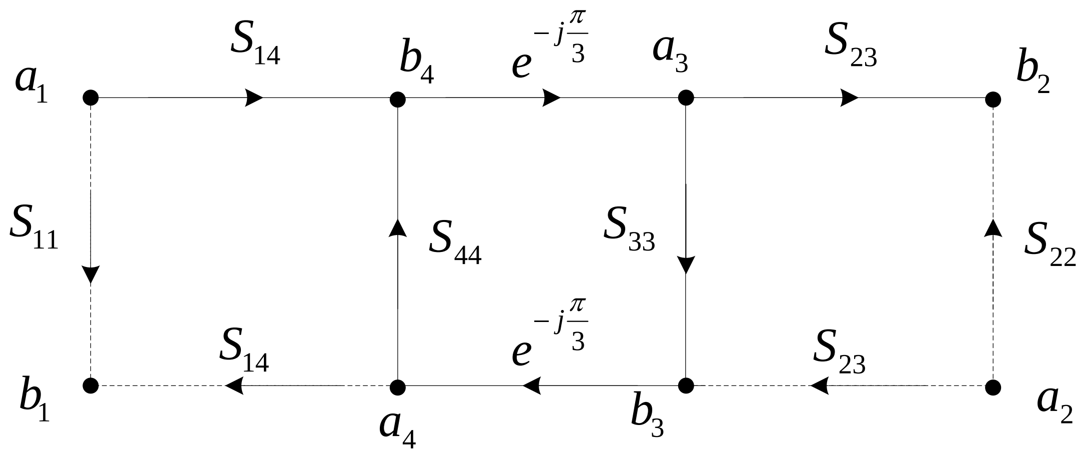


Connecting Ports 3 and 4 with a matched transmission line (60° length):

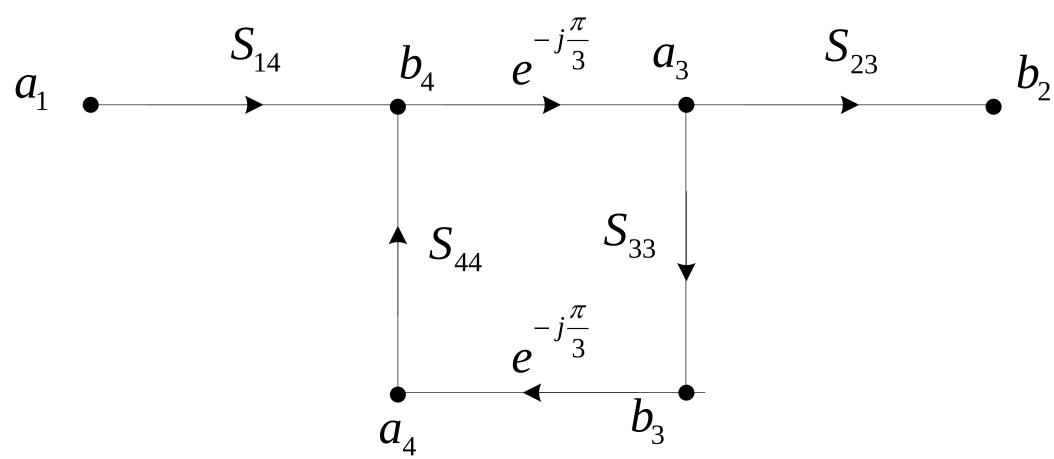


What ratio gives the resultant $\tilde{S}_{21}$?

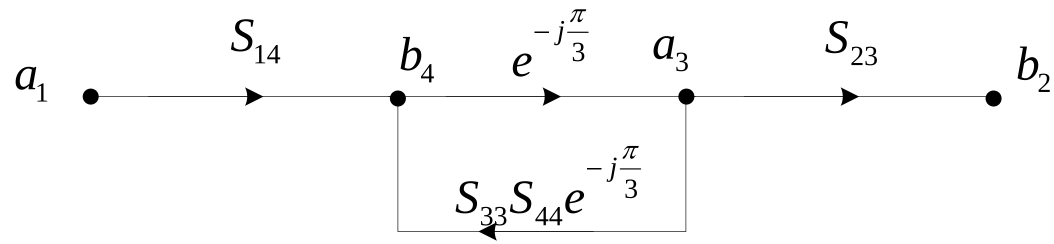
Finding b_2 / a_2:



- Note: B/c port 2 is matched, and port 1 is driven with a matched source, the dashed branches cannot contribute to b_2 when the network is driven by a_1. Thus, these branches can be removed for *this* analysis.

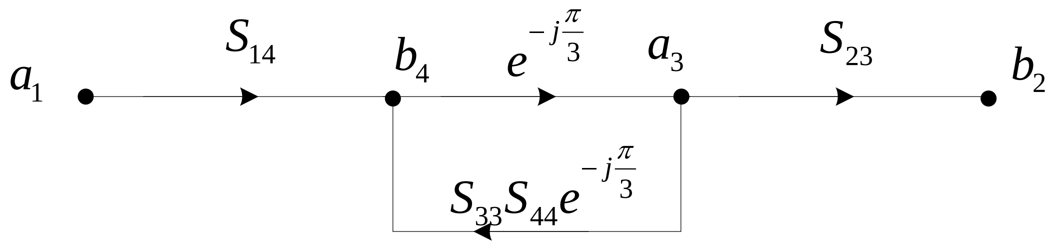


- Applying a series rule:

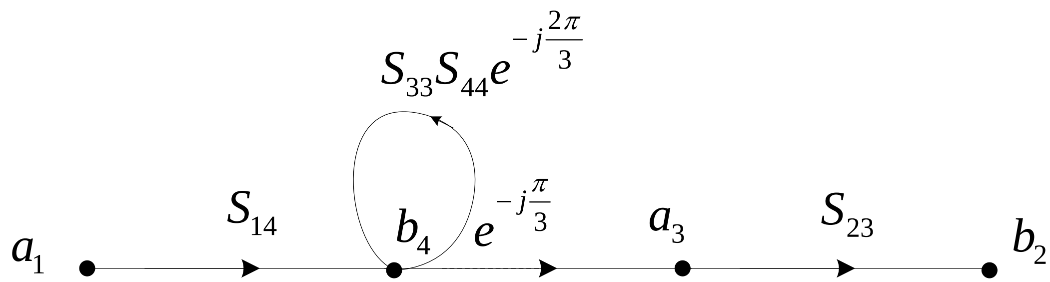


- What next?

- Applying a series rule:

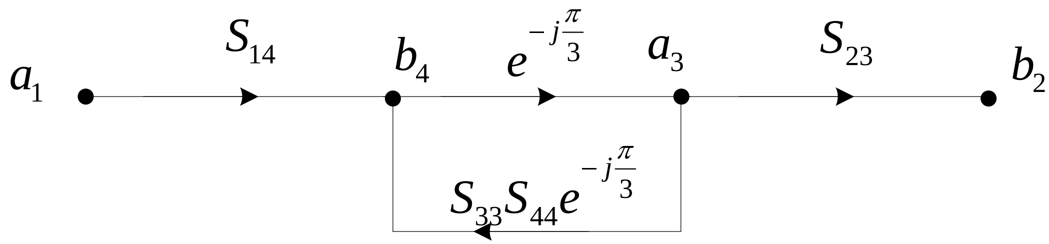


- Splitting rule:

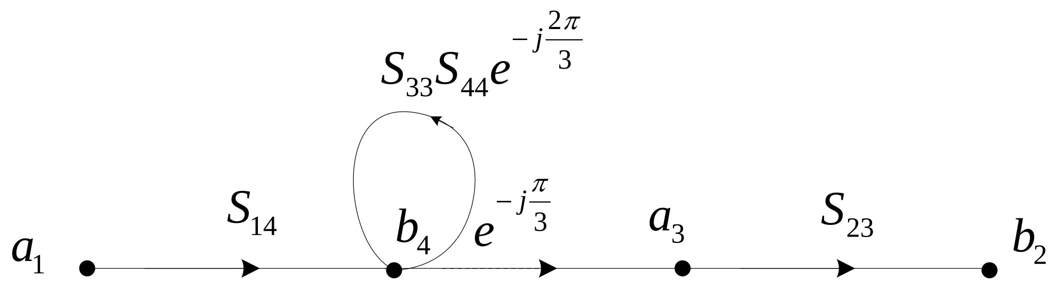


- What Next?

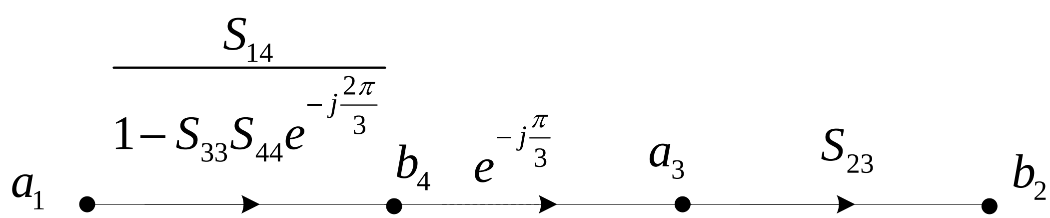
- Applying a series rule:

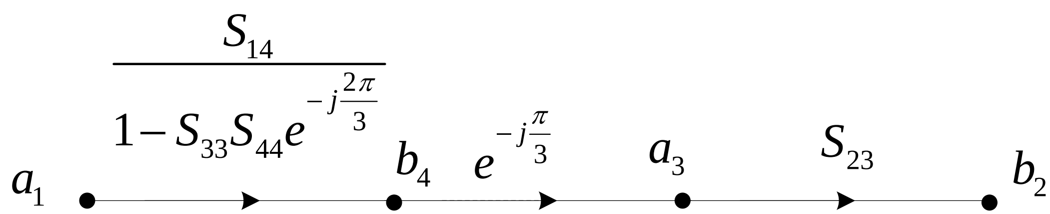


- Splitting rule:



- Self-Loop Rule:

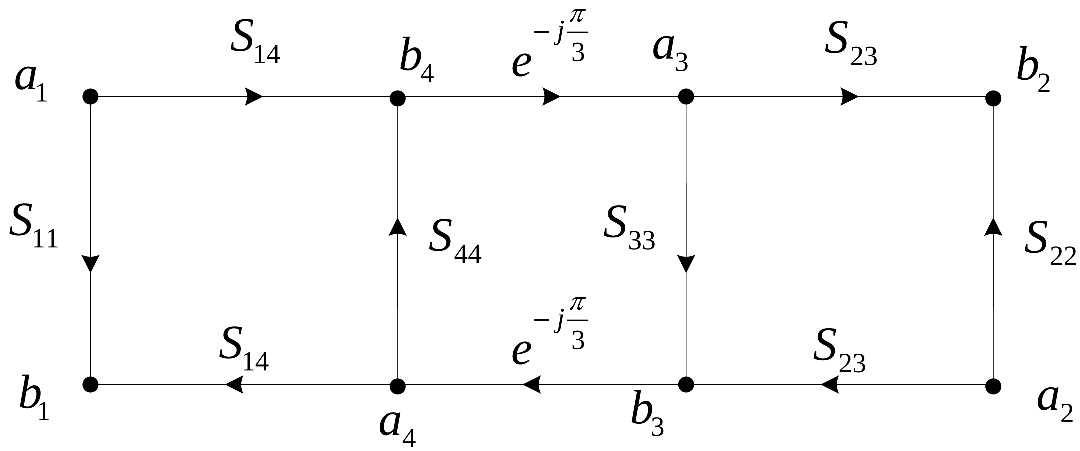




- Series Rule:

$$\frac{b_2}{a_1}=\frac{S_{14}S_{23}e^{-j\frac{\pi}{3}}}{1-S_{33}S_{44}e^{-j\frac{2\pi}{3}}}$$

Mason's Rule



$$P_1 =$$

$$P_2 =$$

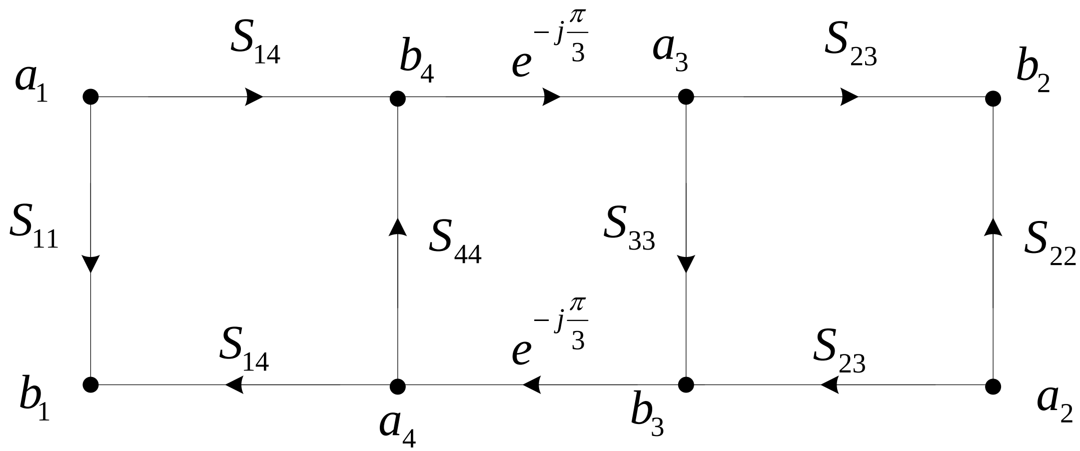
$$\sum L(1) =$$

$$\sum L(2) =$$

$$\sum L(1)^1 =$$

etc

Mason's Rule



$$P_1 = S_{14} e^{-j\frac{\pi}{3}} S_{23}$$

$$P_2 = 0$$

$$\sum L(1) = S_{44} e^{-j\frac{\pi}{3}} S_{33} e^{-j\frac{\pi}{3}}$$

$$\sum L(2) = 0$$

$$\sum L(1)^1 = 0$$

$$\frac{b_2}{a_1} = \frac{S_{14} e^{-j\frac{\pi}{3}} S_{23} (1-0)}{1 - S_{44} e^{-j\frac{\pi}{3}} S_{33} e^{-j\frac{\pi}{3}}} = \frac{S_{14} S_{23} e^{-j\frac{\pi}{3}}}{1 - S_{33} S_{44} e^{-j\frac{2\pi}{3}}}$$