MATCHING NETWORKS

- Matching networks provide a transformation of impedance to a desired value to maximize the power dissipated by a load. For example, the figure below illustrates the matching networks for a transistor amplifier. The matching networks ensure that the proper impedance is seen by the amplifier. One such matching method may be a conjugate match of the impedance.
- We will discuss many useful types of matching networks in class, including:
  - L-section matching
  - Quarter wave transformers
  - Single stub tuners

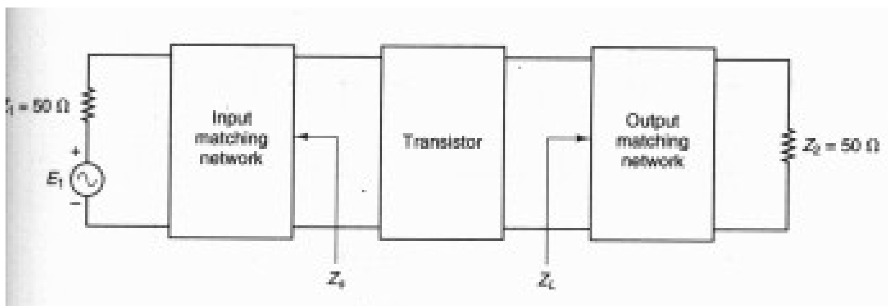


Figure 2.4.1 Block diagram of a microwave amplifier.

L-Section Matching Networks

- L-sections utilize purely reactive components such that no power is dissipated in the matching network
- Smith Charts are an extremely useful manner by which to design L-section matching networks

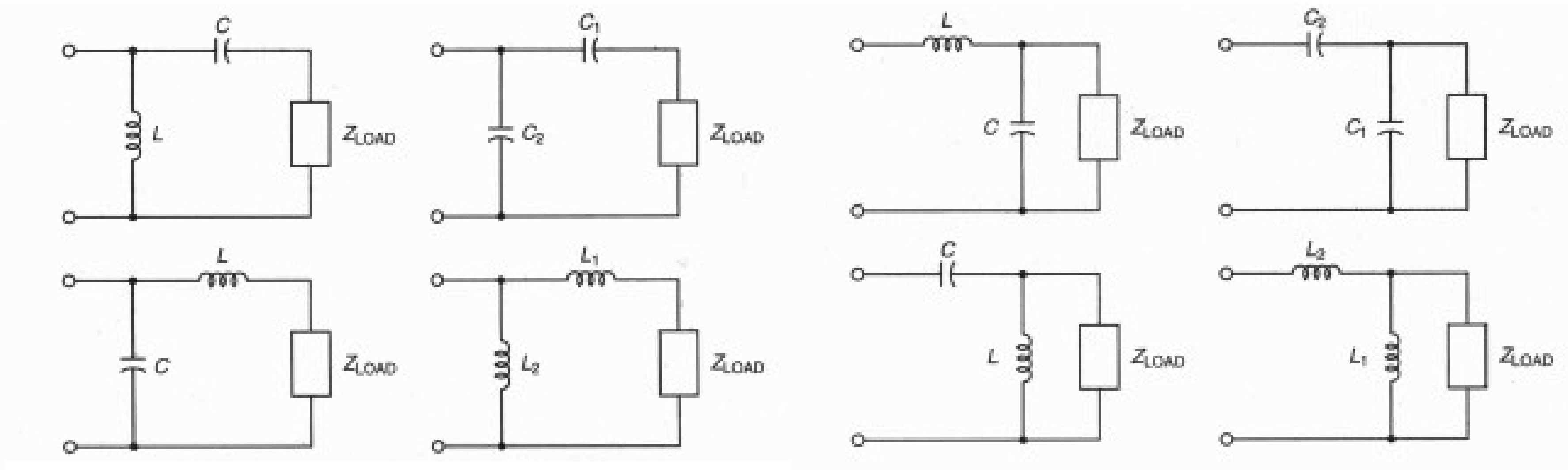


Figure 2.4.2 Matching networks.

- L-section design is best performed on an Admittance/Impedance Smith chart.
- Adding series reactive loads will modify the impedance by adding negative reactance (series C), or positive reactance (series L)
- Adding shunt reactive loads will modify the admittance by adding negative susceptance (shunt C), or positive susceptance (shunt L).
- Note that a solution for a given L-section is not guaranteed. The Smith chart provides visual insight into the feasibility of design.

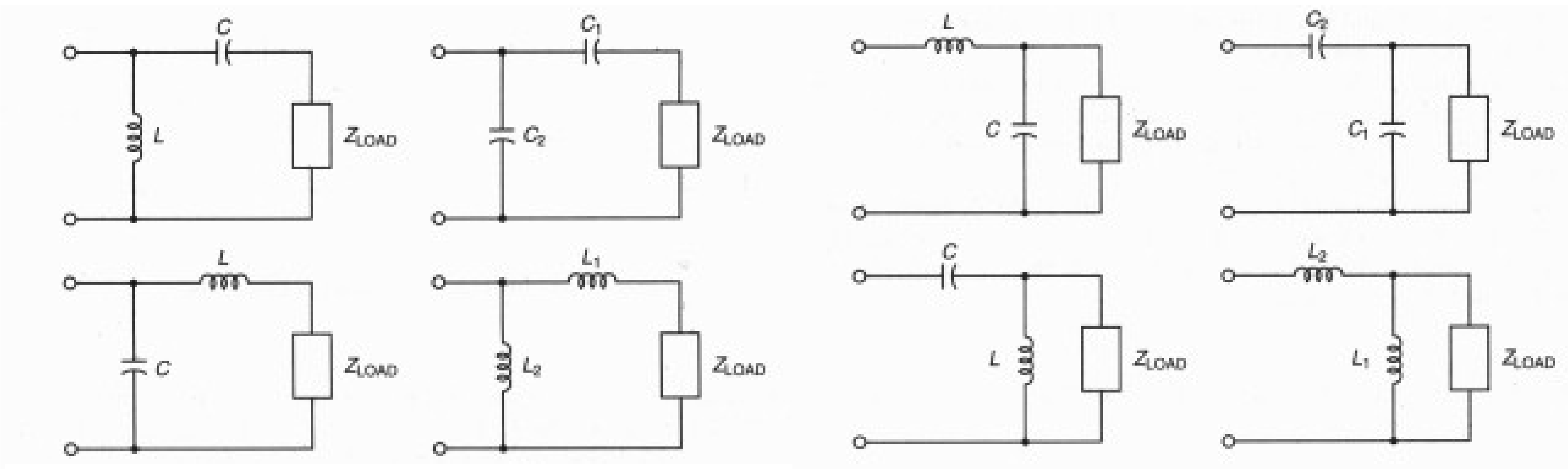


Figure 2.4.2 Matching networks.

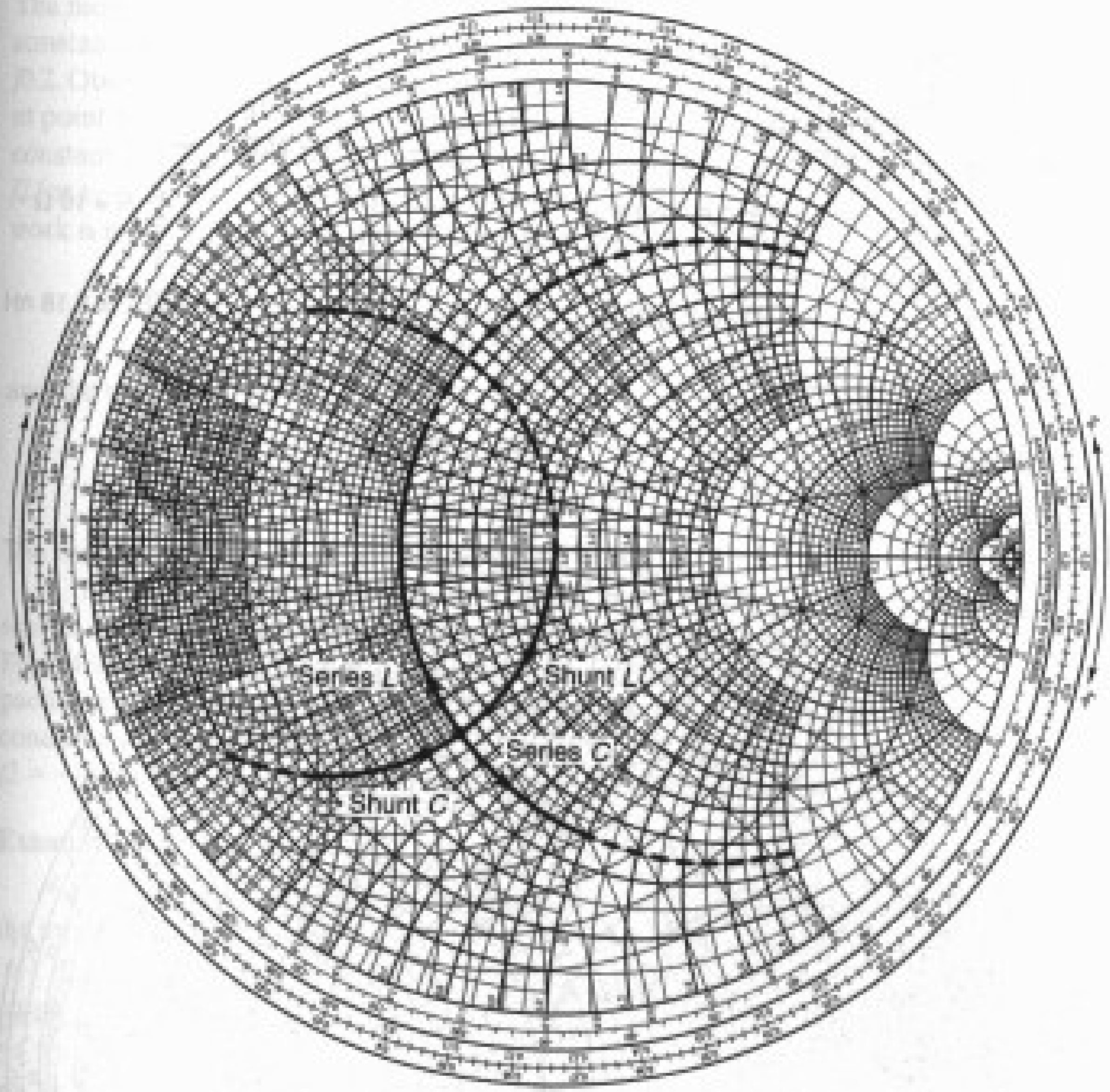


Figure 2.4.7 Effect of adding series and shunt elements in the *ZY* Smith chart.

- Example:A load $Z_L = 10 + j\,10\ \Omega$ is to be matched to a 50 Ω line. Design L-section matching networks at 500 MHz using:
  a) a series L, shunt C
  b) a series C, shunt L

Solution:

a) Given the normalized load: $0.2 + j\,0.2$, move along a constant-resistance circle until the unit conductance circle is intersected. This adds $j\,0.2$ normalized reactance. The normalized admittance at this point is $1 - j\,2$. Thus, the normalized capacitor admittance must be $j\,2$. This brings us to the origin.
Finally, the values for L and C are computed at 500 MHz:

$$L = \frac{0.2 \cdot 50}{2\pi(500 \times 10^6)} = 3.18\,\text{nH}$$

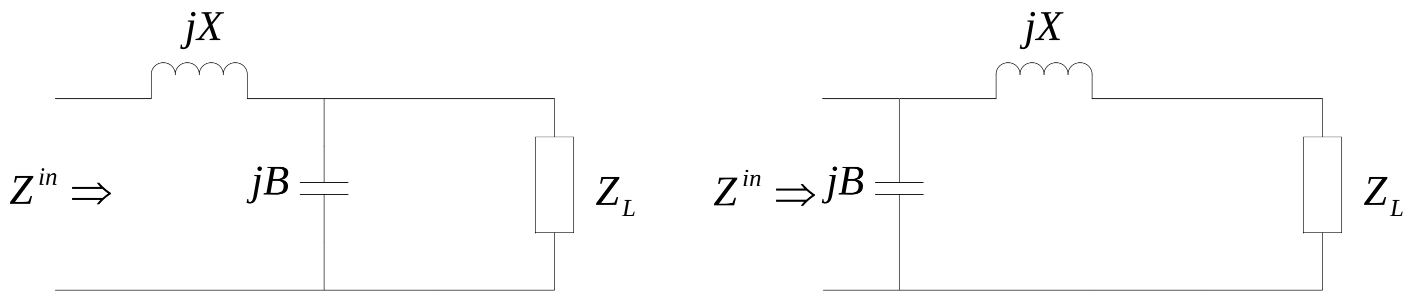
$$C = \frac{2}{50} \cdot \frac{1}{2\pi(500 \times 10^6)} = 12.74\,\text{pF}$$

b) For the series capacitor, we move along the constant resistance circle in the opposite direction until the unit conductance circle is intersected. Thus, the normalized impedance of the capacitor is $-j\,0.6$. The normalized admittance at this point is $1 + j\,2$. Thus, the normalized admittance of the inductor must be $-j\,2$. Thus, at 500 MHz:

$$C = \frac{1}{0.6 \cdot 50} \cdot \frac{1}{2\pi(500 \times 10^6)} = 10.6\,\text{pF} \quad L = \frac{50}{2} \cdot \frac{1}{2\pi(500 \times 10^6)} = 7.95\,\text{nH}$$

Analytical Solution for the L-Matching Network

- A simple matching network can be achieved with only 2 reactive elements
  - Transforms both the real and the imaginary part of the input impedance
- A common configuration of the 2 reactive elements is referred to as the L-section matching network (or "el-section")
  - Two types:



Network #1 Network #2

    - where, $Z_L = R_L + jX_L$
  - Consider network #1:

$$Z^{in} = jX + \frac{1}{jB + \frac{1}{R_L + jX_L}} = jX + \frac{R_L + jX_L}{1 + jBR_L - BX_L}$$

- We desire that $Z^{in} = R_s$. Therefore, we can write the equation:

$$R_s = jX + \frac{R_L + jX_L}{1 + jBR_L - BX_L}$$

- We essentially have two degrees of freedom to solve for: B and X
  - Multiply both sides by the denominator:

$$(R_s + jBR_L R_s - BX_L R_s) = (jX - XBR_L - jXBX_L) + R_L + jX_L$$

  - Equate the real and imaginary terms:

$$\text{Real: } B\left(XR_L - X_L\right)R_s = R_L - R_s$$

$$\text{Imag: } X\left(1 - BX_L\right) = BR_s R_L - X_L$$

  - From the imaginary term, find X as a function of B:

$$X = \frac{1}{B} + \frac{X_L R_s}{R_L} - \frac{R_s}{BR_L}$$

  - Plug this into the real. Then, solving for B:

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{R_s}}\sqrt{R_L^2 + X_L^2 - R_s R_L}}{R_L^2 + X_L^2}$$

  - We can solve for B, and then calculate X from B.

➡ Note, that there are 2 solutions available

- Choose the design that makes sense physically, and is easiest to construct
- From Network #1:

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{R_s}}\sqrt{R_L^2 + X_L^2 - R_s R_L}}{R_L^2 + X_L^2}$$

- Note that in the radical term, the argument can be negative. Typically, Network #1 is only used in the case when $R_L > R_s$.
- In this case, the argument will always be positive.

Network #2 is used when $R_L < R_s$

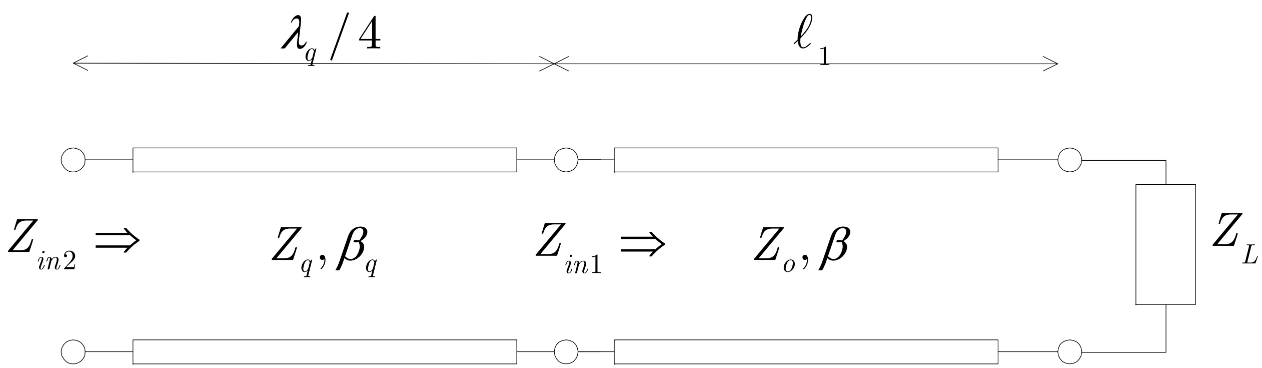
Following the same procedure of equating $Z^{in} = R_s$ for Network #2, we derive:

$$X = \pm\sqrt{R_L\left(R_s - R_L\right)} - X_L$$

$$B = \pm\frac{\sqrt{\left(R_s - R_L\right)/R_L}}{R_s}$$

The Quarter Wave Transformer

- The quarter wave transformer is another useful narrow band matching technique that allows the use of a quarter-wavelength of transmission line with controllable impedance connected to a real load
- Useful for waveguides for which one can control the dimensions and hence the characteristic line impedance (printed waveguides) when fabricating the network
- Useful Theorem:
  $Z\left(x \pm \lambda/4\right) Z(x) = Z_o^2$



Methodology

- Given a complex load Z_L, choose ℓ_1 such that Z_{in1} is purely real, i.e., $Z_{in1} = R$
- Choose Z_q such that $Z_{in2} = Z_0$. Thus,

$$\boxed{Z_o R = Z_q^2 \Rightarrow Z_q = \sqrt{Z_o R}}$$

- Note that in practice, one needs to design both Z_q and ℓ_q such that $Z_q = \sqrt{Z_o R}$ and $\ell_q = \lambda_q / 4$

Example

Design a QWT to match a 100+j50 ohm load to a 50 ohm transmission line. To design the QWT, move along the SWR circle to the nearest real axis crossing (Max or Min?).

Determine $\Gamma_L = 0.447e^{j0.464}$

Recall:

$$d_{\max} = \frac{\lambda}{2}\left(\frac{\phi}{2\pi}\right) = \frac{\lambda}{2}\left(\frac{0.464}{2\pi}\right) = 0.037\lambda \quad (\text{for } n = 0)$$

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} = 130.9\,\Omega$$

Finally,

$$Z_q = \sqrt{Z_{in} Z_o} = 80.9\,\Omega$$

Check:

$$Z_{inq} = Z_q \frac{Z_{in1} + jZ_q \tan(\pi)}{Z_q + jZ_{in1} \tan(\pi)} = 50\,\Omega$$

General Design Rule

- Given a complex load Z_L, choose ℓ_1 such that Z_{in1} is purely real, i.e., $Z_{in1} = R$
- If $R < Z_o$ (typically, if $\mathrm{Im}\left(Z_L\right) < 0$, then

$$d_{\min} = \frac{\lambda}{2}\left(\frac{1}{2} + \frac{\phi}{2\pi}\right) = \ (\text{for } n = 0)$$

$$Z_{q_{\min}} = \sqrt{Z_o Z_{in}} = \sqrt{Z_o^2\left(\frac{1-\left|\Gamma_L\right|}{1+\left|\Gamma_L\right|}\right)}$$

- If $R > Z_o$ (typically, if $\mathrm{Im}\left(Z_L\right) > 0$, then

$$d_{\max} = \frac{\lambda}{2}\left(\frac{\phi}{2\pi}\right) = \ (\text{for } n = 0)$$

$$Z_{q_{\min}} = \sqrt{Z_o Z_{in}} = \sqrt{Z_o^2\left(\frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}\right)}$$

Bandwidth

- The QWT is only truly matched at the resonant frequency. However, one can determine a frequency bandwidth over which the QWT yields a reflection coefficient below a desirable threshold.
- Assume, $Z_L = R$. Then, then input impedance to the QWT is:

$$Z_{in} = Z_q \frac{R + jZ_q \tan\left(\beta_q \ell_q\right)}{Z_q + jR\tan\left(\beta_q \ell_q\right)} = Z_q \frac{R + jZ_q t}{Z_q + jRt}$$

- This results in the reflection coefficient:

$$\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{Z_q\left(R - Z_o\right) + jt\left(Z_q^2 - Z_o R\right)}{Z_q\left(R + Z_o\right) + jt\left(Z_q^2 + Z_o R\right)}$$

- Since, $Z_q^2 = Z_o R$, this reduces to:

$$\Gamma = \frac{\left(R - Z_o\right)}{\left(R + Z_o\right) + j2t\sqrt{Z_o R}}$$

- It can be shown that:

$$|\Gamma| = \frac{1}{\sqrt{1 + \left[\frac{4Z_o R}{\left(R - Z_o\right)^2}\right]\sec^2\theta}}, \text{ where } \theta = \beta_q \ell_q$$

- Define a maximum allowable reflection coefficient Γ_m. Then, let $\theta_m = \theta$ for which $|\Gamma| = \Gamma_m$. Namely,

$$\theta_m = \cos^{-1}\left\{\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_o R}}{|R - Z_o|}\right\}$$

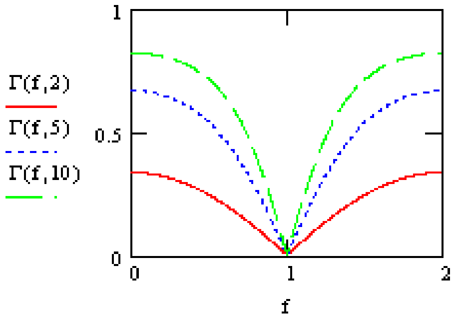
- Finally, the bandwidth is defined as:

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$

- Since $\theta = \beta\ell = \frac{2\pi f}{c}\ell$, and $\theta_m = \beta_m\ell = \frac{2\pi f_m}{c}\ell$

$$\Rightarrow \frac{\Delta f}{f_o} = \frac{2\left(f_o - f_m\right)}{f_o} = 2 - 2\frac{f_m}{f_o} = 2 - 4\frac{\theta_m}{\pi}$$

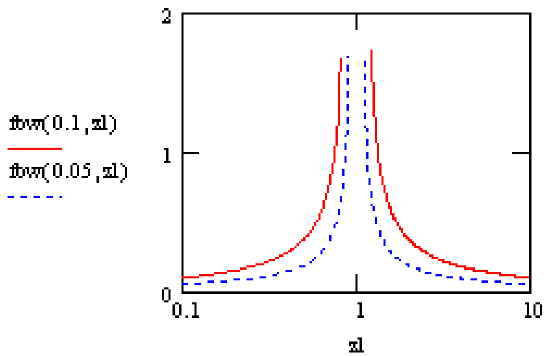
$|\Gamma|$ vs. f / f_o for various values of Z_L / Z_o



Fractional Bandwidth versus Z_L / Z_o given Γ_m

$$\theta m(\Gamma m, zl) := \operatorname{acos}\left(\frac{\Gamma m}{\sqrt{1-\Gamma m^2}} \cdot \frac{2\sqrt{zl}}{|zl-1|}\right)$$

$$fbw(\Gamma m, zl) := 2 - 4 \cdot \frac{\theta m(\Gamma m, zl)}{\pi}$$



Single Stub Tuning

- An alternate method of matching is to reactively load the line with a shunt load rendering the net line impedance equal to the characteristic impedance.
- A single stub tuner is best illustrated on a smith chart using the following procedure:
  - Plot the normalized load impedance on the smith chart
  - Draw the SWR circle and determine the line admittance
  - Move toward the load until you cross the r = 1 circle
  - At this point, the line admittance = 1 + jb
  - Add in a shunt load with input admittance = -jb
    - Note that a purely imaginary input admittance can be achieved by a short or open circuited line with the proper line length
  - At this point, the normalized admittance = 1. Thus, the line is matched to the load

- As with the QWT, the geometry of the line is simply modfied to manipulate the line impedance and reach a matched condition. No lumped loads are needed.
- In essence, the stub line cancels out the reactive power stored in the standing wave between the load and the line. Thus, all power is delivered to the load.

Analytical Solution

- A Smith chart will solve the SST approximately. Simple analytic solutions can also be derived.
- Assume a load impedance (admittance):

$$Z_L = R_L + jX_L;\ Y_L = 1/Z_L$$

- A distance *d* from the load, the line admittance is given as:

$$Y = G + jB = \frac{1}{Z_o}\frac{Z_o + j\left(R_L + jX_L\right)t}{\left(R_L + jX_L\right) + jZ_o t}$$

where $t = \tan \beta d$

- Evaluating the real and imaginary parts:

$$G = \frac{R_L\left(1+t^2\right)}{R_L^2 + \left(X_L + Z_o t\right)^2},\quad B = \frac{R_L^2 t - \left(Z_o - X_L t\right)\left(X_L + Z_o t\right)}{Z_o\left[R_L^2 + \left(X_L + Z_o t\right)^2\right]}$$

- Solving for *d* such that $G/Y_o = 1$

$$t = \begin{cases} \dfrac{X_L \pm \sqrt{R_L\left[\left(Z_o - R_L\right)^2 + X_L^2\right]/Z_o}}{R_L - Z_o}, & \text{if } R_L \neq Z_o \\ -\dfrac{X_L}{2Z_o}, & \text{if } R_L = Z_o \end{cases}$$

- Finally, there are two solutions for *d*:

$$\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t, & (t \geq 0) \\ \dfrac{1}{2\pi}\left(\pi + \tan^{-1} t\right), & (t < 0) \end{cases}$$

- At this point, the normalized line admittance = $1 + jB/Y_o$
- Thus, we need to add in a shunt stub tuner to cancel out the reactive part.
- Assume that the stub is terminated by a short circuit. Then:

$$Z_{in_{sc}} = jZ_o \tan\left(\beta\ell_{sc}\right) \Rightarrow Y_{in_{sc}} = -j\frac{1}{Z_o}\cot\left(\beta\ell_{sc}\right)$$

- It is desired that: $Y_{in_{sc}} = -jB$
- Therefore:

$$-jB = -j\frac{1}{Z_o}\cot\left(\beta\ell_{sc}\right) \Rightarrow \frac{\ell_{sc}}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{1}{Z_o B}\right)$$

- Similarly, for an open circuit stub:

$$Z_{in_{oc}} = -jZ_o \cot\left(\beta\ell_{oc}\right) \Rightarrow Y_{in_{oc}} = j\frac{1}{Z_o}\tan\left(\beta\ell_{oc}\right)$$

- Hence

$$\frac{\ell_{oc}}{\lambda} = -\frac{1}{2\pi}\tan^{-1}\left(BZ_o\right)$$

Example:

Design a QWT and an open-circuit single stub tuner to match a parallel resistor capacitor load to a $50\,\Omega$ TEM transmission line ($\varepsilon_r = 1$) at 1 GHz, given $R = 100\,\Omega$, $C = 1\,\mathrm{pF}$.

Solution (QWT):

$f := 1\cdot10^{9}$ $c := 3\cdot10^{8}$ $Zo := 50$ $R := 100$ $C := 1\cdot10^{-12}$

$\lambda := \frac{c}{f}$ $\lambda = 0.3$ $Xc := \frac{1}{j\cdot 2\cdot\pi\cdot f\cdot C}$ $Xc = -159.155i$

$ZL := \frac{R\cdot Xc}{R + Xc}$ $RL := \mathrm{Re}(ZL)$ $XL := \mathrm{Im}(ZL)$ $RL = 71.696$ $XL = -45.048$

Quarter Wave Transformer Design:

$\Gamma := \frac{ZL - Zo}{ZL + Zo}$ $\Gamma = 0.277 - 0.268i$

Note that due to the negative reactance, we first expect a Voltage Minimum:

$dmin := 0.5\cdot\left[\frac{1}{2} + \frac{(\arg(\Gamma))}{2\cdot\pi}\right]$ $dmin = 0.189$

$Zin := Zo\cdot\frac{(ZL + j\cdot Zo\cdot\tan(2\cdot\pi\cdot dmin))}{Zo + j\cdot ZL\cdot\tan(2\cdot\pi\cdot dmin)}$ $Zin = 22.186$

$Zq := \sqrt{Zin\cdot Zo}$

$Zq = 33.306$ Ω $dmin\cdot\lambda = 0.057$ m

If instead, we move to the first voltage maximum:

$dmax := 0.5\cdot\left[1 + \frac{(\arg(\Gamma))}{2\cdot\pi}\right]$ $dmax = 0.439$

$Zin := Zo\cdot\frac{(ZL + j\cdot Zo\cdot\tan(2\cdot\pi\cdot dmax))}{Zo + j\cdot ZL\cdot\tan(2\cdot\pi\cdot dmax)}$ $Zin = 112.684$

$Zq := \sqrt{Zin\cdot Zo}$

$Zq = 75.061$ Ω $dmax\cdot\lambda = 0.132$ m

Solution (SST):

$f := 1\cdot 10^{9}$ $c := 3\cdot 10^{8}$ $Zo := 50$ $R := 100$ $C := 1\cdot 10^{-12}$

$\lambda := \frac{c}{f}$ $\lambda = 0.3$ $Xc := \frac{1}{j\cdot 2\cdot\pi\cdot f\cdot C}$ $Xc = -159.155i$

$ZL := \frac{R\cdot Xc}{R+Xc}$ $RL := Re(ZL)$ $XL := Im(ZL)$ $RL = 71.696$ $XL = -45.048$

Single Stub Tuner Design (RL != Zo):

Solution 1:

$$t1 := \frac{XL + \sqrt{RL\cdot\frac{\left[(Zo-RL)^2 + XL^2\right]}{Zo}}}{RL - Zo}$$ $t1 = 0.683$

$$d1 := \frac{1}{2\cdot\pi}\cdot atan(t1)\cdot\lambda$$ $d1 = 0.029$ m

$$B1 := \frac{RL^2\cdot t1 - (Zo - XL\cdot t1)\cdot(XL + Zo\cdot t1)}{Zo\cdot\left[RL^2 + (XL + Zo\cdot t1)^2\right]}$$ $B1 = 0.017$

$$ls1 := \frac{1}{2\cdot\pi}\cdot atan\left(\frac{1}{Zo\cdot B1}\right)\cdot\lambda$$ $ls1 = 0.042$ m

Solution 2:

$$t2 := \frac{XL - \sqrt{RL\cdot\frac{\left[(Zo-RL)^2 + XL^2\right]}{Zo}}}{RL - Zo}$$ $t2 = -4.836$

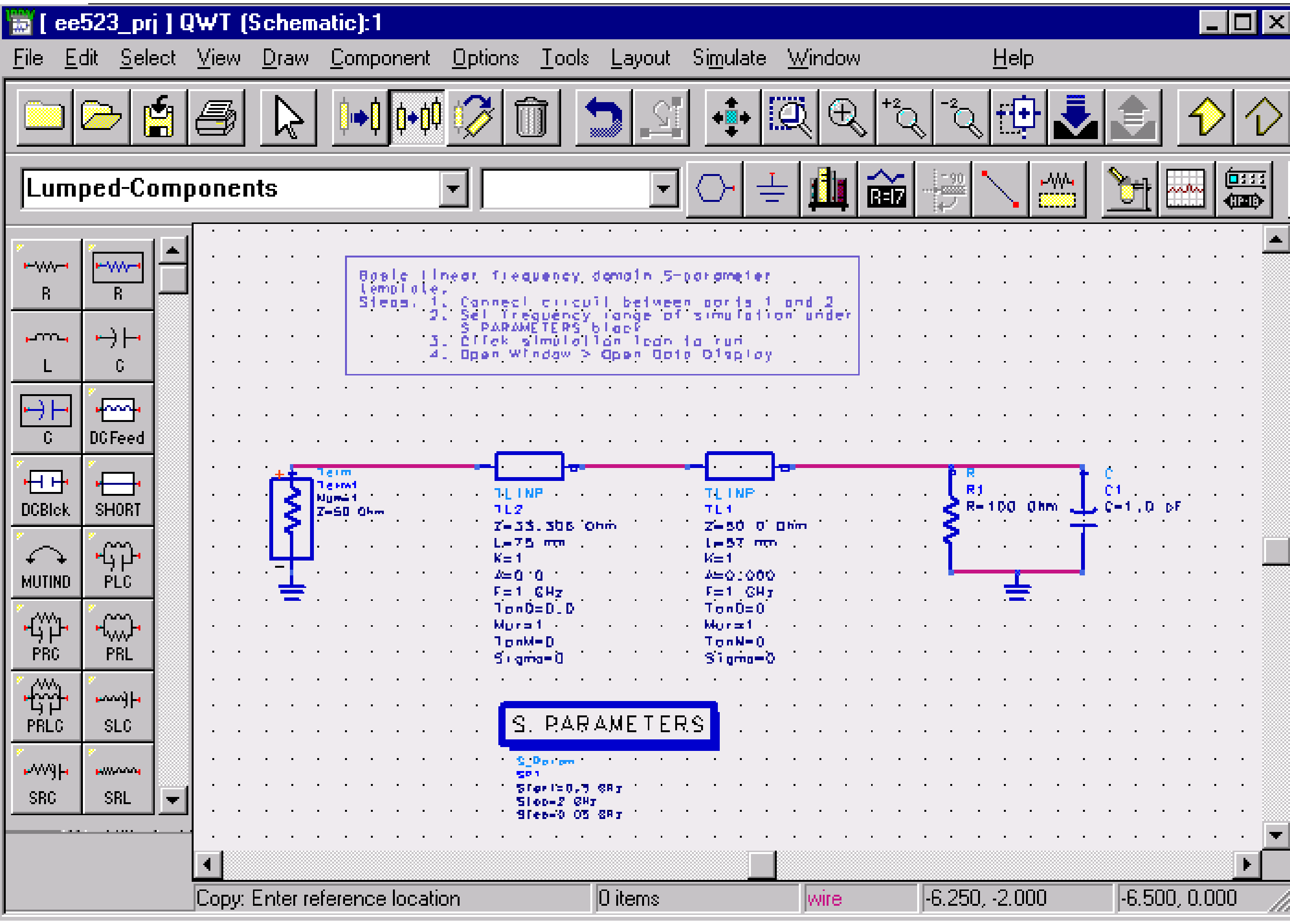
$$d2 := \frac{1}{2\cdot\pi}\cdot(\pi + atan(t2))\cdot\lambda$$ $d2 = 0.085$ m

$$B2 := \frac{RL^2\cdot t2 - (Zo - XL\cdot t2)\cdot(XL + Zo\cdot t2)}{Zo\cdot\left[RL^2 + (XL + Zo\cdot t2)^2\right]}$$ $B2 = -0.017$

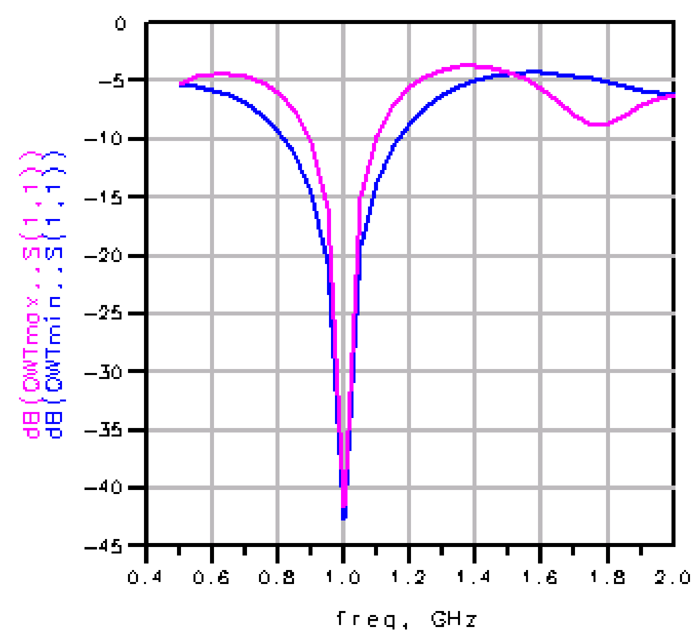
$$ls2 := \frac{1}{2\cdot\pi}\cdot atan\left(\frac{1}{Zo\cdot B2}\right)\cdot\lambda$$ $ls2 = -0.042$ m

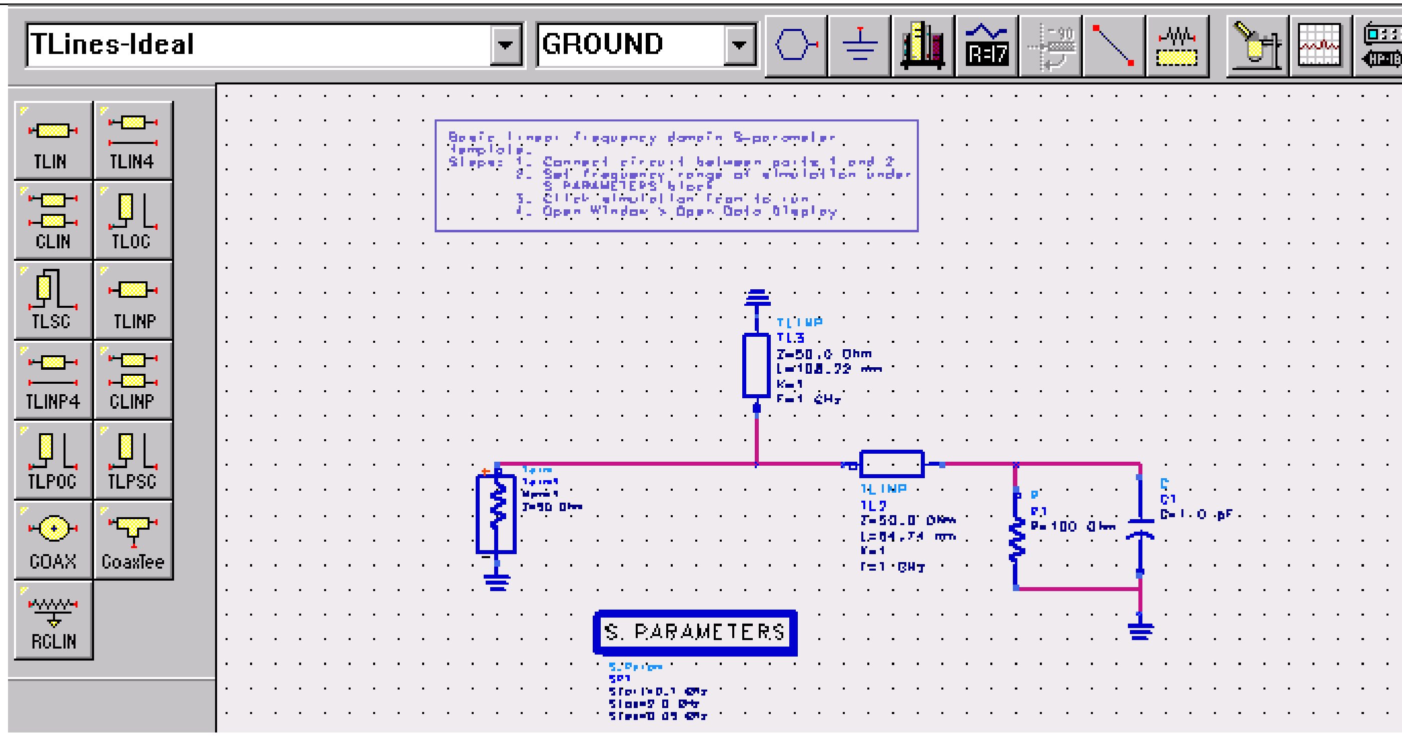
Note that since ls2 is negative, we need to add λ/2 to get a positive result:

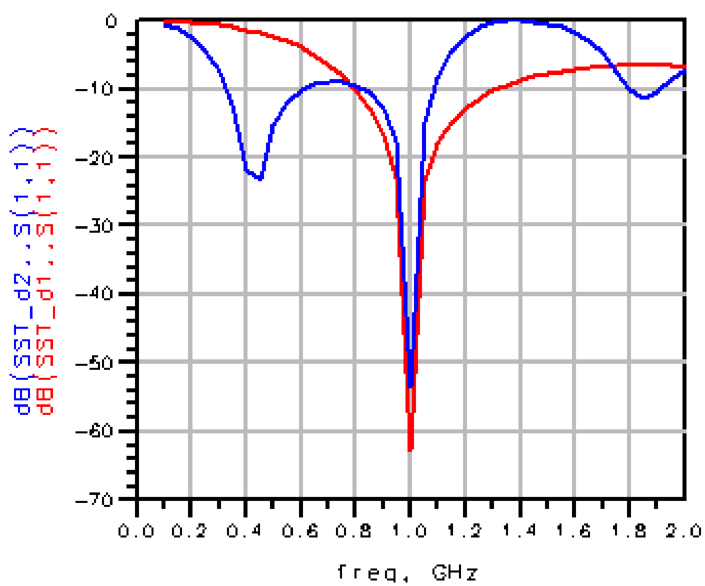
$$ls2 := ls2 + \frac{\lambda}{2}$$ $ls2 = 0.108$ m


[ee523_prj] QWT (Schematic):1
File Edit Select View Draw Component Options Tools Layout Simulate Window Help
Lumped-Components
R
R
L
C
C
DC Feed
DCBlck
SHORT
MUTIND
PLC
PRC
PRL
PRLC
SLC
SRC
SRL
Basic linear frequency domain S-parameter template.
Steps: 1. Connect circuit between ports 1 and 2
2. Set frequency range of simulation under S PARAMETERS block
3. Click simulation icon to run
4. Open Window > Open Data Display
Term
Term1
Num=1
Z=50 Ohm
TLINP
TL2
Z=35.355 Ohm
L=75 mm
K=1
A=0.0
F=1 GHz
TanD=0.0
Mur=1
TanM=0
Sigma=0
TLINP
TL1
Z=50.0 Ohm
L=57 mm
K=1
A=0.000
F=1 GHz
TanD=0
Mur=1
TanM=0
Sigma=0
R
R1
R=100 Ohm
C
C1
C=1.0 pF
S_PARAMETERS
Copy: Enter reference location
0 items
wire
-6.250, -2.000
-6.500, 0.000

|Γ| in dB


0
−5
−10
−15
−20
−25
−30
−35
−40
−45
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
freq, GHz
dB(QWTmax..S(1,1))
dB(QWTmin..S(1,1))


TLines-Ideal
GROUND
TLIN
TLIN4
CLIN
TLOC
TLSC
TLINP
TLINP4
CLINP
TLPOC
TLPSC
COAX
CoaxTee
RCLIN
S_PARAMETERS


dB(SST_d2..S(1,1))
dB(SST_d1..S(1,1))
0
-10
-20
-30
-40
-50
-60
-70
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
freq, GHz