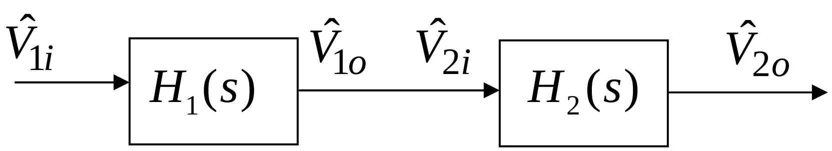
Cascading Filter Stages:

Given two active op-amp filter circuits with transfer functions $H_1(s)$ and $H_2(s)$. By definition, the op-amp circuit has a large input and output impedance.

In the limit that the input and output impedances tend to infinity, then cascading the filters yields a transfer function equal to the product of the individual circuit transfer functions:

$$H_T(s) = H_1(s)H_2(s)$$



$$H_T(s) = \frac{\hat{V}_{2o}}{\hat{V}_{1i}} = \left(\frac{\hat{V}_{1o}}{\hat{V}_{1i}}\right)\left(\frac{\hat{V}_{2o}}{\hat{V}_{2i}}\right) = H_1(s)H_2(s)$$

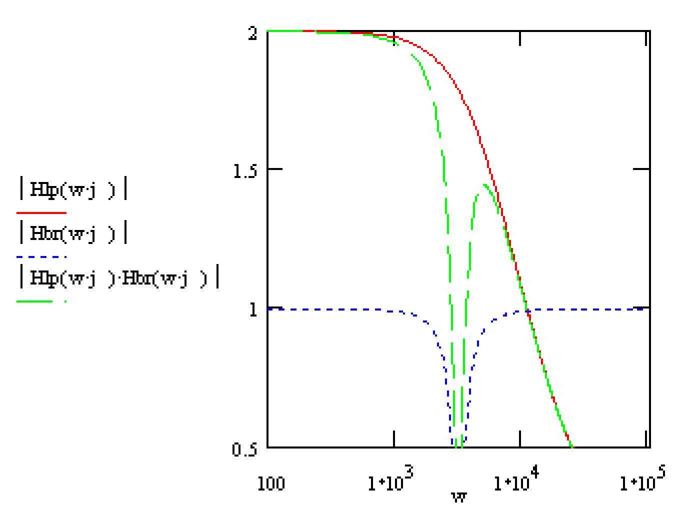
(If the output impedance of *H*1, or the input impedance of *H2* are not large, this relationship is no longer true)

Example: Sketch the transfer function magnitude of a first-order low-pass filter (with $A_v = 2$ and f_c = 1 kHz) in series with a second order band-reject filter (with f_0 = 500 Hz, B = 200 Hz, and A_v 1).

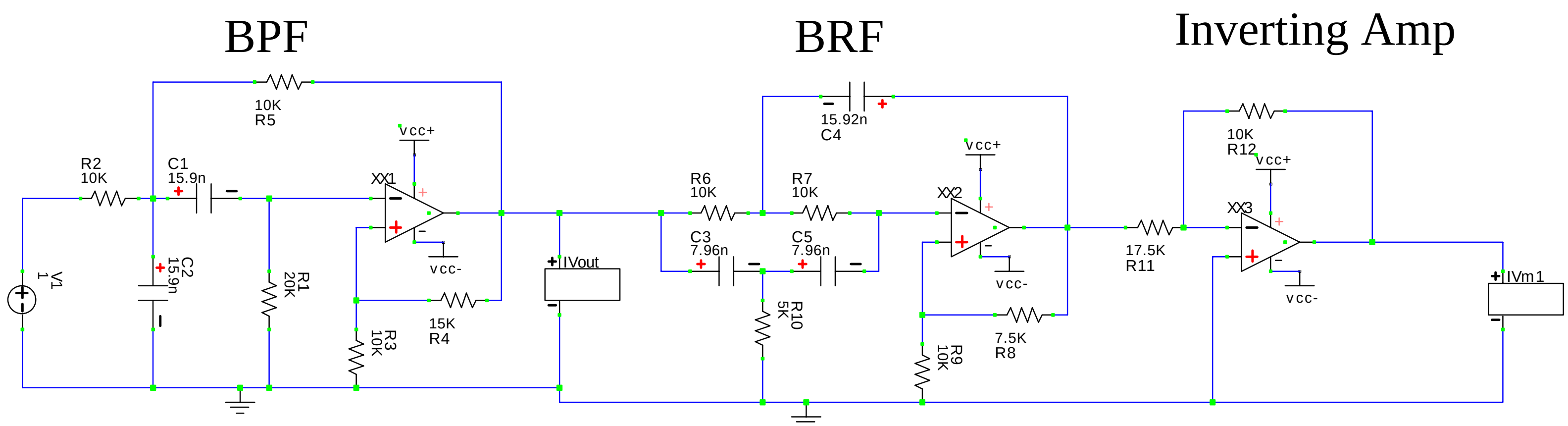
Solution:

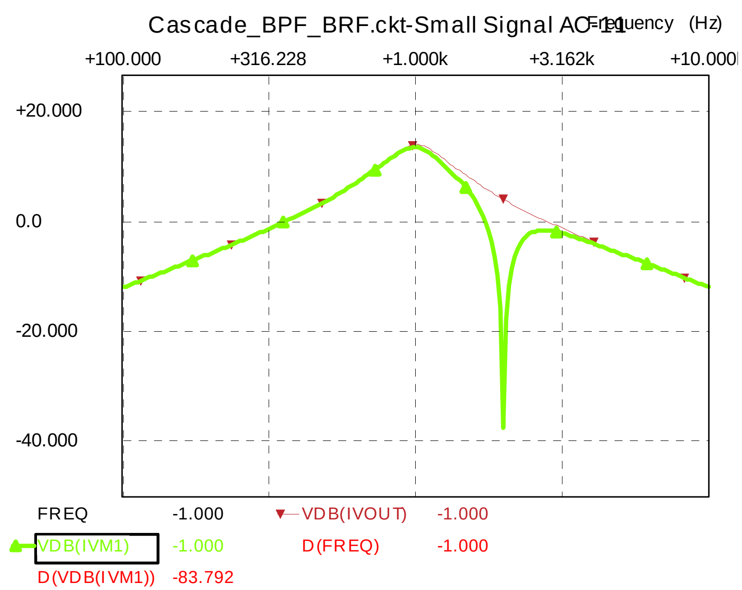
$$H_{LP}(s) = \frac{A_v}{1 + \dfrac{s}{\omega_c}} = \frac{2}{1 + \dfrac{s}{2\pi \cdot 1 \times 10^3}},$$

$$H_{BR}(s) = \frac{(s^2 + \omega_o^2) A_v}{s^2 + \dfrac{\omega_o}{Q} s + \omega_o^2} = \frac{(s^2 + (2\pi \cdot 500)^2) \cdot 1}{s^2 + 2\pi \cdot 200 s + (2\pi \cdot 500)^2}$$



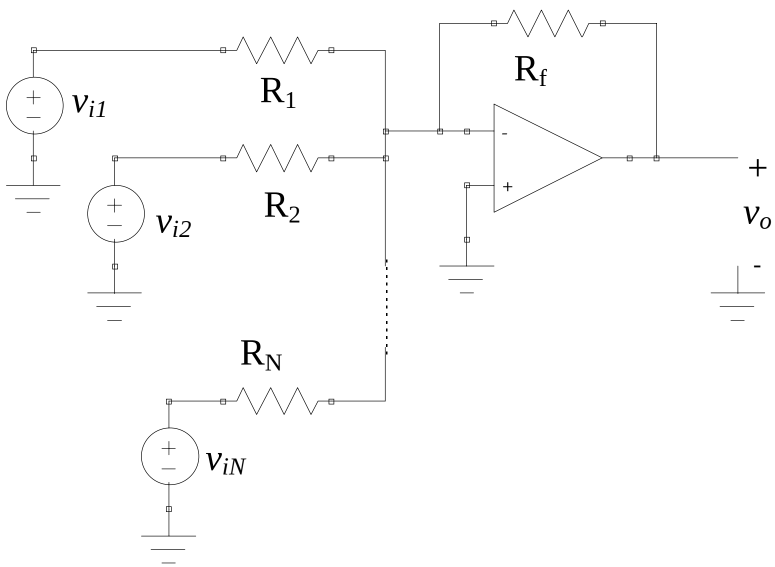
Example: Cascade a Second-Order Sallen Key BPF (f_o = 1 kHz, Q = 2, A_v = 14dB) in series with a second order Sallen Key band-reject filter (with f_0 = 2 kHz, Q = 2, and A_v = 1.75). The total gain is adjusted to 14 dB through an inverting amp.





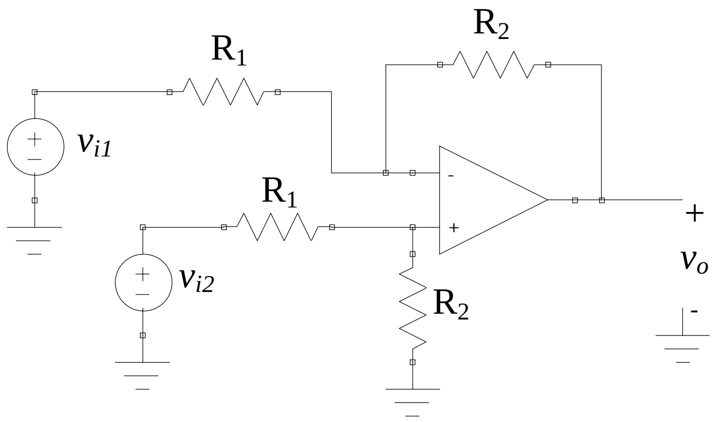
Parallel Combination of Filters

Summing Amplifier:



$$v_o = -\left(\frac{R_f}{R_1}v_{i1} + \frac{R_f}{R_2}v_{i2} + \cdots + \frac{R_f}{R_N}v_{iN}\right)$$

Differential Amplifier:

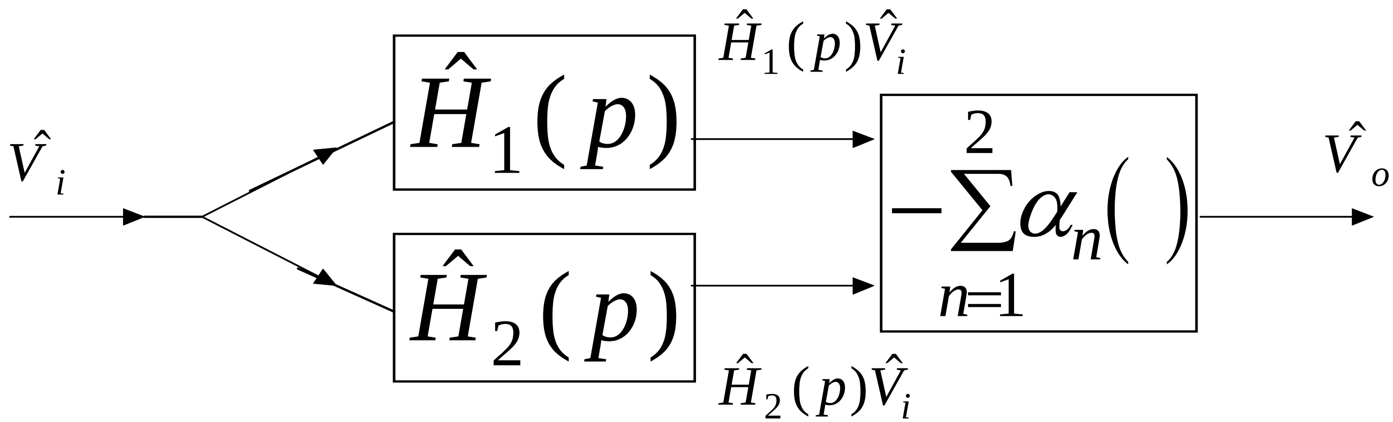


$$v_o = \frac{R_2}{R_1}\left(v_{i2} - v_{i1}\right)$$

Adding Outputs of Filter Stages:

Given two filter circuits with transfer functions $H_1(s)$ and $H_2(s)$, a circuit composed of these circuits connected in parallel with a common input and their outputs summed together (through a summing amp) has a transfer function equal to the scaled sum of the individual transfer functions $H_T(s) = \alpha_1 H_1(s) + \alpha_2 H_2(s)$, provided that the connection of these circuits does not significantly alter the output resistance or input resistance of these circuits (i.e. no loading effects).

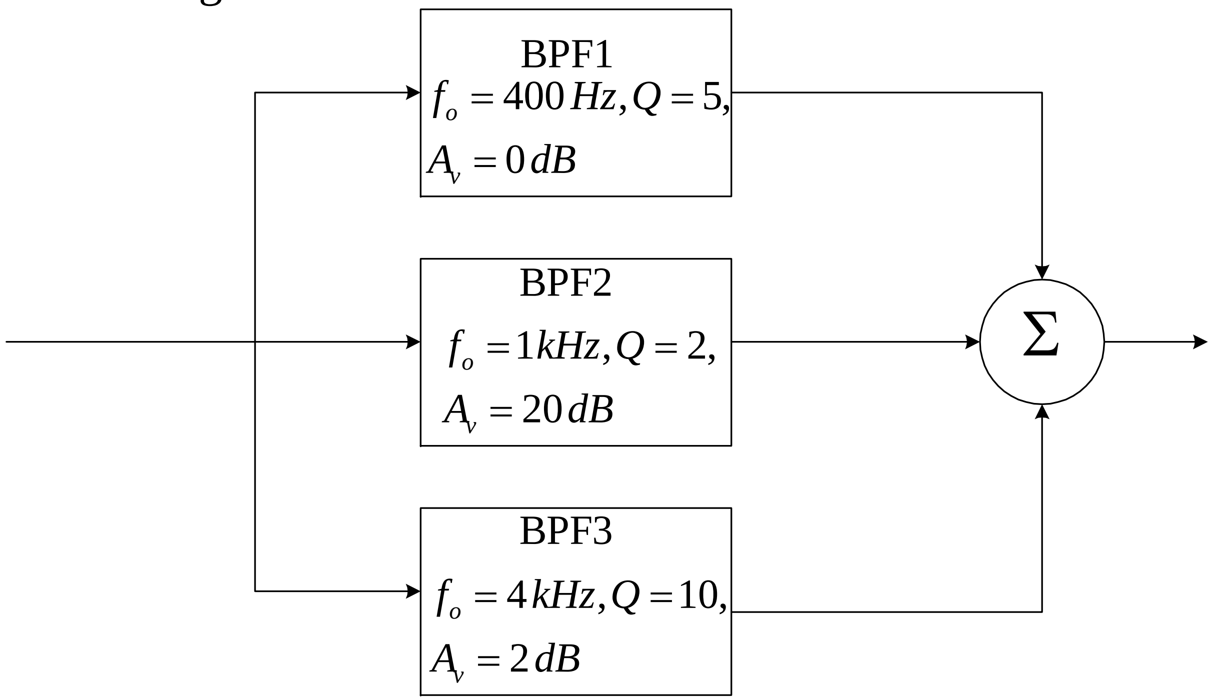
$$\hat{V}_o = -\alpha_1 H_1(s)\hat{V}_i - \alpha_2 H_2(s)\hat{V}_i$$



$$\Rightarrow \quad H_T(s) = \frac{\hat{V}_o}{\hat{V}_i} = -\alpha_1 H_1(s) - \alpha_2 H_2(s)$$

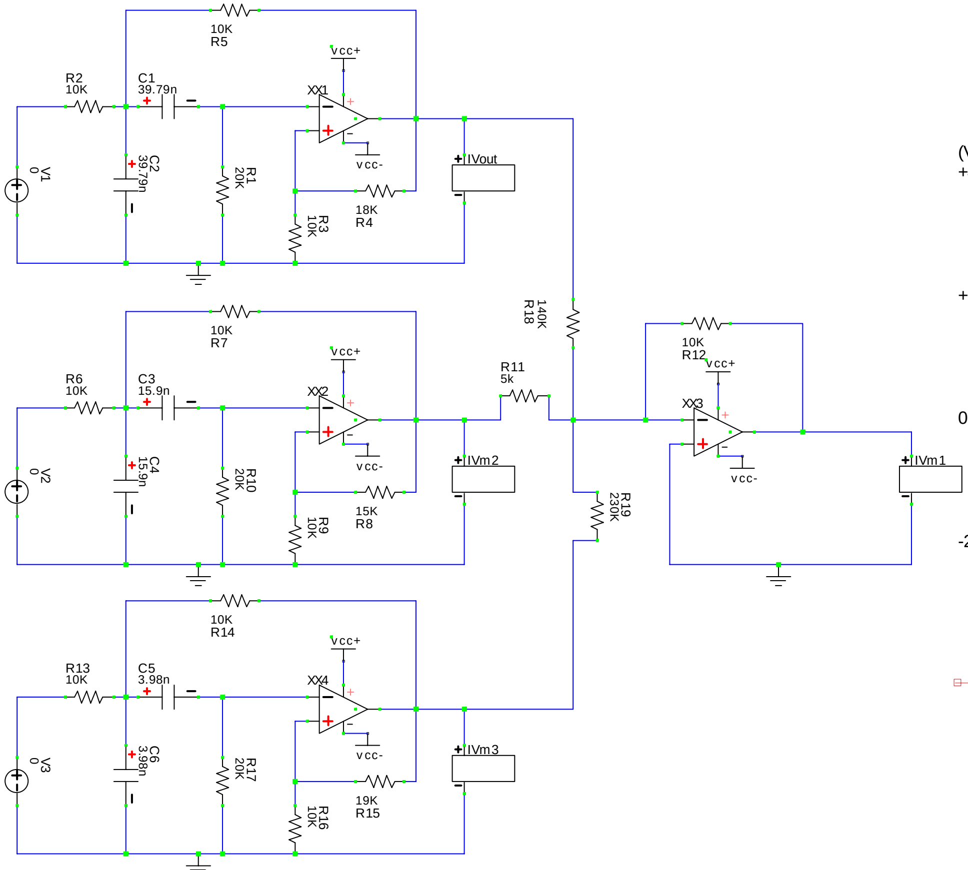
Example:
You are to Sum three BPF's using Sallen-Key Filters

Top Level Design:

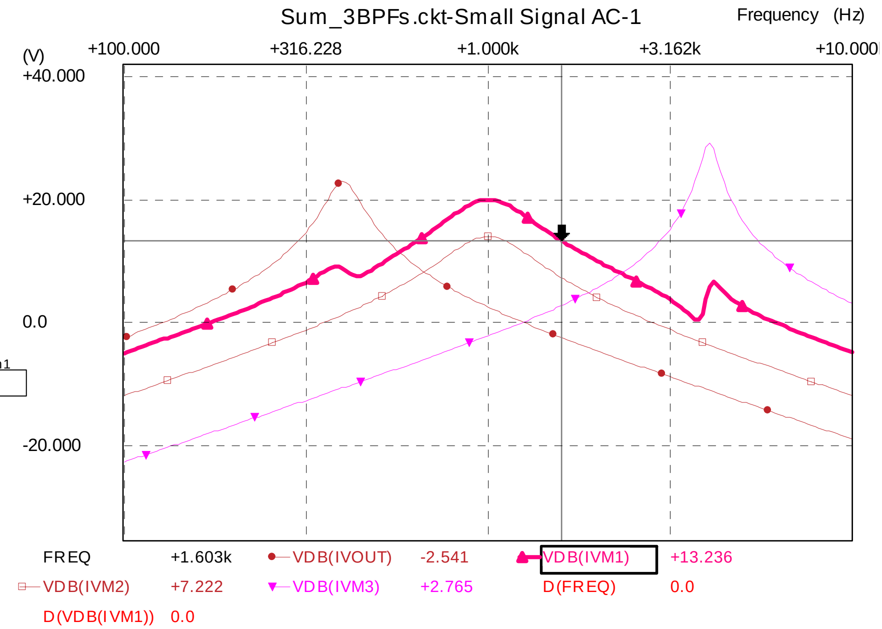


Design ($\omega_o = 1/RC, Q = 1/(3-A)$, $A_v = A/(3-A)$):

BPF1: R = 10 K, C = 39.79 nF, A = 3-1/Q = 2.8, $A_v = 14$. Therefore, Σ-amp needs gain 1/14.
BPF2: R = 10 K, C = 15.92 nF, A = 3-1/Q = 2.5, $A_v = 5$. Therefore, Σ-amp needs gain 2
BPF3: R = 10 K, C = 3.98 nF, A = 3-1/Q = 2.9, $A_v = 29$. Therefore, Σ-amp needs gain 1/23

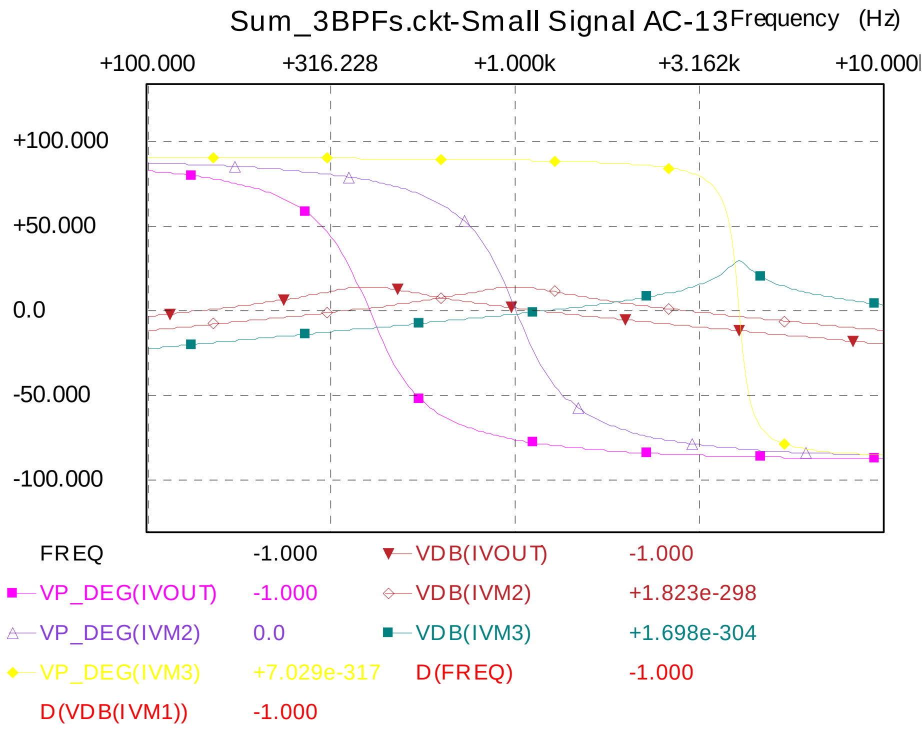


10K
R5
R2
10K
C1
39.79n
XX1
vcc+
vcc-
C2
39.79n
R1
20K
V1
0
R3
10K
18K
R4
IVout
10K
R7
R6
10K
C3
15.9n
XX2
C4
15.9n
R10
20K
V2
0
R9
10K
15K
R8
IVm2
10K
R14
R13
10K
C5
3.98n
XX4
C6
3.98n
R17
20K
V3
0
R16
10K
19K
R15
IVm3
140K
R18
R11
5k
10K
R12
XX3
R19
230K
IVm1



Sum_3BPFs.ckt-Small Signal AC-1
Frequency (Hz)
(V)
+100.000
+316.228
+1.000k
+3.162k
+10.000
+40.000
+20.000
0.0
-20.000
FREQ +1.603k
VDB(IVOUT) -2.541
VDB(IVM1) +13.236
VDB(IVM2) +7.222
VDB(IVM3) +2.765
D(FREQ) 0.0
D(VDB(IVM1)) 0.0

Observe the phase of the Transfer functions:



Phase = 0 at resonant frequency. It is positive for $f < f_o$ (inductive) and negative for $f > f_o$ (capacitive).

The total complex signal is summed, not just the amplitudes. With phase reversal, there can be cancellation when summing in ranges of frequencies between resonances.