EE211 Name So lution

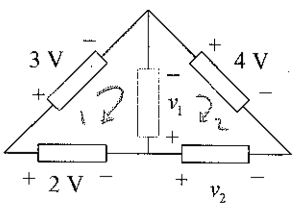
Final Exam May 3, 2007, 1 – 3 pm

Instructions: There are a total of 13 questions on this exam. Read each question carefully before answering. Show *all* work on the final. If more room is needed, write on the back page, or request an additional sheet of paper from the proctor. As necessary, *indicate units* when writing an answer. Circle your final answers in each problem. If an answer is not circled, it is assumed that no final answer was derived.

P1	P2	P3	P4	P5	P6	P7
7 *7 points*	8 *8 points*	8 *8 points*	7 *7 points*	7 *7 points*	8 *8 points*	8 *8 points*
P8	**P9**	**P10**	**P11**	**P12**	**P13**	**Total**
8 *8 points*	7 *7 points*	8 *8 points*	8 *8 points*	8 *8 points*	8 *8 points*	100 *100 points*

P1. *Basic Laws:*

a) Given the network below, determine the branch voltages v_1 and v_2.



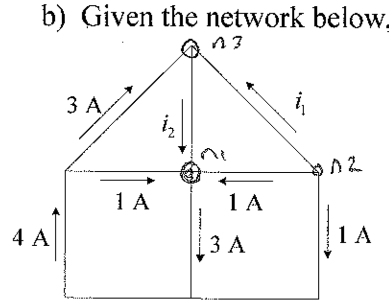
Use KVL

Loop 1 : $+3V - V_1 - 2V = 0 \Rightarrow \boxed{V_1 = 1V}$

Loop 2 : $4V - V_2 + V_1 = 0$

$\boxed{V_2 = 5V}$

b) Given the network below, determine the branch currents i_1 and i_2.



KCL : n1 : $i_2 + 1A + 1A = 3A$

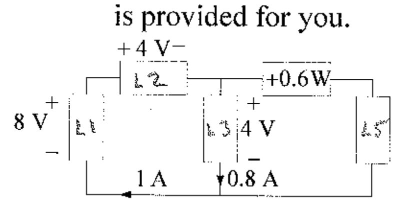
$\boxed{i_2 = 1A}$

n2 : $i_1 + 1A + 1A = 0$ $\boxed{i_1 = -2A}$

Check : n3: $3A + i_1 = i_2$

$3A - 2A = 1A$ ✓

c) For the network below, determine the power dissipated (or delivered) by each branch element. Write the power in the box representing each branch element. The power of 1 branch element is provided for you.



Power :

L1 : $P1 = 8V \times (-1A) = -8W$

L2 : $P2 = 4V \times (+1A) = +4W$

L3 : $P3 = 4V \times (0.8A) = +3.2W$

Power Conservation :

$P1 + P2 + P3 + 0.6W + P5 = 0$

$-8W + 4W + 3.2W + 0.2W + P5 = 0$

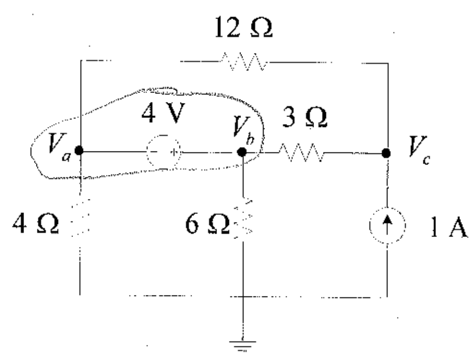
$\Rightarrow \boxed{P5 = 0.2W}$

EE211 Name ______________________

Final Exam May 3, 2007, 1 – 3 pm

P2. *Nodal Analysis*

You are to analyze the circuit below via a nodal analysis. Write down the equations necessary to solve for the node voltages using a nodal analysis. Express these in a matrix form. **Do not solve the equations.**



① Voltage constraint

$$\boxed{V_b - V_a = 4V}$$

② KCL

node C : $\boxed{\frac{V_c - V_b}{3\Omega} + \frac{V_c - V_a}{12\Omega} - 1A = 0}$

$$\Rightarrow -\frac{V_a}{12} + \frac{5V_c}{12} - \frac{V_b}{3} = 1A$$

Supernode : $\frac{V_a - 0}{4} + \frac{V_a - V_c}{12} + \frac{V_b - 0}{6} + \frac{V_b - V_c}{3} = 0$

$$\Rightarrow \boxed{\frac{1}{3}V_a + \frac{1}{2}V_b - \frac{5}{12}V_c = 0}$$

Answer:

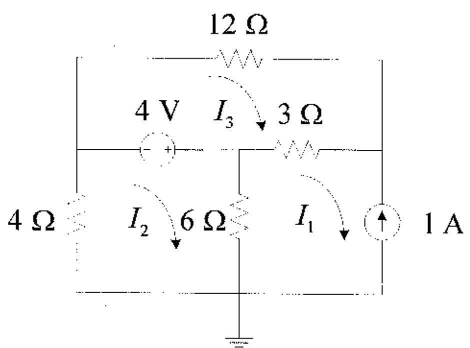
$$\begin{pmatrix} -1 & 1 & 0 \\ -1/12 & -1/3 & 5/12 \\ 1/3 & 1/2 & -5/12 \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$$

EE211 Name ______________________

Final Exam May 3, 2007, 1 – 3 pm

P3. *Mesh Analysis*

You are to analyze the circuit below via a mesh analysis. Write down the equations necessary to solve for the mesh currents using a mesh. Express these in a matrix form. **Do not solve the equations.**



Current Constraint

$I_1 = -1A$

KVL:

Loop 2: $4\Omega(I_2) - 4V + 6\Omega(I_2 - I_1) = 0$

$$\boxed{-6 I_1 + 10 I_2 = 4V}$$

Loop 3: $12 I_3 + 3\Omega(I_3 - I_1) + 4V = 0$

$$\boxed{-3 I_1 + 15 I_3 = -4}$$

Answer:

$$\begin{pmatrix} 1 & 0 & 0 \\ -6 & 10 & 0 \\ -3 & 0 & 15 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -4 \end{pmatrix}$$

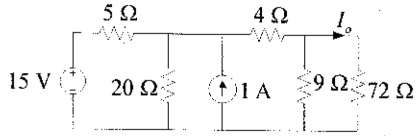
EE211 Name ____________________

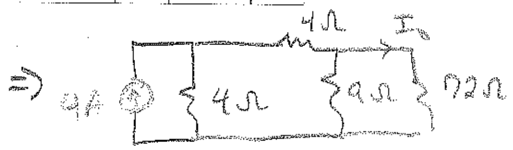
Final Exam May 3, 2007, 1 – 3 pm

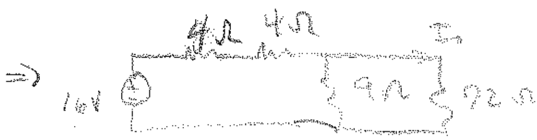
P4. *Source Transformation*

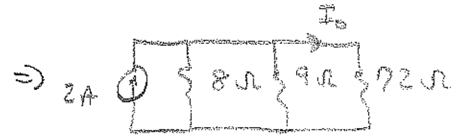
Applying appropriate source transformations, determine the current I_o in the circuit below assuming a DC steady state response.

$\boxed{I_o = \frac{1}{9} A}$









Current division

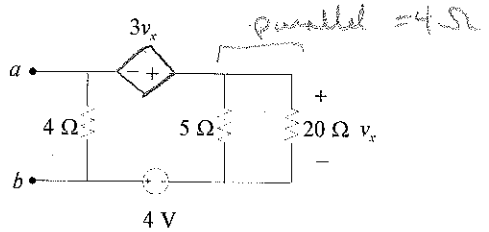
$$I_o = 2A \times \frac{\frac{1}{72}}{\frac{1}{8}+\frac{1}{9}+\frac{1}{72}}$$

$$= 2A \times \frac{1}{9+8+1}$$

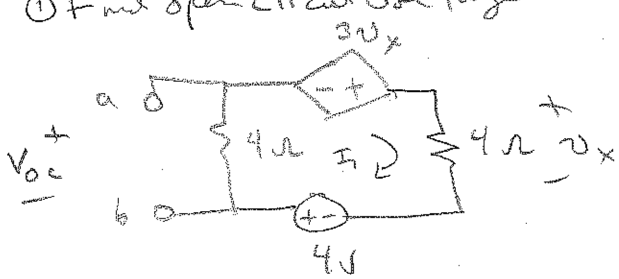
$$\boxed{I_o = \frac{1}{9} A}$$

P5. *Thevenin/Norton Equivalence*

Determine the Thevenin equivalent circuit relative to terminals *a-b* in the DC circuit below.



① Find open circuit voltage



KVL: $-4V + 4I_1 - 3v_x + 4I_1 = 0$

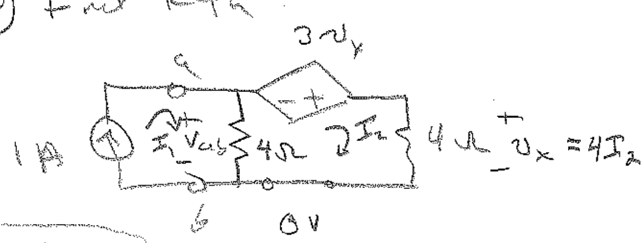
$v_x = 4I_1$

$\Rightarrow 4V = 4I_1 - 3(4I_1) + 4I_1$

$4V = -4I_1$ $\boxed{I_1 = -1A}$

$$\boxed{V_{oc} = -I_1 \times 4\Omega = +4V}$$

② Find R_{Th}



current constraint $I_1 = 1A$

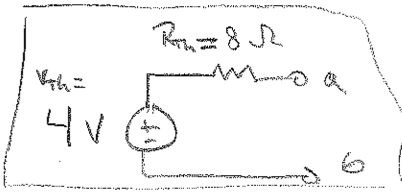
KVL: $4(I_2 - I_1) - 3v_x + v_x = 0$

$4(\underbrace{I_2 - I_1}_{=1A}) - 2(\underbrace{4I_2}_{v_x}) = 0$

$4I_2 - 4 - 8I_2 = 0$

$\therefore \boxed{I_2 = -1A}$

$$\boxed{R_{Th} = \frac{V_{ab}}{1A} = \frac{4(I_1 - I_2)}{1A} = \frac{8V}{1A} = 8\Omega}$$

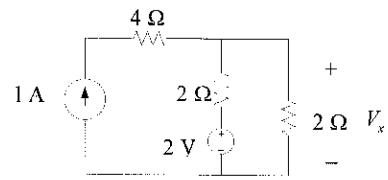


EE211 Name ___________________________

Final Exam May 3, 2007, 1 – 3 pm

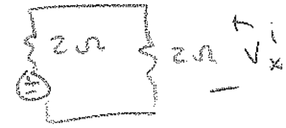
P6. *Superposition*

Determine the voltage V_x in the network below by using *superposition*.



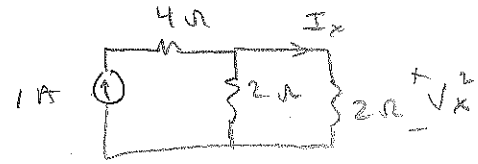
① open ckt 1A source

Voltage division:



$$\boxed{V_x^1 = 2V \times \frac{2\Omega}{2\Omega + 2\Omega} = 1V}$$

② Short ckt 2V srce ; current div:



$$I_x = 1A \times \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}} = \frac{1}{2}A$$

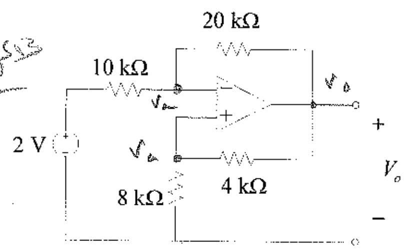
$$\therefore V_x^2 = 2\Omega \times \frac{1}{2}A = 1V$$

$$\boxed{V_x = V_x^1 + V_x^2 = 2V}$$

P7. *Op-Amp Circuit*

The network below contains an *ideal operational-amplifier*. Find the output voltage V_o.

Nodal analysis



KCL: a^+ : $\frac{V_a}{8k\Omega} + \frac{V_a - V_o}{4k} = 0$

$$\Rightarrow V_a\left(\frac{1}{8k} + \frac{1}{4k}\right) = \frac{V_o}{4k}$$

$$\Rightarrow \boxed{V_a = \frac{2}{3}V_o} \quad \text{or} \quad \boxed{V_o = \frac{3}{2}V_a}$$

KCL at a^- : $\left(\frac{V_a - 2V}{10k} + \frac{V_a - V_o}{20k} = 0\right) 20k$

$$\Rightarrow (2V_a - 4V) + V_a - V_o = 0$$

$$3V_a - V_o = 4V$$

from above,

$$V_a = \frac{2}{3}V_o \Rightarrow \cancel{3}\left(\frac{2}{\cancel{3}}V_o\right) - V_o = 4V$$

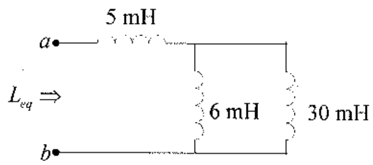
$$\therefore \boxed{V_o = 4V}$$

EE211 Name ______________________

Final Exam May 3, 2007, 1 – 3 pm

P8. *Energy Storage Elements*

a) Given the network below, find the total network inductance.

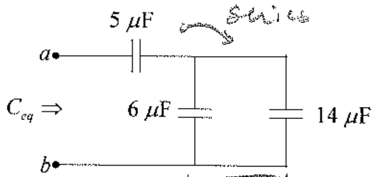


L_p

$$L_p = \frac{1}{\frac{1}{6mH} + \frac{1}{30mH}} = \frac{30mH}{5+1} = 5mH$$

$$L_{eq} = 5mH + 5mH = \boxed{10mH = L_{eq}}$$

b) Given the network below, find the total network capacitance.



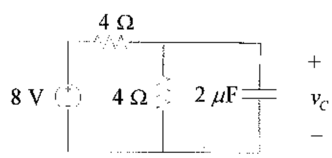
Series

$$C_{eq} = \frac{1}{\frac{1}{5\mu F} + \frac{1}{20\mu F}} = \frac{20\mu F}{4+1} = 4\mu F$$

$C_p = 20\mu F = 6\mu F + 14\mu F$

$$\boxed{C_{eq} = 4\mu F}$$

c) In the circuit below, the DC source has been on for a very long time. What is the energy stored in the capacitor?

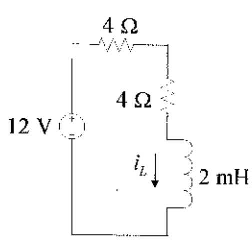


$$V_C(\infty) = \frac{4}{4+4} \times 8V = 4V$$

$$W_C = \frac{1}{2} C V^2 = \frac{1}{2} \times 2\mu F \times 4^2$$

$$\boxed{W_C = 16\mu J = 16 \times 10^{-6} J}$$

c) In the circuit below, the DC source has been on for a very long time. What is the energy stored in the inductor?



$$i_L(\infty) = \frac{12V}{8\Omega} = \frac{3}{2} A$$

$$W_L = \frac{1}{2} L I^2 = \frac{1}{2} \times 2mH \times \left(\frac{3}{2}A\right)^2$$

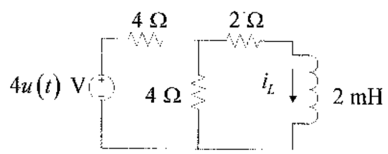
$$\boxed{W_L = \frac{9}{4} mJ = \frac{9}{4} \times 10^{-3} J}$$

EE211 Name ____________________________

Final Exam May 3, 2007, 1 – 3 pm

P9. *First-Order Transient Circuits*

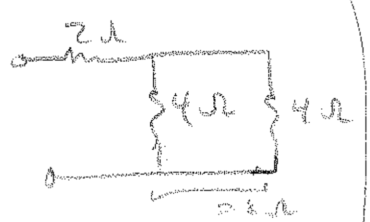
The network below is driven by a transient voltage source $v_s(t) = 4u(t)$. Determine the inductor current $i_L(t)$.



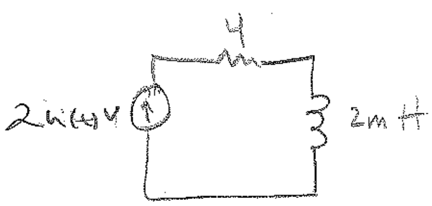
Find the Thevenin equivalent circuit

$V_{oc} = 2u(t)\,V$

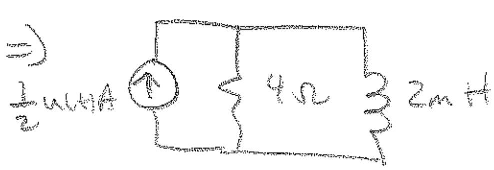
$R_{Th} \Rightarrow$



$R_{Th} = 4\,\Omega$



$\Rightarrow$



$i_L(0^-) = 0$

$i_L(\infty) = \frac{1}{2}\,A$

$R = 4\,\Omega, \quad \tau = \frac{L}{R} = \frac{2\,mH}{4\,\Omega} = \frac{1}{2}\,ms$

$\tau = \frac{1}{2} \times 10^{-3}\,s \; ; \; \frac{1}{\tau} = 2000\,s^{-1}$

$i(t) = \left\{ \left(i_L(0) - i_L(\infty) \right) e^{-t/\tau} + i_L(\infty) \right\} u(t)$

$\therefore \boxed{i(t) = \left\{ \frac{1}{2}\left(1 - e^{-2{,}000t}\right) \right\} u(t)}$

or

$$\boxed{i(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}\left(1 - e^{-2000t}\right), & t > 0 \end{cases}}$$

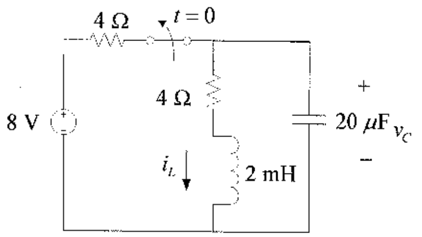
EE211 Name ______________________

Final Exam May 3, 2007, 1 – 3 pm

P10. *Second-Order Transient Circuits*

In the circuit below, the switch connecting the DC source to the network has been closed for a very long time. The switch then opens at $t = 0$, disconnecting the source.

a) Determine the inductor current at time $t = 0$.
b) Determine the capacitor voltage at time $t = 0$.
c) Determine the derivative of the capacitor voltage at time $t = 0^+$.
d) Determine the steady state capacitor voltage at time $t = \infty$.
e) Is the series RLC circuit (for time $t > 0$) under-damped, critically damped, or over damped?



($t > 0$) steady state w/ switch closed.

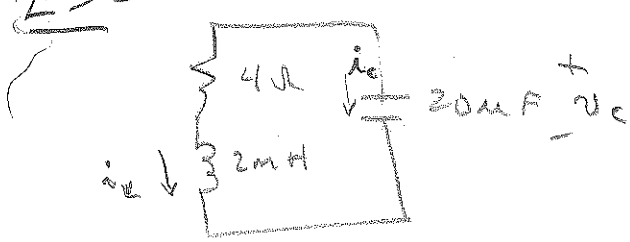
inductor = short ckt

capacitor = open ckt

a) $i_L = \frac{8V}{8\Omega} = 1A \quad \therefore \boxed{i_L(0^-) = 1A}$

b) $\boxed{v_c(0^-) = 4\Omega \times 1\Omega = 4V}$

$t > 0$



c) $i_c = C v_c' = -i_L$

$\therefore C v_c'(0^+) = -i_L(0^+) = -i_L(0^-)$

$v_c'(0^+) = \frac{-i_L(0^-)}{C}$

$v_c'(0^+) = \frac{-1A}{20 \times 10^{-6} F} = 50 \times 10^3$

$\boxed{v_c'(0^+) = -50 \times 10^3}$

d) $v_c(\infty) = 0$

e) $\alpha = \frac{R}{2L}$ (series RLC) $= \frac{4\Omega}{2 \times 2 \times 10^{-3}} = 1000\ s^{-1}$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 10^{-3} \times 20 \times 10^{-6}}} = \frac{1}{\sqrt{4 \times 10^{-8}}} = \frac{1}{2} 10^4 = 5{,}000\ rad/s$

$\omega_0 > \alpha \quad \therefore \boxed{\text{under damped}}$

EE211 Name ____________________

Final Exam May 3, 2007, 1 – 3 pm

P11. *AC-Steady State – Phasors and Impedance*

A network is driven by an AC steady state sinusoidal voltage source $v_s(t) = 4\cos(2t - \pi/3)$ V.

a) Express the time-dependent sinusoidal source voltage as a phasor:

$$\hat{V}_s = 4e^{-j\pi/3} = \boxed{4\angle -60^\circ \text{ V}}$$

b) There is a 4 H inductor in the network. Determine the impedance of the inductor.

$$\omega = 2 \text{ rad/s} \quad Z_L = j\omega L = j \times 2 \text{ rad/s} \times 4 \text{ H} = \boxed{+j8\,\Omega}$$

c) There is a 50 mF capacitor in the network. Determine the impedance of the capacitor.

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 2 \times 50 \times 10^{-3}} = \boxed{-j10\,\Omega}$$

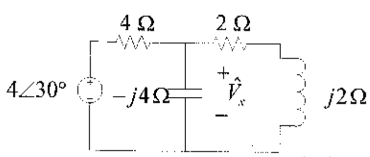
d) A phasor voltage in the network is $\hat{V}_x = 5\angle 45^\circ$. Determine the actual time-dependent voltage $v_x(t)$.

$$v_x(t) = \mathrm{Re}\left\{\hat{V}_x e^{j2t}\right\} = \mathrm{Re}\left\{5e^{j\pi/4}e^{j2t}\right\}$$

$$\boxed{v_x(t) = 5\cos(2t + \pi/4) \text{ V}}$$

P12 *AC-Steady State Analysis*

The network below is expressed in the phasor domain. Determine the phasor voltage $\hat{V}_x$.



series $= 2 + j2\,\Omega$

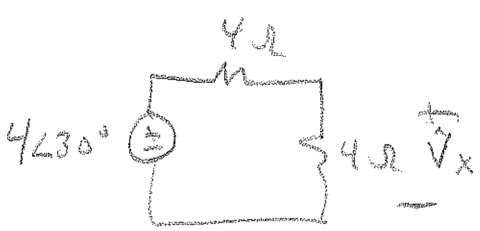
Parallel: Z_P

$\underline{Z_P = 4\,\Omega}$

$$Z_P = \frac{1}{\frac{1}{-4j} + \frac{1}{2+2j}} = \frac{-4j}{1 - \frac{2j}{1+j}}$$

$$= \frac{-4j(1+j)}{1+j-2j} = \frac{4-4j}{1-j}\frac{(1+j)}{(1+j)}$$

$$= \frac{4 + \cancel{4j} - \cancel{4j} + 4}{1+1} = \frac{8}{2} = 4\,\Omega$$



By voltage division

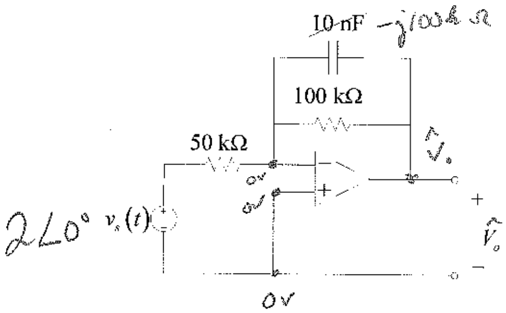
$$\boxed{\hat{V}_x = 2\angle 30^\circ}$$

EE211 Name ______________________

Final Exam May 3, 2007, 1 – 3 pm

P13 *AC-Steady State Analysis*

The network below contains an *ideal operational-amplifier* driven by an AC steady-state sinusoidal source $v_s(t) = 2\cos(1000t)$ V. Find the sinusoidal output voltage $v_o(t)$.



$\omega = 1000$

$$Z_c = \frac{1}{j \cdot 1000 \cdot 10\times10^{-9} F} = -j\,100k\,\Omega$$

nodal analysis:

$$\text{KCL at } 0^-: \left(\frac{0 - 2\angle 0}{50k} + \frac{0 - V_o}{100k} + \frac{0 - V_o}{-j100k} = 0 \right) \times 100k$$

$$-4\angle 0^\circ - V_o - \frac{V_o}{-j} = 0$$

$$V_o(-1-j) = 4\angle 0^\circ$$

$$\frac{4(-1+j)}{(-1-j)(-1+j)} = \frac{-4+4j}{2}$$

$$V_o = \frac{4}{-1-j} = \frac{-4+4j}{2} = -2+2j$$

$$\boxed{\hat{V}_o = 2\sqrt{2}\angle 135^\circ}$$

$$v_o(t) = \mathrm{Re}\left(2\sqrt{2}\, e^{j3\pi/4}\, e^{j2t}\right) \text{ V}$$

$$\boxed{v_o(t) = 2\sqrt{2}\cos\left(\cancel{2}t + \frac{3\pi}{4}\right) \text{ V}}$$

1000