

**CE 382**

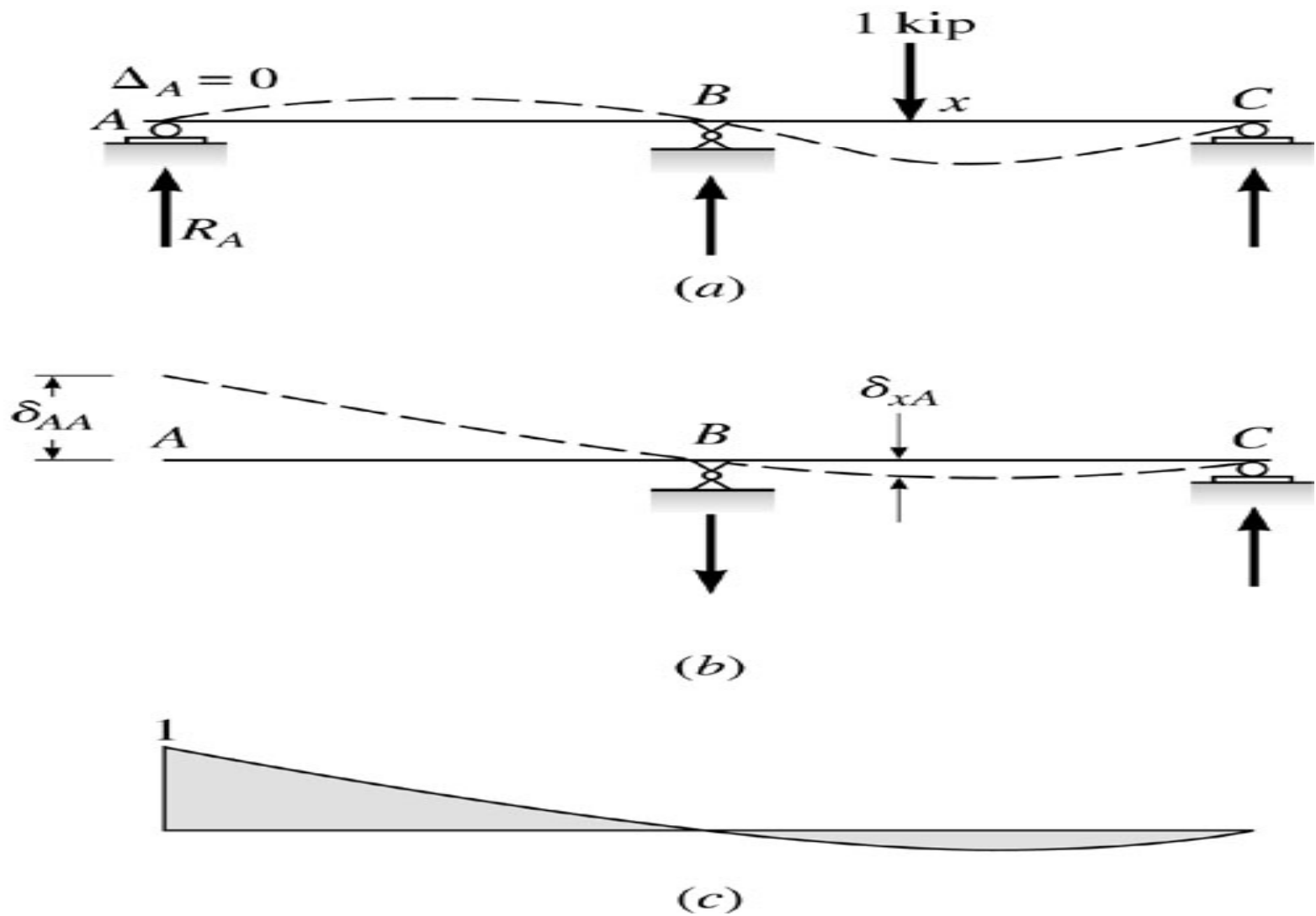
**Exam 3 Review Session**

# Qualitative Influence Lines for Statically Indeterminate Structures: Muller-Breslau's Principle

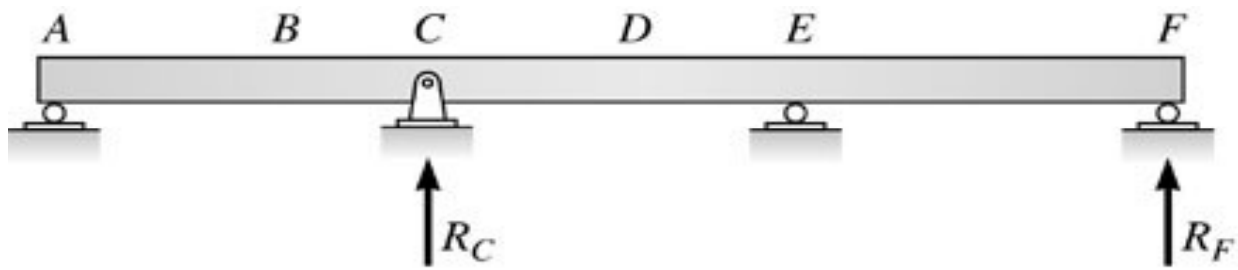
*The influence line for a force (or moment) response function is given by the deflected shape of the released structure by removing the displacement constraint corresponding to the response function of interest from the original structure and giving a unit displacement (or rotation) at the location and in the direction of the response function.*

Procedure for constructing qualitative influence lines for indeterminate structures is: **(1)** remove from the structure the restraint corresponding to the response function of interest, **(2)** apply a unit displacement or rotation to the released structure at the release in the desired response function direction, and **(3)** draw the qualitative deflected shape of the released structure consistent with all remaining support and continuity conditions.

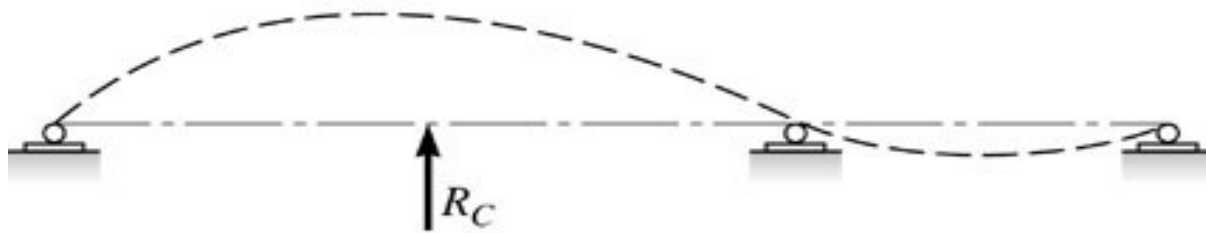
**Uniformly distributed live loads are placed over the positive areas of the ILD to maximize the drawn response function values.**



QILD for  $R_A$



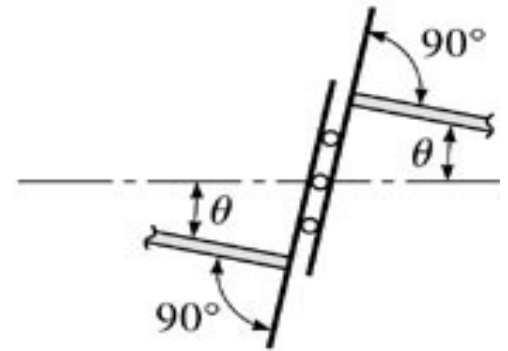
(a)



(b)



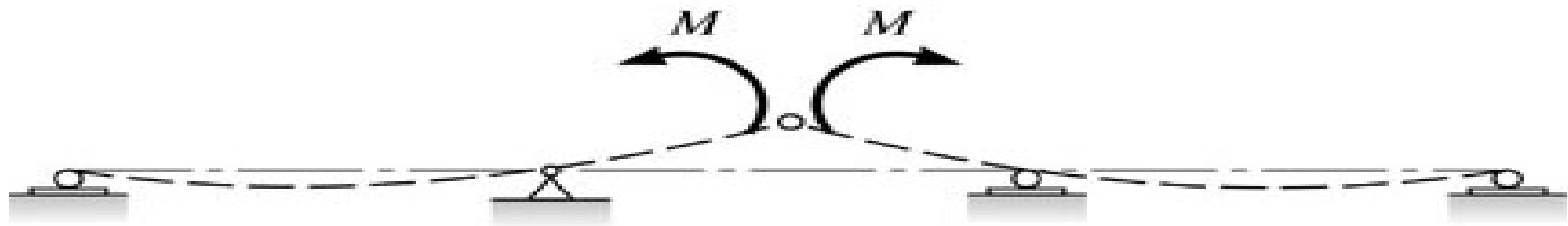
(c)



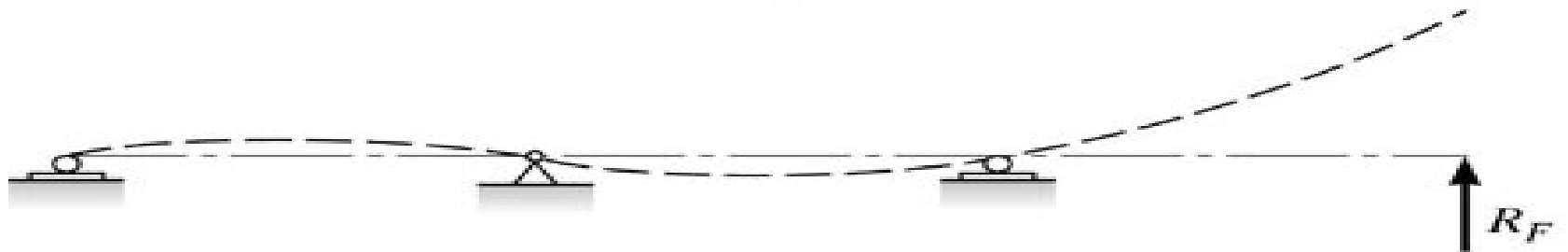
QILD's for  $R_C$  and  $V_B$



(d)



(e)



(f)

QILD's for  $(M_C)^-$ ,  $(M_D)^+$  and  $R_F$

# Concluding Points

- Rotation at a fixed support is zero unless you are sketching the qualitative influence line diagram for the support moment – equals one in this case.
- Moment at an end simple support is zero, thus the qualitative influence line diagram for such a case is a zero displacement line.

# Approximate Analysis of Statically Indeterminate Structures

## BEAMS:

$$I_b = N_{bR} - C - 2$$

$N_{bR}$  = **number of bending reactions**

$C$  = **number of equations of condition in bending**

**Each inflection point position introduces one equation of condition to the static equilibrium equations.** Three strategies are used to approximate the location of the inflection points:

1. qualitative displacement diagrams of the beam structure,
2. qualitative bending moment diagrams (**preferred method**), and
3. location of exact inflection points for some simple statically indeterminate structures.

Recall that **zero moment locations correspond to the inflection point locations.**

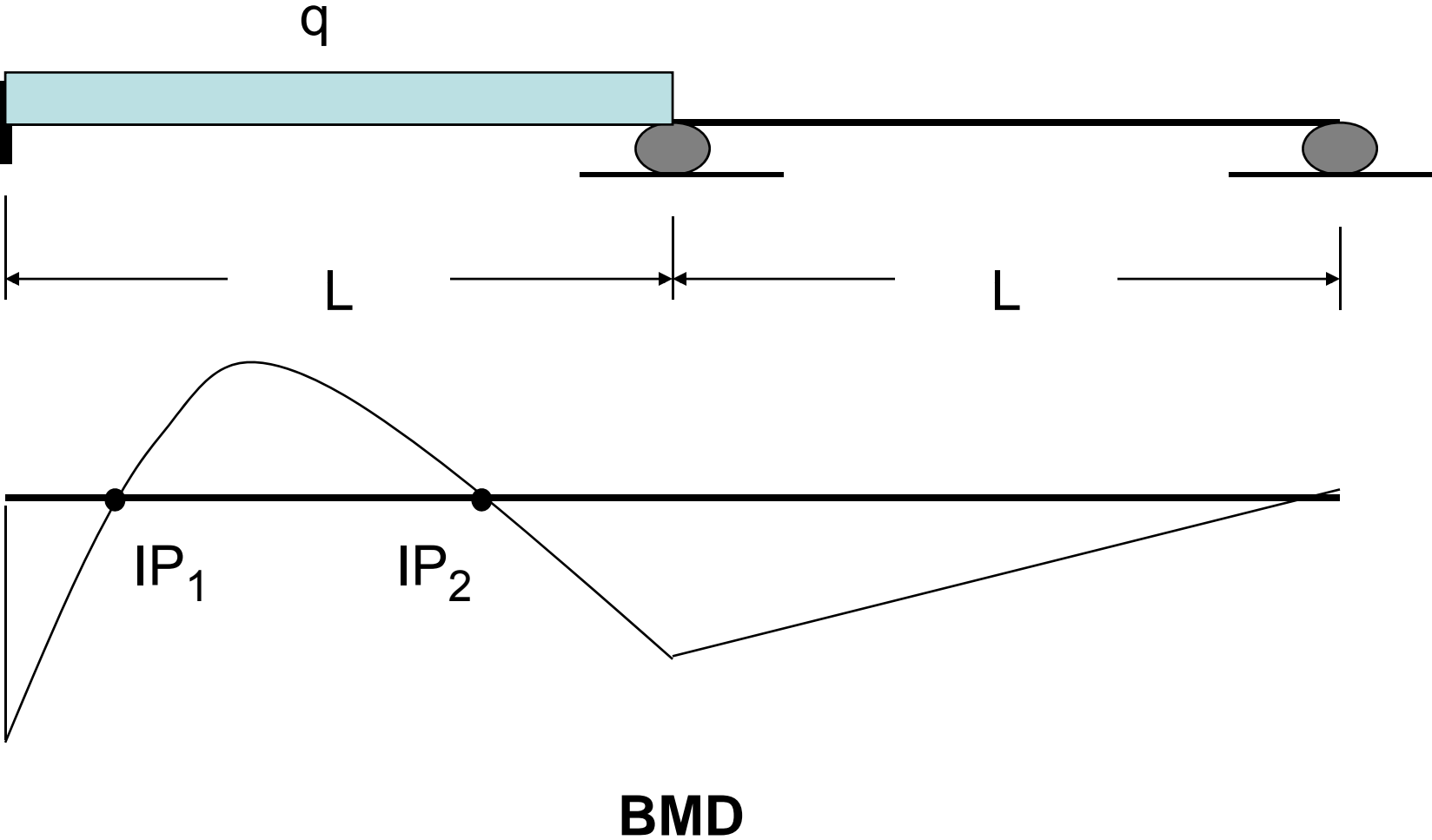
$$\text{Number of Inflection Points} = I_b$$

Check out the exact inflection point location figures, page 1 of the exam 3 supplemental material.

When considering problems that do not match the exact values given, some useful guides are:

- **Inflection points move towards positions of reduced stiffness,**
- **No more than one inflection point can occur in an unloaded span, and**
- **No more than two inflection points will occur in a loaded span.**

# Continuous Beam Example



# DOUBLE DIAGONAL TRUSS STRUCTURES

$$C = \alpha T$$

**Draw section FBD:**

For parallel chord trusses – use force equilibrium to calculate the tension and compression diagonal forces

For non-parallel chord trusses – use moment equilibrium to calculate the tension and compression diagonal forces

# Force Method Procedure

1. Determine the degree of static indeterminacy.
  - Number of **releases** equal to the degree of static indeterminacy are applied to the structure.
  - **Released structure** is referred to as the *primary structure*.
  - Primary structure must be chosen such that it is geometrically stable and statically determinate.
  - Redundant forces should be carefully chosen so that the primary structure is easy to analyze.

# Force Method – con't

- 2.** Calculate “errors” (displacements) at the primary structure redundants. These displacements are calculated using the method of virtual forces (or using tabulated results).
- 3.** Determine displacements in the primary structure due to unit values of redundants (method of virtual forces). These displacements are required at the same location and in the same direction as the displacement errors determined in step 2.

# Force Method – con't

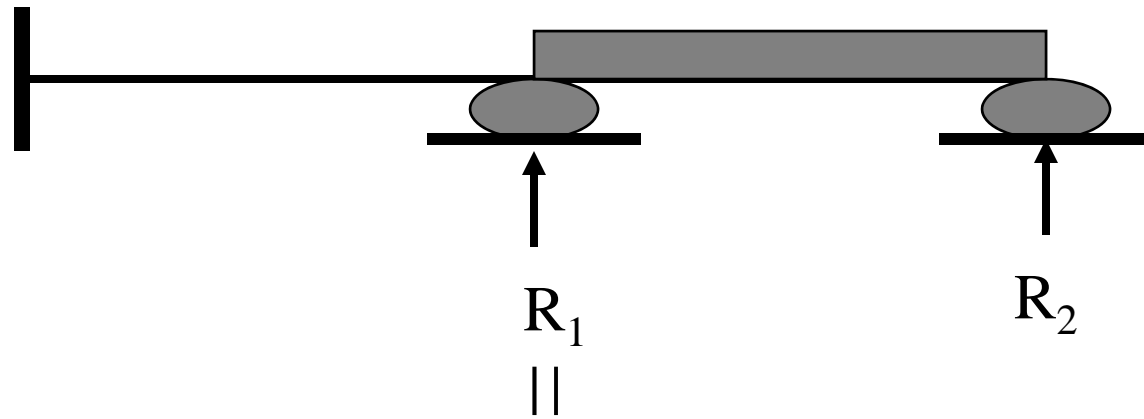
4. Calculate redundant forces to eliminate displacement errors.
  - Use superposition equations in which the effects of the separate redundants are added to the displacements of the released structure.
  - Displacement superposition results in a set of  $n$  linear equations ( $n$  = number of releases) that express the fact that there is zero relative displacement at each release.

# Force Method – con't

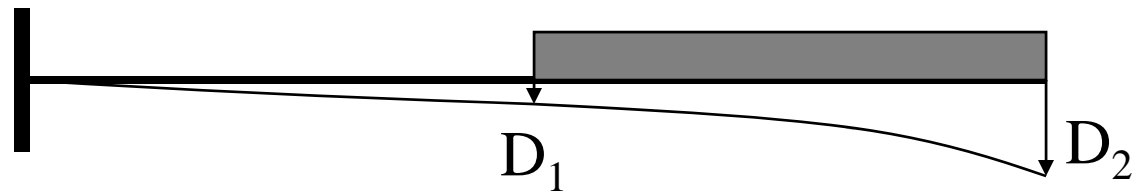
- These **compatibility equations** guarantee a final displaced shape consistent with known support conditions, i.e., the structure fits together at the  $n$  releases with no relative displacements.
- 5. Hence, we find the forces on the original indeterminate structure. They are the sum of the correction forces (redundants) and forces on the released structure.

# Flexibility Analysis (Superposition)

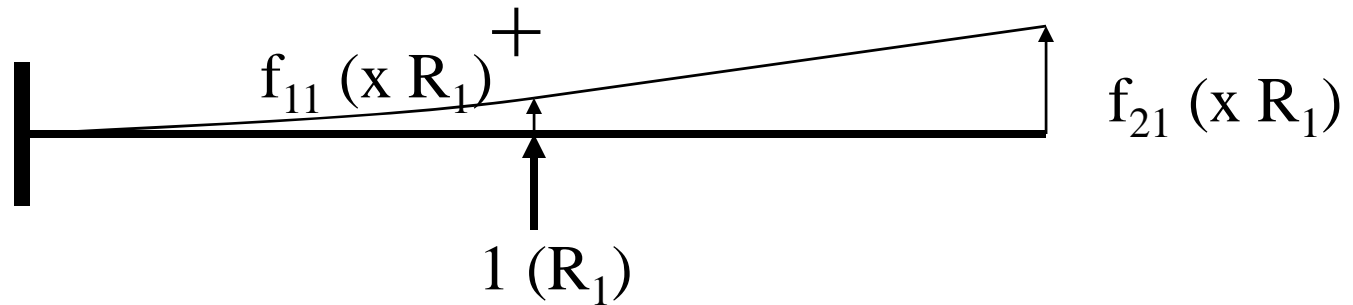
(1)



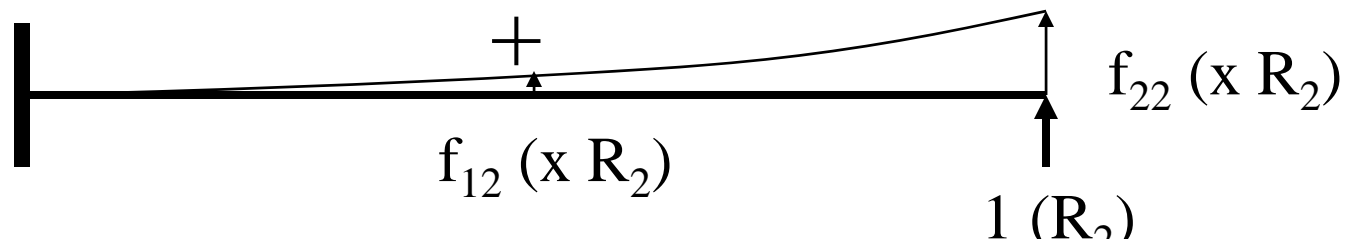
(2)



(3)



(3)



$$\begin{aligned} f_{11} R_1 + f_{12} R_2 + D_1 &= 0 \\ f_{21} R_1 + f_{22} R_2 + D_2 &= 0 \end{aligned} \quad (4)$$

Solve for  $R_1$  and  $R_2$ .

$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \frac{-1}{\det[F]} \begin{Bmatrix} f_{22}D_1 - f_{12}D_2 \\ -f_{21}D_1 + f_{11}D_2 \end{Bmatrix}$$

$$\det [F] = f_{11}f_{22} - f_{12}f_{21}$$

**Know how to use and interpret the displacement figures from Ghali and Neville in your Exam 3 Supplements.**

## **Nonmechanical Loading:**

$$[F]\{R\} = -(\{D\} + \{D^{\Delta}\})$$

$$\{D^{\Delta}\} = \langle D_1^{\Delta} \ D_2^{\Delta} \ \dots \ D_n^{\Delta} \rangle^T$$

*Displacements due to dimension changes are all relative displacements*, as are all displacements corresponding to releases. They are *positive when they are in the same vector direction as the corresponding release (redundant)*.

Calculation of the **non-redundant forces  $A_i$**  (support reactions, internal shears and moments, truss member forces) can be expressed using **superposition** as

$$A_i = A_i^p + \sum_{j=1}^{N_R} (A_{ui})_j R_j \quad (5)$$

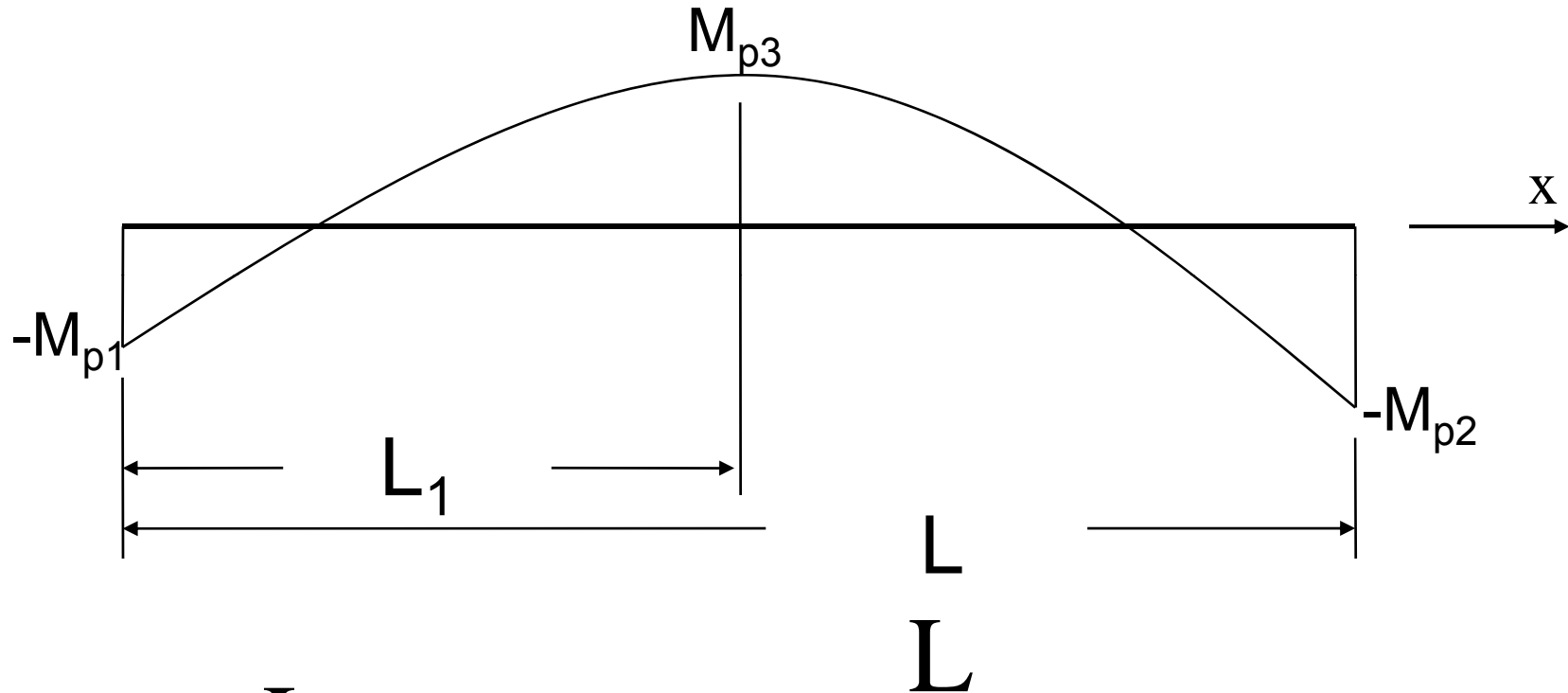
where  $A_i^p =$  **desired action  $A_i$  on the primary structure due to the applied loading**;  $(A_{ui})_j =$  **action  $A_i$  on the primary structure due to a unit virtual force at redundant  $R_j$**  and  $N_R =$  **number of redundants**.

# Plastic Analysis of Continuous Beams

- **At an end simple support, the moment is zero and there will not be a plastic hinge.**
- **At a fixed support, the support moment will form a plastic hinge if it is part of the mechanism being considered.**
- **If there is a discontinuity in the plastic moment capacity at an interior support, choose the smaller value as the plastic moment capacity at that support.**

# Uniform Load:

$$M_{p1} \leq M_{p3}; \quad M_{p2} \leq M_{p3}$$



$$\therefore L_1 = \frac{L}{1 + \sqrt{1 - \left( \frac{M_{p1} - M_{p2}}{M_{p1} + M_{p3}} \right)}}$$