EE 422G - Signals and Systems Laboratory

Lab 9 PID Control

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November 28, 2011

Objectives:

- Identify the components of a PID controller and derive its impact on the transfer function of a total feedback system.
- Derive state-space models of systems and understand their relationship to transfer functions.
- Simulate a feedback control system in LabVIEW and use it to verify a design of PID controller.

1. Background

This laboratory exercise requires the design of a PID controller for changing the response of a second order plant. LabVIEW software will be used to simulate the performance of the controller and compare its performance with other controllers.

PID stands for Proportional, Integral, and Derivative. Controllers are designed to eliminate the need for continuous operator attention. Examples of common controllers include cruise control for maintaining a constant automobile velocity and a thermostat for maintaining a constant temperature. These controllers automatically adjust a system input (i.e. flow of fuel) based on feedback to hold the measurement to maintain the process at the desired set-point. For a constant reference input (such as speed or temperature), the error is defined as the difference between set-point and measurement:

$$e = R - Y \tag{1}$$

where e is the error, R is the desired set-point, and Y is the measurement of the output being controlled. The variable being adjusted is called the manipulated variable (u) which usually is equal to the output of the controller (or input to the system to be controlled). The relationship between these variables and a feedback control system is illustrated in Fig. 1.

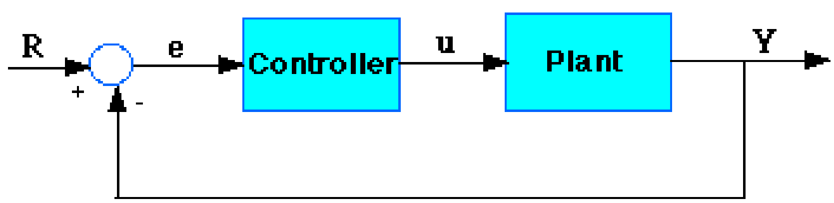

Figure 1. A plant controlled in a feedback system.

The output of PID controllers will change in response to **changes** in the set-point. So for the cruise control example, the measurement *Y* is the speedometer reading, *R* is the value the driver sets through the cruise control unit (desired speed), *E* is the difference between the desired and actual. The controller input, *e*, will dynamically increase and decrease based on other system inputs (such as change incline, weight, wind friction, and driver setting a different velocity value) in addition to known properties of the system, such as the system inertia. The *e* value is processed to dynamically modify the variable *u* (i.e. the flow of fuel or pressure on the break depending on the size, direction, and dynamic of *e*).

The three-term controller

The transfer function of the PID controller can be modeled in the Laplace domain as:

$$K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \qquad (2)$$

where the following values must be determined to meet the design criteria:

- K_p = Proportional gain
- K_i = Integral gain
- K_d = Derivative gain

Consider how the PID controller works in a closed-loop system using the schematic shown in Fig. 1. The variable *e* represents the tracking error, which is the difference between the desired input value *R* and measured output *Y*. This error signal *e* is the input for the PID controller, and the controller computes both the derivative and the integral of this error signal. The controller output *u* is given by:

$$u = K_p e + K_I \int e dt + K_d \frac{de}{dt} \qquad (3)$$

where the dependence of *e* and *u* on time *t* is implied. The plant accepts input *u* (Fig. 1) which modifies output *Y* and is sent back through the feedback loop to compute the new *e*. This process continues with the objective to drive *e* toward 0.

The characteristics of P, I, and D controllers

While there are several ways to consider control system performance, this laboratory assignment only considers performance metrics associated with a step response or step disturbance. This models the case when the desired value changes or a disturbance pushes the system away from the desired response. The performance metrics in this case describe how well the system returns to the desired behavior after a disturbance or change. These metrics are illustrated in Fig.

2. Three metrics deal with the speed of the response. The **delay time** t_d, is the amount of time it takes the system to achieve 50% of its steady-state value. The definition of **rise time** t_r can vary with the application. Here it is taken as the time for the system response to go between 10% and 90% of the steady-state value. The **settling time** t_s can also vary with the application. In Fig.2 it is defined as the time it takes for the system to remain within 5% of the steady-state response. Lastly, 2 metrics deal with the deviation of the dynamic response from the ideal step response. The steady-state response may not converge on the desired value. In that case a **steady-state** error E_{ss} occurs, which is the difference between the desired and steady-state value of the system response. The **overshoot error** is the maximum deviation resulting from the system overshooting the steady-state response. Note that only under-damped systems will have overshoot. Critically-damped and over-damped systems will not have an overshoot error.

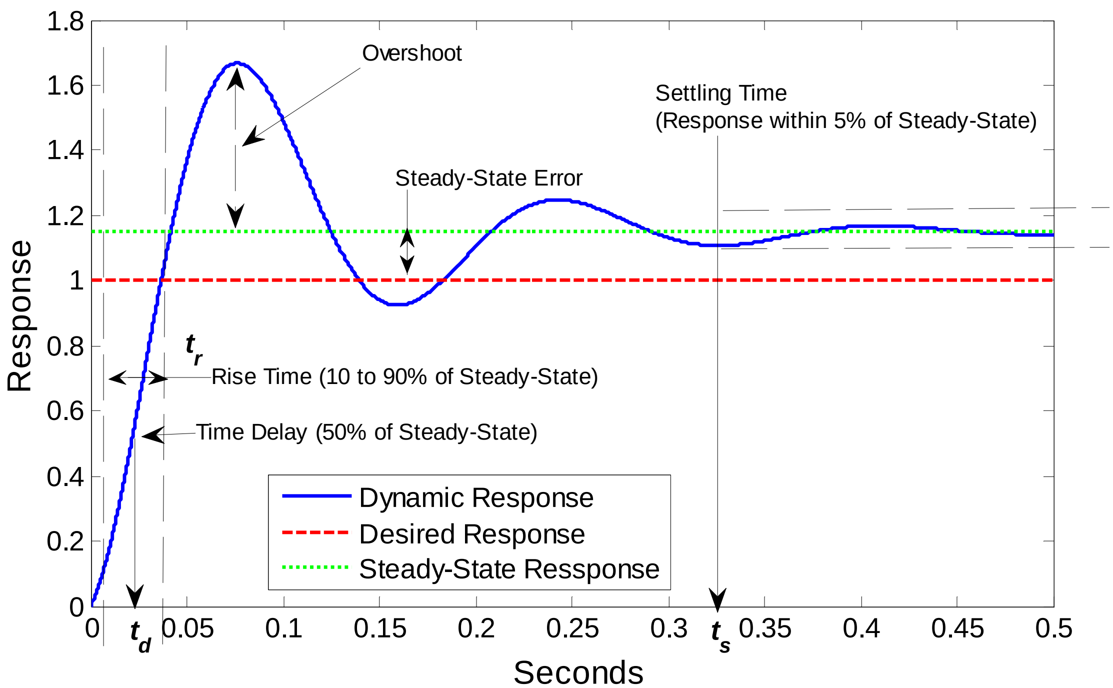

Figure 2. Performance metrics to describe the systems ability to return to or achieve the desired response. These include the time delay t_d, rise time t_r, settling time t_s, and steady-state error E_{ss}.

A proportional controller K_p has the ability to reduce the rise time and reduce (but never totally eliminate) the steady-state error. An integral control K_i can eliminate the steady-state error, but it may make the system more inertial (slowing it down and taking longer to settle). A derivative control K_d can impact the stability of the system, reduce overshoot, and improve speed of the response.

Effects of each of controller components on a closed-loop system are summarized in Table 1.

Table 1. General impact of controller parameters

Controller Parameter	Rise Time	Overshoot	Settling Time	SS Error
K_p	Decrease	Increase	Small Change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small Change	Decrease	Decrease	Small Change

Note that the correlation between the change in parameter and response may not be exact because these parameters are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference for approximate or general behavior when determining the values for K_i, K_p, and K_d.

2. Pre-Laboratory Exercises

This lab considers the plant modeled as a second order system represented by (implemented) the circuit of Fig. 3.

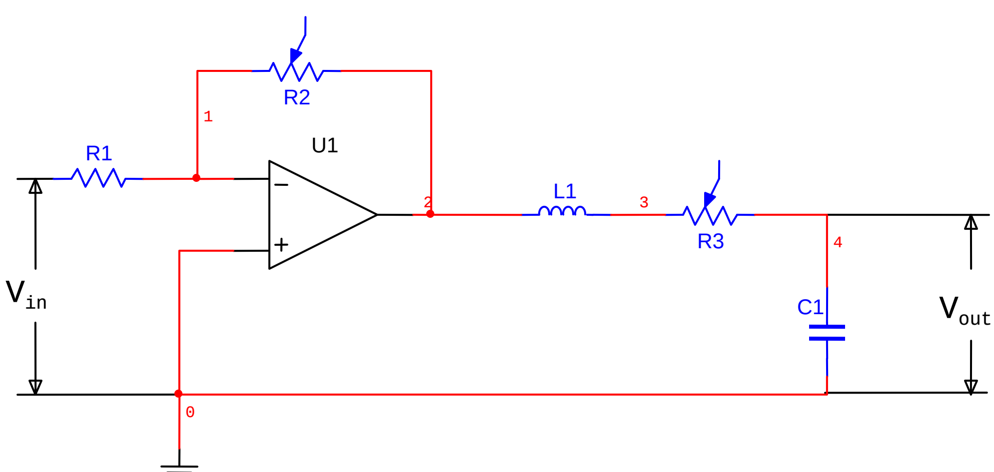

Figure 3. A circuit modeling the system to be controlled

1. Verify that the transfer function with output V_{out} and input V_{in} in the system for Fig. 3 is given by:

$$\frac{V_{out}}{V_{in}} = \frac{-R_2 / R_1}{L_1 \cdot C_1 \cdot s^2 + R_3 C_1 s + 1}$$

You must show your work, i.e. start with nodal equation and derive the transfer function indicated above.

2. Derive the state-space representation for the system in the previous problem. Let the state-vector consist of the voltage on the capacitor (first element) and the current in the inductor (second element).

3. For the system in the previous problems let $R_1 = 1\Omega, R_2 = 1\Omega, R_3 = 10\Omega, L_1 = 5H, C_1 = 0.5F$.
   Substitute in these numbers and express the transfer function and state-space expression in terms of numbers. Verify the poles of the transfer function are equal to the eigenvalues of the state-space representation.

4. Derive the transfer function for the system in Fig. 1 where the controller is given by Eq. (2) and the plant is given by the system derived in the previous problem. Express the transfer function in terms of a monic rational polynomial with coefficients being a function of K_i, K_p, and K_d.

5. Use the expression in the previous problems to compute the transfer functions for the case of unity feedback (no controller, i.e. let $K_i = 0$, $K_p = 1$, and $K_d = 0$). Comment on the stability of this system without the controller.

6. Repeat problems 1 through 5 given $R_1 = 1\Omega, R_2 = 1\Omega, R_3 = -10\Omega, L_1 = 0H, C_1 = 0.5F$ (*i.e.* no inductor and an active element for R_3) and use K_i=5, K_p=11, and $K_d = 0$ for the controller. If you derived the transfer function in terms of the components, you can just substitute in to get the new numerical transfer function. The state-space model is a little more involved since setting the inductor to 0 reduces the system to a first order system and the matrices will reduce in dimension. You may want to rederive from the circuit again, if this confuses you.

3. Laboratory Exercises (just demonstrate results to TA)

1. Read and do the tutorial in LabVIEW for using its simulation module. When LabVIEW is launched click on the *List of All New Features* link to get the help screen to appear. Then under Contents expand the "Control Design and Simulation Module" then click on link *Tutorial: Getting Started with Simulation*. You can also access this directly from National Instrument via their web page:

   http://zone.ni.com/reference/en-XX/help/371894F-01/lvsimhowto/sim_h_gs/

2. It is expected that the module requires about 30 minutes. Demonstrate to the TA the final simulation result in the tutorial (plots of position and velocity using the modularized block). Credit for this exercise is solely based on showing the TA the final example.

3. Given $R_1 = 1\Omega, R_2 = 1\Omega, R_3 = 10\Omega, L_1 = 5H, C_1 = 0.5F$ (same numbers as in first set of pre-lab problems) for the transfer function (plant), use the LabVIEW simulator to generate a unit step response for the feedback system shown below in Fig. 4 (where *R* is the unit step). In the LabVIEW simulation module there is a step function under signal generation and a system transfer function implementation under linear systems. Let the step start at *t*=1 seconds and its amplitude jump from 0 to 1. Let the simulation run initially from 0 to 20 seconds. You can reduce or increase that depending on how long the transient lasts after the step. Plot the output *Y* and the error signal *e*. Show results to TA and comment on this response. Is this what you would expect from the transfer function computed in problem 2 of the prelab (i.e. for unity feedback K_i = 0, K_p = 1, and K_d = 0).

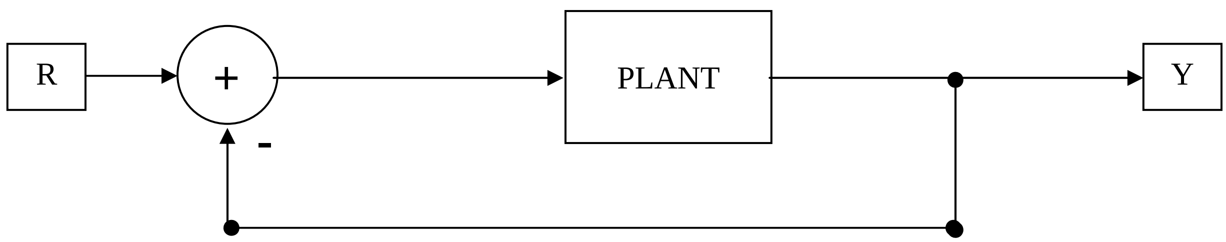

Figure 4. Block diagram for system of Exercise 3.

4. Create a proportional (P) gain controller (K_i = 0 and K_d = 0) in the close loop system as shown in Fig. 5. Try several values for K_p. Indicate the range for K_p values where the system is unstable. Record the error plot for this case. Find the range of K_p values where the system is stable, but with real poles, and find the range of K_p values where the system has complex poles. Show the TA how these ranges were found and plot the error and output for examples of the 3 ranges (unstable, stable with real poles, stable with complex poles). Also find the value of K_p to drive the steady-state error to 0, if it exists. Use the transfer function to support you claim (Hint: use final value theorem in Laplace)). Also for the systems where oscillations can occur, indicate to the TA whether increasing the frequency of the oscillatory behavior changes the settling time.

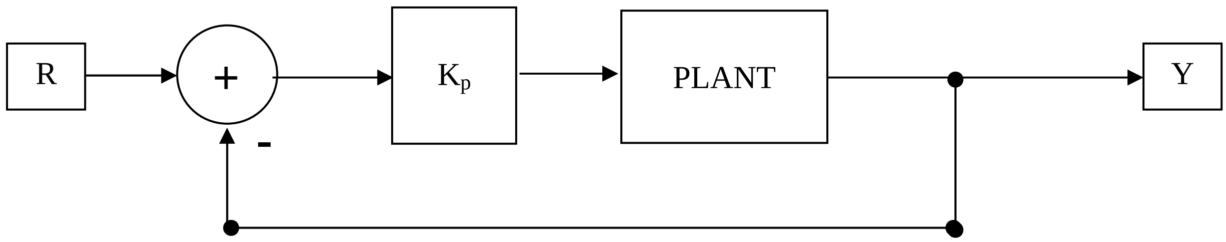

Fig. 5. Block diagram for system of Exercise 3.

5. Create a proportional and integral (PI) gain controller ($K_d = 0$) in the close loop system (extension of the system in Fig. 5). Set K_i so that the constant coefficient in the denominator of the compensated system is 1 and try several values for K_p. Determine the approximate value for K_p where the system transitions between stable and unstable. Show figures to the TA associated with these values to support the ranges you determined. Clearly indicate to the TA what features of the graphs suggest stability/instability and what this implies for the pole placements.

6. For the PI controller plot the error output for several values of K_p. How should the steady-state error differ between the controller of this exercise and the proportional one in the previous (P) system? Create a couple of plots comparing the error from the P to PI controller to show this difference (Hint. Keep the poles near the critically damped response). Determine the range of values for K_p where the dominate poles become complex. Show the TA plots that correspond to these cases and indicate the K_p values used.

7. Repeat 3 through 6 for $R_1 = 1\Omega, R_2 = 1\Omega, R_3 = -10\Omega, L_1 = 0H, C_1 = 0.5F$ (same numbers as in second set of pre-lab problems)