Phasor Circuit Analysis

Phasor Diagrams, Voltage and Current Division
Phasor Diagram

Phasors are denoted by vectors in 2-D space. Phasor diagrams graphically illustrate the summation of complex values.

Consider the following summation of complex numbers resulting from setting up a KVL or KCL equation:

\[ 7 \angle 0 = 9 \angle -45^\circ + 6.4 \angle 84.29^\circ \]

\[ 7 = 9 \cos(-\pi 45/180) + j9 \sin(-\pi 45/180) + 6.4 \cos(\pi 84.29/180) + j6.4 \sin(\pi 84.29/180) \]

\[ 7 = 6.36 - j6.36 + 0.64 + j6.36 \]
Phasor Diagram Example

Draw Currents $i_s$, $i_r$, $i_l$, $i_c$ in a phasor diagram to show $i_s = i_r + i_l + i_c$
where $i_s(t) = \sin(1000t)$ A
By substituting impedance in for resistance (or admittance for conductance), the current division formula can be generalize to:

\[ I_k = I_s \left( \frac{1}{\hat{Z}_1} + \frac{1}{\hat{Z}_2} + \cdots + \frac{1}{\hat{Z}_k} + \cdots + \frac{1}{\hat{Z}_N} \right) = I_s \left( \frac{\hat{Y}_1 + \hat{Y}_2 + \cdots + \hat{Y}_k + \cdots + \hat{Y}_N}{\hat{Y}_1 + \hat{Y}_2 + \cdots + \hat{Y}_k + \cdots + \hat{Y}_N} \right) \]
Voltage Division

By substituting impedance in for resistance, the voltage division formula can be generalize to:

\[
\hat{V}_k = \hat{V}_s \frac{\hat{Z}_k}{\hat{Z}_1 + \hat{Z}_2 + \cdots + \hat{Z}_k + \cdots + \hat{Z}_N}
\]
In-Phase and Out-of-Phase

Voltages and currents are considered *in phase* if the phase angle between their phasor quantities is zero, otherwise they are *out of phase* by an amount equal to their phase difference.

Which quantities are in phase?

\[ \hat{V}_1 \angle 45^\circ \quad \hat{V}_2 \angle 0^\circ \quad \hat{V}_2 \angle -90^\circ \quad \hat{I}_1 \angle 270^\circ \quad \hat{I}_2 \angle -45^\circ \quad \hat{I}_2 \angle 45^\circ \]
Out of phase voltages and currents are considered to either lead or lag each other. If the phase of quantity 1 is subtracted from the phase of quantity 2 and the result is positive, then quantity 1 leads quantity 2 or equivalently quantity 2 lags quantity 1.

Given
\[ \hat{V}_1 \angle 45^\circ \quad \hat{V}_2 \angle 0^\circ \quad \hat{V}_3 \angle -90^\circ \quad \hat{I}_1 \angle 270^\circ \quad \hat{I}_2 \angle -45^\circ \quad \hat{I}_3 \angle 45^\circ \]

Describe the phase difference between
- \( v_1 \) and \( i_1 \)
- \( v_2 \) and \( i_2 \)
- \( v_3 \) and \( i_3 \)
Example

Find $R$ such that the $v_1$ leads $i_1$ by 15 degrees, if $v_1 = \cos(400t) \text{ V}$

Show $R = 15 \Omega$