Simple Models That Illustrate Dynamic Matrix Control

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Abstract

Dynamic Matrix Control (DMC) is one of the most popular methods of model predictive control. It is especially powerful for multiple input multiple output (MIMO) control systems. A way to have students explore the nature of DMC control is to use it on a simulated process. This paper details a series of online instructional modules that allow students to compare the performance of DMC controllers to conventional control schemes using PID control. Examples of the Behavior of DMC and comparisons to PID are presented.

Introduction

The Online Widener Laboratories (OWL) is an online series of instructional modules for various aspects of Chemical Engineering written as JAVA applets running client side(1,2). A portion of OWL is dedicated to process control including Model Predictive Control (MPC) especially dynamic matrix control (DMC). Simulations of Single Input Single Output (SISO) process and a Multiple Input Multiple Output (MIMO) process are used to illustrate and compare MPC to classic PID control. The modules illustrate both DMC and conventional PID control of the same processes, allowing students to perform a variety of interesting and instructive process control experiments; several of which are discussed in this paper.

The goals of the Process Control Laboratory are
1. To provide a realistic experience in which students can explore the concepts of process control
2. To provide an opportunity for students to develop skills in tuning controllers
3. To provide simple examples of a variety of classical PID and modern IMC and DMC control schemes in a simulated real world setting.

This paper will focus on the experiments associated with Dynamic Matrix Control. The modules in the Process Control Laboratory monitor students’ performance and reports their actions to the web server, similar to the way most PLC software stores history files. This data is available to the instructor on line. Evaluation of the Virtual Process Control Laboratory is the subject of another paper (3).
Simulation of a Single Input Single Output (SISO) Process

This process consists of a constant volume tank in which a solvent and solute are mixed. A steady dead time is caused by delay in analysis of the mixed stream. The screen for the process is shown in figure 1.

The valve response to a change in the signal can be approximated by a first order lag.

\[
\frac{dS}{dt} = K(MV - S)
\]

where \( K \) = the valve time constant
\( MV \) = the manipulated variable (i.e. the signal from the controller)
\( S \) = the stem position of the I-th valve

The flow is assumed to be proportional to the valve position. Therefore \( F_{\text{solute}} = K'S \) where \( K' \) is a constant of proportionality

The change in concentration of solute is obtained from a material balance

\[
V \frac{dC_{\text{out}}}{dt} = C_{\text{in}} F_{\text{solute}} - C_{\text{out}} (F_{\text{solute}} + F_{\text{solvent}})
\]

where \( C_{\text{in}} \) = the concentration of the solute in the solute stream
\( C_{\text{out}} \) = the concentration of solute in the exit stream
\( F_{\text{solute}} \) = the flow rate of the solute
\( F_{\text{solvent}} \) = the flow rate of the solvent
\( V \) = the volume of the stirred tank
\( t \) = time
when $F_{\text{solute}} \ll F_{\text{solvent}}$ equation 2a can be simplified to

$$V \frac{dC_{\text{out}}}{dt} \approx C_{\text{in}} F_{\text{solute}} - C_{\text{out}} F_{\text{solvent}} \quad (2b)$$

The delay represents time for an analysis. This yields the familiar FOPDT transfer function for the CST. The FOPDT parameters for the mixer are gain = 1, dead time = 10 and lag constant = 10. The valve’s time constant is fast and can be neglected when analyzing the process.

Noise is simulated as normally distributed white noise using equation 3 (see ref. 4)

$$n = \frac{1.961\sigma(x - 0.5)}{[(x + 0.002432)(1.002432 - x)]^{0.203}} \quad (3)$$

where $n$ = the white noise component of the controlled variable (y)
$x$ = a uniformly distributed random number between 0 and 1
$\sigma$ = the standard deviation (a user controlled variable).

**Simulation of a Multiple Input Multiple Output (MIMO) Process**

The second model process is a multiple input multiple output (MIMO) process shown in figure 2 and is a constant volume mixer in which two streams with different concentrations of a solute are mixed.

![Figure 2](image-url)

The controlled variables are the exit flow and the solute concentration. The manipulated variables are the signals to the two feed valves. The equations that model this process are:

$$F_{\text{out}} = -F_1 - F_2 \quad (4)$$
\[
V \frac{dC_{\text{out}}}{dt} = F_1 C_1 + F_2 C_2 - (F_1 + F_2) C_{\text{out}} \quad (5)
\]

where \(C_1\) and \(C_2\) are the feed concentrations
\(C_{\text{out}}\) = the output concentration from the mixer
\(F_1\) and \(F_2\) are the feed flows
\(F_{\text{out}}\) = the output flow from the mixer
\(V\) = the volume of the mixer

As before, the valves are treated as first order lags, each with a different time constant and the flow is assumed to be linear to the stem position. No lag is assumed in the measurement of flow or concentration. Note that this is a nonlinear process.

Both simulated processes are controlled by conventional PID controllers and DMC controllers in separate modules. The MIMO mixer has two PID’s connected to The control scheme for the This setup allows students to compare

**DMC Control**

The original work developing dynamic matrix control was developed by Cutler and others (5). DMC control is based on a discrete time step response model that calculates a desired value of the manipulated value that remains unchanged during the next time step. Details of the derivation of the formulae are derived in the course. The new value of the manipulated variable the value that gives the smallest sum of squares error between the set point and the predicted value predicted values of the controlled variable. The number of time steps the DMC uses for its prediction is called the “Prediction Horizon”. The dynamic model used to predict the future values of the controlled variable is represented by a vector, \(A\), whose elements are defined as

\[
a_i = \frac{\Delta y(t_i)}{\Delta u(t_0)} \quad (6)
\]

where \(\Delta y(t_i) = y(t_i) - y(t_0)\)
\(y(t)\) = the value of the controlled variable at time \(t\)
\(\Delta u(t_0)\) = the change in the manipulated variable at \(t_0\)

Thus, the response of a process to a step change, \(\Delta u\), in the manipulated variable at \(t_0\) \((\Delta u(t_0)\) is given by

\[
\begin{bmatrix}
\Delta y(t_1) \\
\Delta y(t_2) \\
\Delta y(t_3) \\
\vdots \\
\Delta y(t_n)
\end{bmatrix} =
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
\vdots \\
\ a_n
\end{bmatrix}
\begin{bmatrix}
\Delta u(t_0)
\end{bmatrix}
\quad (7)
\]
Acquiring the model (the dynamic matrix A)

A key step in this process is the determination of the dynamic matrix, A. Often, this step is accomplished offline by the designer who then enters the values of \( a_i \) into the controller. Several workers have explored the idea of having the controller acquire the dynamic matrix from measurements of a step change in the manipulated variables. The DMC controllers in OWL have an automatic “Acquire” feature that allows the user to have the simulation acquire the model automatically by subjecting the process to predefined step changes in the manipulated variables. The controller then performs the math to determine the values of the matrix used to calculate the new value of the manipulated variable. Note that because the noise level can be controlled in the simulation, an ideal model can be given to the controller merely by setting the noise level to 0.

Experiment 1 PID Control of the Process

The first several experiments will now be described to illustrate how these modules can be used when teaching process control.

The first experiment is control of the SISO process using conventional PID control using Cohen and Coon PID and PI settings calculated from the FOPDT constants. The purpose is to illustrate the influence of unfiltered noise on control of the process. A step change in set point from 50 to 40 is made. Typical results shown Figure 3 illustrate the problems associated with the use of derivative action on a noisy signal. Students can also see this by examining error measurements associated with this step change. The ISE (integral of the sum of the squares of the error) for the period 100 seconds after the step change is shown in Figure 4. Further experiments can be used to explore digital filters. For this paper, skip to DMC control.

Figure 3
Conventional Control of FOPDT Process with white noise (\( \sigma = 3 \))

<table>
<thead>
<tr>
<th>PID Control</th>
<th>PI Control</th>
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<tr>
<td>[Graph showing PID control with white noise]</td>
<td>[Graph showing PI control with white noise]</td>
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Time (sec) | Time (sec)
---|---
0 | 0
10 | 10
20 | 20
30 | 30
40 | 40
50 | 50
60 | 60
70 | 70
80 | 80
90 | 90
100 | 100
110 | 110
120 | 120
130 | 130
140 | 140
150 | 150

Signal (0-100) | Signal (0-100)
Figure 4
ISE for PI and PID controller of FOPDT Process as a function of Noise

Experiment 2: Demonstration of Dynamic Matrix Control

The next experiment demonstrates DMC control. With the noise and digital filter turned off, the controller acquires an accurate model of the system. The DMC settings are input horizon = 10 and the output horizon = 4 and the time interval is 5. The controller is then placed into the automatic mode and the set point is changed from 50 to 40. The controller calculates an adjustment for 1 time interval (5 seconds) that will bring the controlled variable to the new set point, then moves the manipulated variable to the new equilibrium value, all before the controlled variable starts to react. The result (Figure 5) is an impressive demonstration of DMC’s capabilities.

Figure 5
Demonstration of DMC Controller
Experiment 3: Effect of Noise on a SISO DMC Controller

Experiment 3 illustrates the effect of noise on DMC control. Noise has two influences on DMC control: 1) while controlling the process, and 2) as a source of error in the dynamic matrix (a kind of process model mismatch). Both contributions can be examined. With the noise level at 0, acquire a model by pressing “acquire”. This will cause the controller to acquire a “perfect” model. Then, set the noise level to a value and, switch the controller to automatic, wait to ensure that the controller is working properly and then change the set point from 50 to 40. Observe the response and note the ISE after 100 seconds. Repeat this experiment with the following modification: press acquire after the noise has been adjusted to a new level. This action will cause the controller to acquire a dynamic matrix containing noise. The controller set up in this way can be used to control the process with the noise set to 0 and with the noise maintained at its observed level. The purpose is to illustrate the influence of inaccuracies in the dynamic matrix model (the A matrix). The results are summarized in Figure 6.

Figure 6
ISE Measurements 100 seconds after a –10 Change in set point for DMC Control with a Noisy Signal

These results indicate that although the DMC controller gives very tight control when the sensor contains no noise, the control deteriorates as the amount of noise increases. In this set of data best control was achieved with control whose model was achieved from a noiseless signal (triangles).

These first set of experiments illustrate the potential and problems of DMC control. Successive experiments explore the use of filters and the move suppression factor to achieve good control.
Conclusions

The Virtual Laboratory provides an interesting method for demonstrating and comparing DMC and PID control. The experiments illustrate the power and shortcomings of DMC control and allow students to explore a variety of issues regarding advanced methods of process control.

References

4. Riggs, James, B. Chemical Process Control, Ferret Publishing,

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Prof. Nippert has been on the faculty of the Chemical Engineering Department of Widener University, Chester Pa. since 1980. He is a graduate of Lehigh University and worked for several years at Kawecki-Berylco (now Cabot Specialty Metals). His current interests include process simulation, process control and the development of web-based instructional materials for use in a variety of engineering courses. The material is contained in ‘The Online Widener Laboratories’, found at www2.widener.edu/~crn0001/VirtualLab.html. The site is open to all for examination and use. He is married and has two children.