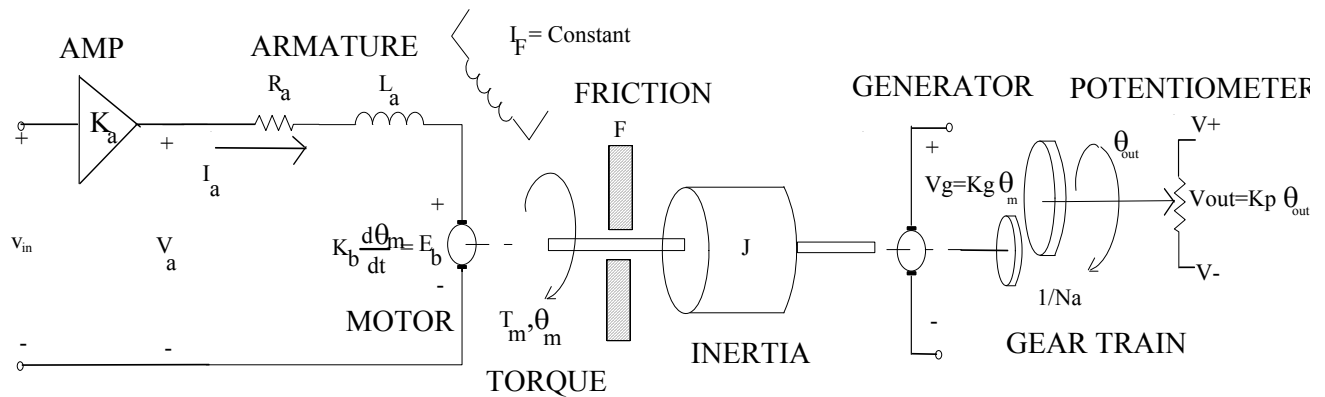


Recall the functional block diagram for our Motomatic armature-controlled DC servomotor in our Controls Lab:



Using methods learned in class, we found the open-loop transfer function of the Motomatic to be:

$$\frac{V_{out}}{V_{in}} = \frac{210.9}{s^2 + 5.27s}$$

1. a) Use Matlab's tf2ss() command to find a continuous-time state space representation of the form  $\dot{x} = Ax + Bw$ ,  $x(0^+)$  for this system. What are values of n,m, and p (i.e., what are the the number of states, inputs, and outputs, respectively)?

Ans: The values of n=number of states = 2, m=number of inputs=1 and p=number of outputs=1. Matlab found the following values for A,B,C, and D:

```
>> [A,B,C,D]=tf2ss(num,den)
```

A =

```
-5.2700    0
 1.0000    0
```

B =

```
1
0
```

C =

```
0 210.9000
```

D =

```
0
```

- a) What are the eigenvalues of the open loop system? How do they relate to the poles of the transfer function? (hint: use the eig() command in Matlab)

Ans: the eigenvalues are -5.27 and 0. They are the same as the open-loop poles of the transfer function.

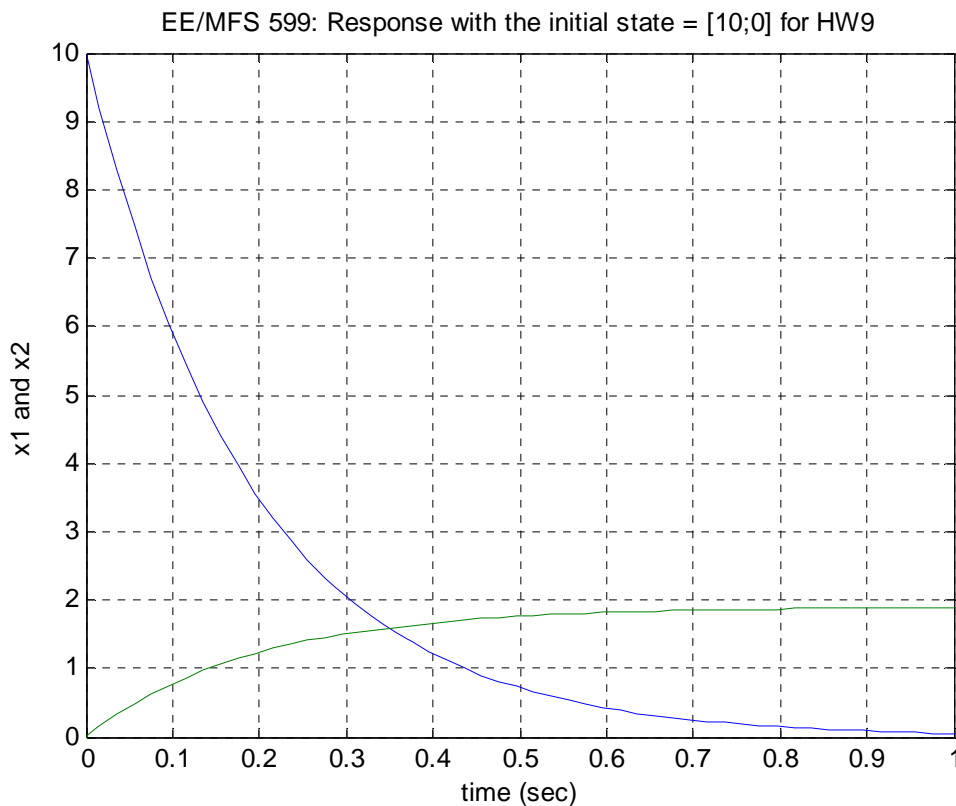
- b) Looking at the open-loop eigenvalues, would you classify our servo as

asymptotically stable, marginally stable, or unstable?

Ans. There is a distinct pole on the  $j\omega$  axis with the rest of the pole(s) in the LHP. Therefore, I would classify it as **MARGINALLY** stable.

- c) Simulate your open-loop state-space system in Simulink using an initial state of  $x(0) = [10 \ 0]^T$  (hint: let C be the 2x2 identity matrix so that the output is the state vector, x. You will also have to make D a 2x1 matrix of zeroes). Given your answer to part b), does the output of your simulation make sense?

Ans: Here is the Simulink simulation of the system. Notice that  $x_2$  does NOT go to zero but goes to a constant value of about 1.95 in steady-state. Thus, the system is marginally stable.



2.

- a) Find the controllability matrix,  $M=[B \ AB \ A^2B \ \dots \ A^{n-1}B]$  and verify that the system is completely controllable by verifying that the rank of the controllability matrix is  $n$  (hint: you can use the `ctrb()` and `rank()` commands in Matlab)

ANS: `>> M=ctrb(A,B)`

M =

```
1.0000 -5.2700
0 1.0000
```

`>> rank(M)`

ans =

2

- b) If the system is completely controllable, then we can use state feedback to completely specify the eigenvalues of the closed-loop system. Use the `place()` command in Matlab to find the value of  $K$  for the feedback control,  $w=-Kx$ , which will set the closed-loop eigenvalues to  $\{-16,-17\}$

Ans: >> p=[-16;-17]

p =

-16  
-17

>> K=place(A,B,p)

K =

27.7300 272.0000

>> eig(A-B\*K)

ans =

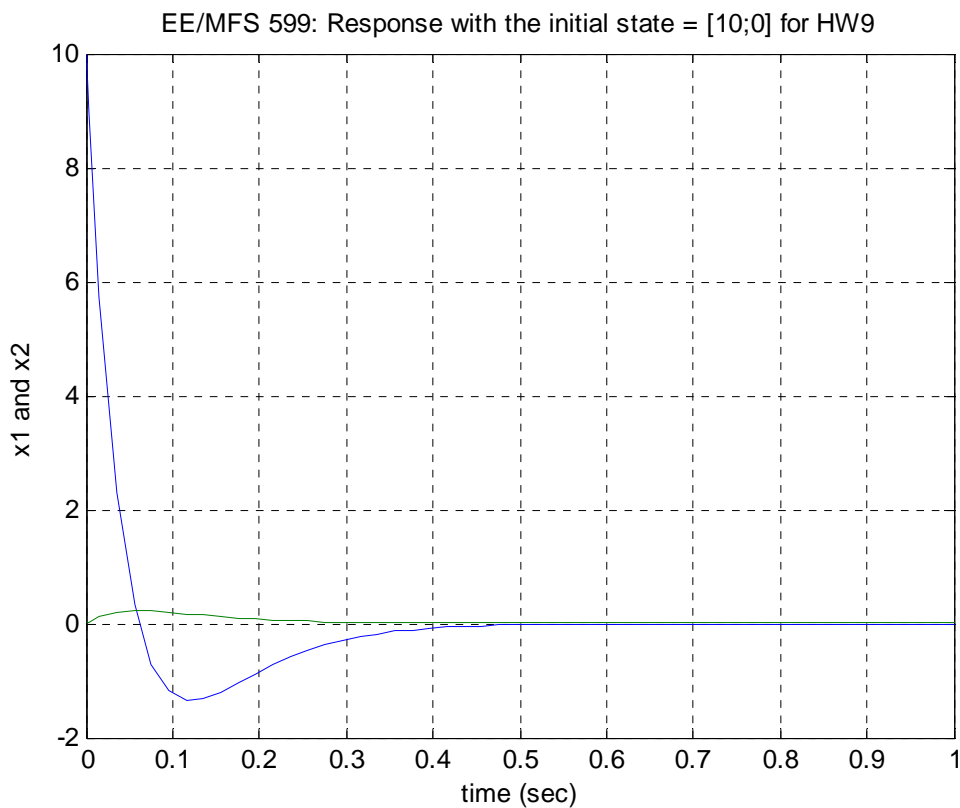
-17.0000  
-16.0000

>>

$K = [27.73 \ 272]$  so  $w = -Kx = -27.73x_1 - 272x_2$

- c) Simulate your closed-loop state-space system in Simulink using an initial state of  $x(0) = [10 \ 0]^T$  (hint: you will need to connect a 2x1 gain block  $-K$  to the output of your Simulink model and feed the output of this block it back to the input )

Ans: Here is the results of the Simulink simulation model using  $w=-Kx$  as feedback:



d) Do your states decay to zero now?

Ans: Yes, both states decay to zero and “settle” in about 0.3 seconds or so