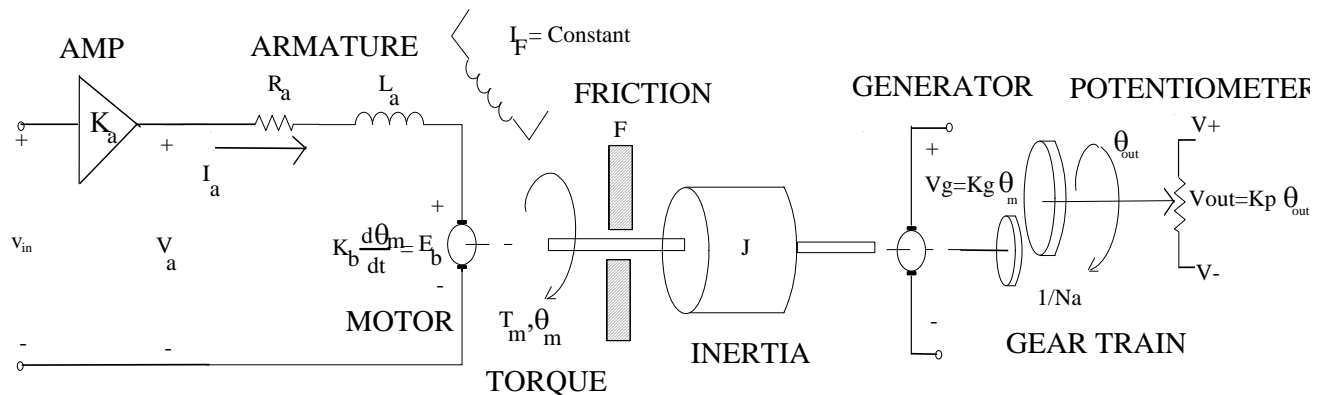


Recall the functional block diagram for our Motomatic armature-controlled DC servomotor in our Controls Lab:



Using methods learned in class, we found the open-loop transfer function of the Motomatic to be:

$$\frac{V_{out}}{V_{in}} = \frac{210.9}{s^2 + 5.27s}$$

1. a) Use Matlab's `tf2ss()` command to find a continuous-time state space representation of the form  $\dot{x} = Ax + Bw$ ,  $x(0^+)$  for this system. What are values of  $n, m$ , and  $p$  (i.e., what are the the number of states, inputs, and outputs, respectively)?
  - a) What are the eigenvalues of the open loop system? How do they relate to the poles of the transfer function? (hint: use the `eig()` command in Matlab)
  - b) Looking at the open-loop eigenvalues, would you classify our servo as asymptotically stable, marginally stable, or unstable?
  - c) Simulate your open-loop state-space system in Simulink using an initial state of  $x(0) = [10 \ 0]^T$  (hint: let  $C$  be the  $2 \times 2$  identity matrix so that the output is the state vector,  $x$ . You will also have to make  $D$  a  $2 \times 1$  matrix of zeroes). Given your answer to part b), does the output of your simulation make sense?
2.
  - a) Find the controllability matrix,  $M = [B \ AB \ A^2B \ \dots \ A^{n-1}B]$  and verify that the system is completely controllable by verifying that the rank of the controllability matrix is  $n$  (hint: you can use the `ctrb()` and `rank()` commands in Matlab)
  - b) If the system is completely controllable, then we can use state feedback to completely specify the eigenvalues of the closed-loop system. Use the `place()` command in Matlab to find the value of  $K$  for the feedback control,  $w = -Kx$ , which will set the closed-loop eigenvalues to  $\{-16, -17\}$
  - c) Simulate your closed-loop state-space system in Simulink using an initial state of  $x(0) = [10 \ 0]^T$  (hint: you will need to connect a  $2 \times 1$  gain block  $-K$  to the output of your Simulink model and feed the output of this block it back to the input )
  - d) Do your states decay to zero now?