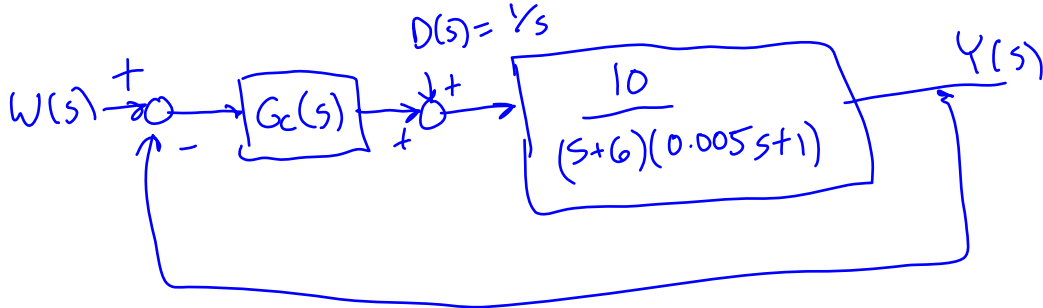


$T_s = 10 \text{ msec}$

1. a) Re-write the problem so it is completely in the s-domain

Solution: $G_{zoh} = 1/[(T_s/2)s+1] = 1/[0.005s+1]$



b) Make the disturbance zero and let $G_c(s) = K$. Find the sensitivity of $Y(s)/W(s)$ to the gain K for both the open-loop and unity feedback case when the nominal value is $K=100$ and the input is a unit step.

open-loop:
$$\sum_K \frac{Y}{W} = \frac{\partial Y}{\partial K} \cdot \frac{K}{Y} \Big|_{Nom} = \frac{G_{zoh}G \cdot K}{K G_{zoh}G} = 100\%$$

closed-loop
$$\sum_K \frac{Y}{W} = \frac{\partial Y}{\partial K} \cdot \frac{K}{Y} = \frac{\partial}{\partial K} \left(\frac{K G_{zoh}G}{1 + K G_{zoh}G} \right) \cdot \frac{K}{\frac{K G_{zoh}G}{1 + K G_{zoh}G}} \Big|_{Nom}$$

$$= \frac{G_{zoh}G(1 + K G_{zoh}G) - G_{zoh}G(K G_{zoh}G)}{(1 + K G_{zoh}G)^2} \cdot \frac{K}{\frac{K G_{zoh}G}{1 + K G_{zoh}G}} = \frac{1}{1 + K G_{zoh}G} \Big|_{Nom}$$

For the nominal case of $s=0$ and $K=100$,
$$\sum_K \frac{Y}{W} = \frac{1}{1 + 100(\frac{10}{6})} = 0.6\%$$

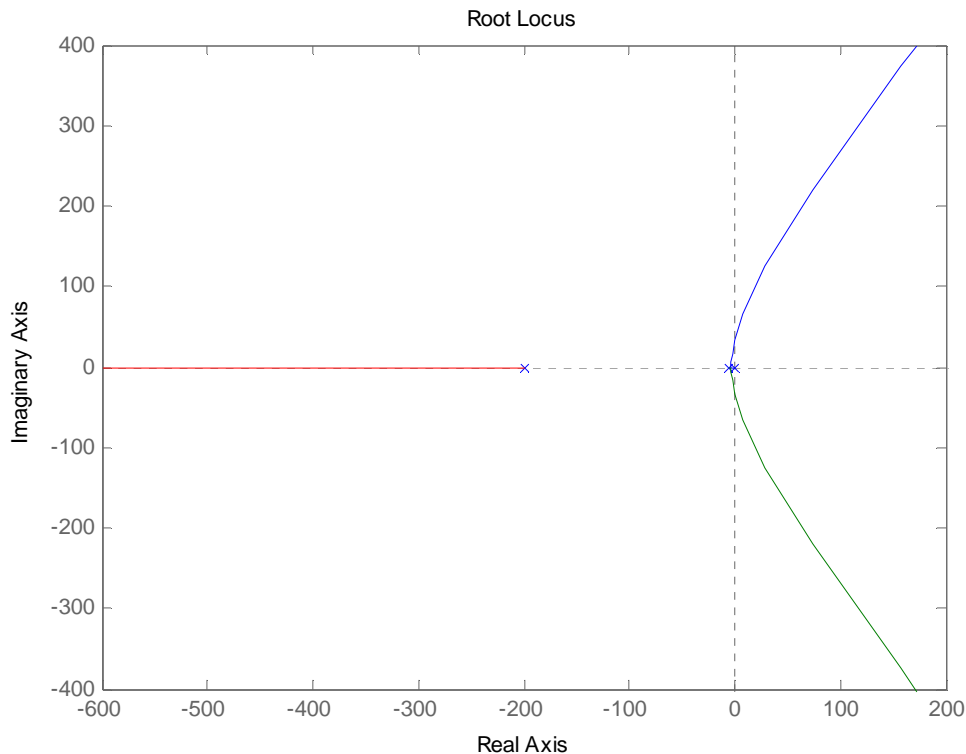
c) Predict how the steady-state response will change if K increases by 10% for both the open-loop and closed-loop configurations.

open-loop: 10% change in K will cause a 10% change in steady-state response
 0.06% change

Closed-loop: 10% change in K will cause a
in Steady-state

2. a) Let the disturbance be a unit step (constant). Design a Robust digital compensator, $G_c(z)$, which will meet the following specifications
1. Closed-loop system is stable
 2. t_s due to a step is less than 2 seconds
 3. e_{ss} due to a step is zero
 4. $M_p < 5\%$ (remember Mohannad's 4.32% means $\zeta = 0.707$)
 5. Steady-state error due to the constant disturbance $d(t)=u(t)$ is minimized

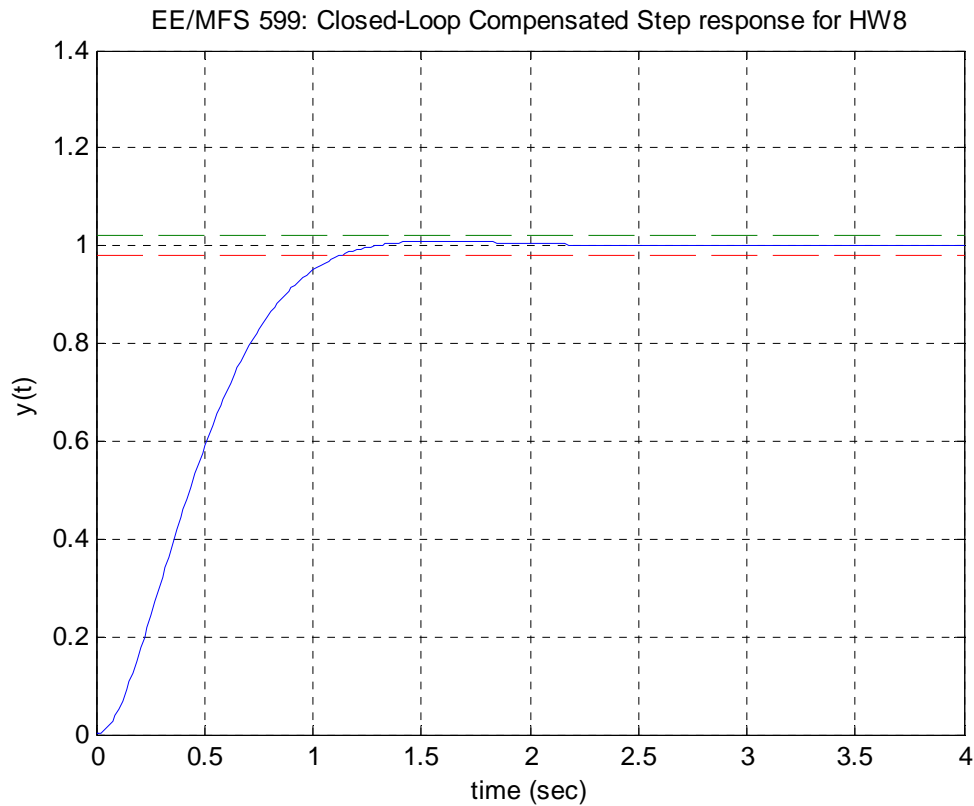
We should address the disturbance and ess specs first. We currently have a Type 0 system. We need a minimum of a Type 1 system to meet ess specs. We also need an integrator to reject the disturbance. We should try $G_c(s) = \text{num}(s)/s$. If we lump the $1/s$ term in with $G_{zoh}G(s)$ to obtain $10/(s(s+6)(0.005s+1))$ and draw the root locus, we find:



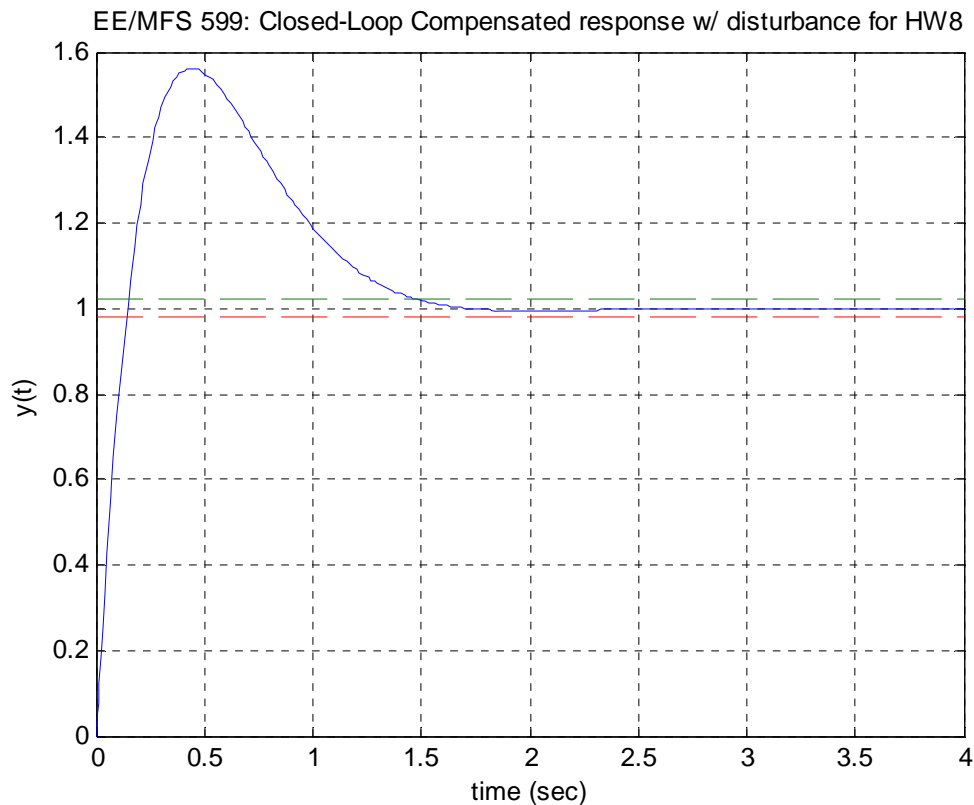
From the transient specifications, we have $s_1 = -3+j2$ will easily meet the specifications. We note that the root locus passes very close to these dominant poles. We can let the $\text{num}(s) = K$ and find K from $K = -1/(G_{zoh}G(s_1)/s_1) = 1.28$. Thus, a compensator which will meet all the above requirements is $G_c = 1.28/s$

- b) Use Simulink to verify your design (in the s-plane using $G_c(s)$). Initially, set the disturbance to zero and measure your transient (t_s and M_p) and steady-state error specs. Then, set the disturbance to a unit step and verify that its effect is negligible

Ans: Here is the compensated step response with no disturbance. As you can see, the system settles in about 1.2 seconds and has less than 2% overshoot (nice response!)

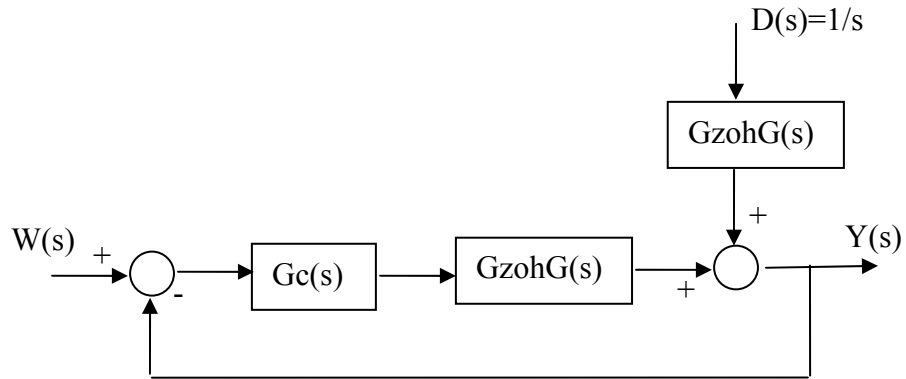


Here is the response with the step disturbance turned on. As you can see, the system rejects the disturbance in steady-state and returns to a final value of 1.

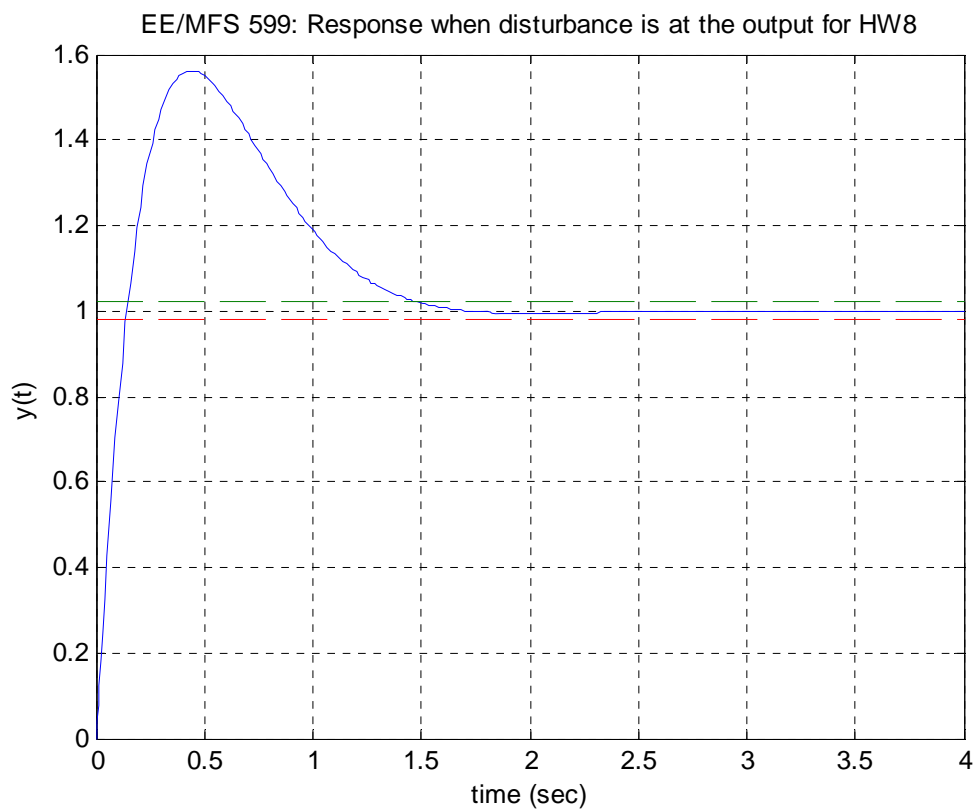


- c) Your book discusses the case when your disturbance occurs at the output. I made the statement in class that the model we use is the same as the model in the book if we simply put $G_{zoh}(s)$ as a pre-filter to the output disturbance. Implement the following diagram on

Simulink using your values of $G_{zoh}(s)$ and $G_c(s)$ to verify the output is the same as in part b)



Ans: The following is the Simulink simulation for the above block diagram. It is identical to the response we obtained in part c)



d) Find $G_c(z)$ using any method you wish (since there is no derivative term, we can use the bilinear transform).

$$G_c(z) = 1.28 / (2(z-1) / [T_s(z+1)]) = 25,600 [z-1] / [z+1] \text{ using the bilinear transformation}$$