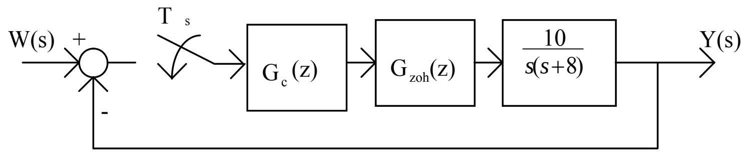
Due Wednesday, February 13 EEMFS 599 HW #16

1. Given the following digital control system for a servo with open-loop transfer function G(s) = 10/(s(s+8)):

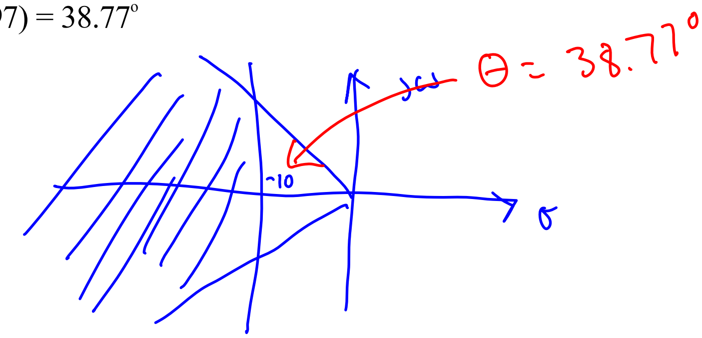


a) Find an S-domain model for the open-loop system including the ZOH if T_s = 10 msec.

Ans: Gzoh(s) = $1/[1+sT_s/2]$ = 1/[1 + 0.005s]. Thus, GzohG(s) = 10/(s(s+8)(0.005s+1))

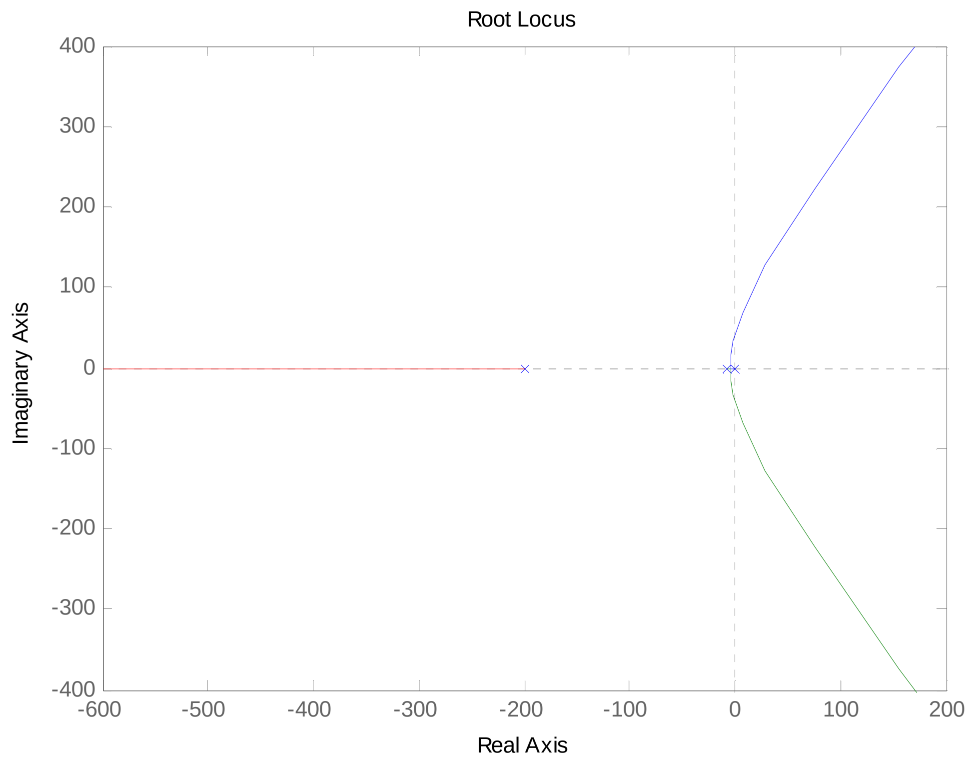
b) Given the following transient specifications: $t_s \leq 0.4$ sec and $M_p \leq 2\%$. Illustrate the region of the s-plane where we must place our dominant poles to satisfy these specs.

Note that ζ=sqrt(log(Mp)^2/(log(Mp)^2+pi^2)) = sqrt(log(.02)^2/(log(.02)^2+pi^2)) = 0.7797
Thus $\theta = \cos^{-1}(0.7797) = 38.77^{\circ}$



c) Pick a pair of dominant closed-loop poles to meet the above specs then sketch the root locus. Does the root locus pass thru the desired poles? If not, design a PD compensator (filter) $G_c(s)$ then find $G_c(z)$ (using the approximation derived in class) to force the root locus to pass thru these poles and meet the specs given in part d).

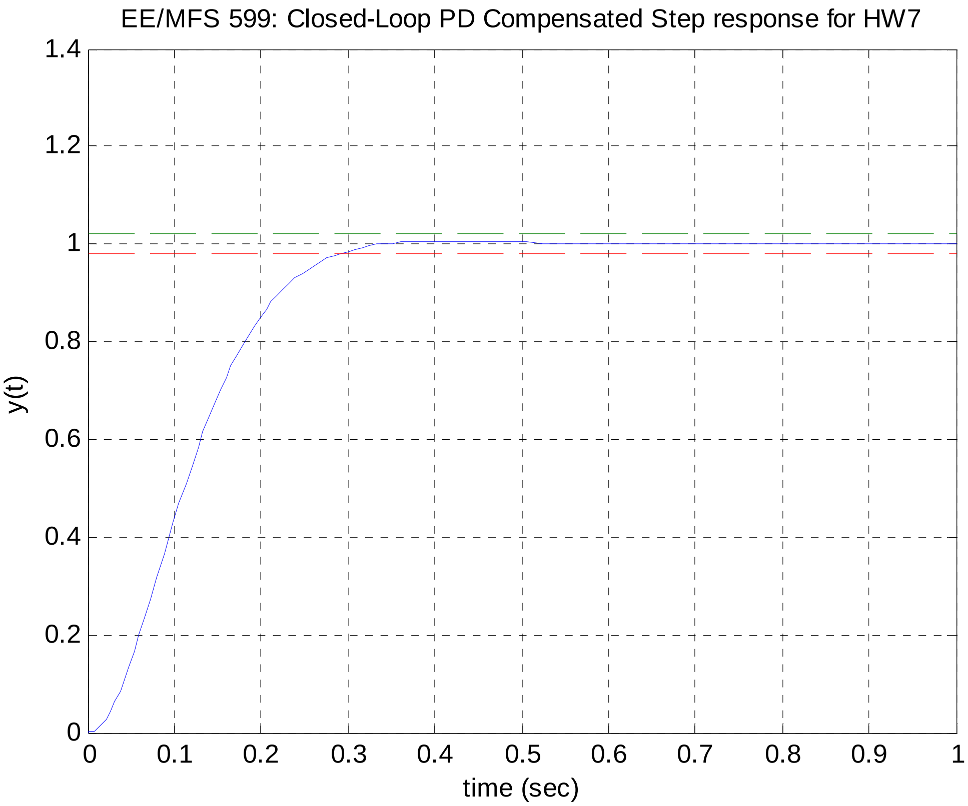
Ans: Pick the poles s1 = -12+j8 and it's conjugate to meet the specifications. Here is the root locus:



It doesn't pass thru the desired dominant poles. c) If no, we have to design a PD compensator to force the root locus to pass thru s1. Let Gpd(s) = Kp + Kds = K(s+z) where K=Kd and z=Kp/Kd. Find values of K and z to make s1 a closed-loop pole of the system

Ans: K(s1+z) = -1/GzohG(s1) = 0.9920 +j12.0960. Thus K = 12.096/8 = 1.5120 and z = 0.992/K +12 = 12.6561. Thus, Gpd(s) = 1.5120 (s+12.6561)= 1.5120s+ 19.1360. So, Kp = 19.1360 and Kd = 1.5120. To find Gpd(z), we should use the rectangular derivative approximation for s=(z-1)/zTs. Thus, Gpd(z) = Kp + Kd(z-1)/zTs = 19.1360 + 151.20(z-1)/z = (170.3360z - 151.20)/z

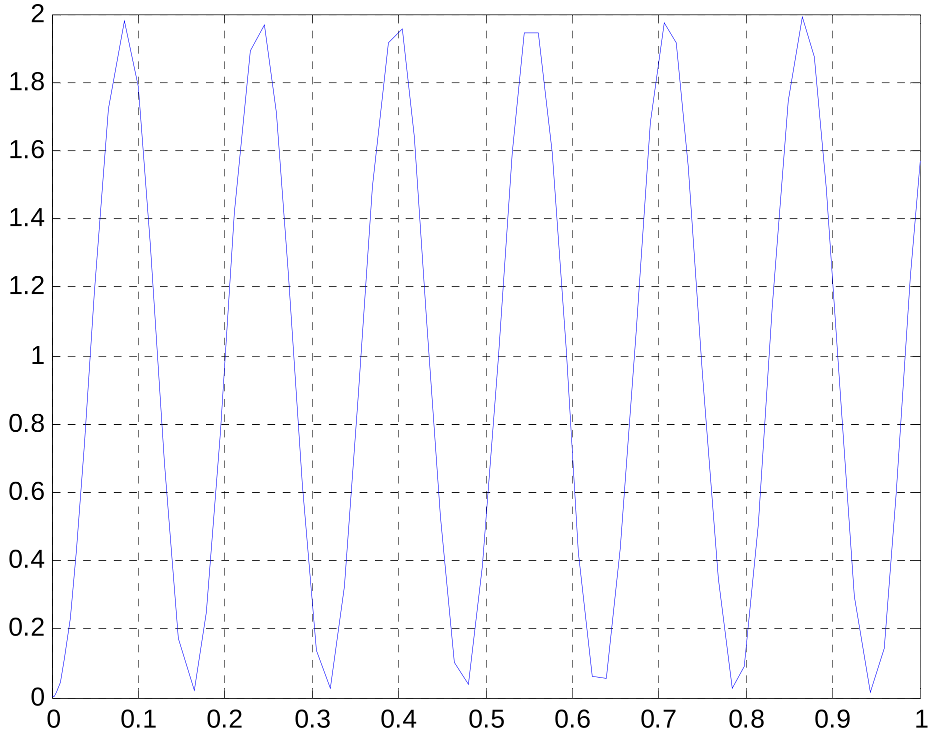
d) Find the closed-loop step response and measure the settling time and overshoot.



The settling time is 0.29 and the overshoot is about 1%. Nice response!

2 a) Use the 2nd Ziegler-Nichols method to tune a PID control for this system

Ans: After several trials, a K0 = 167 produced the following closed-loop marginally stable step response.



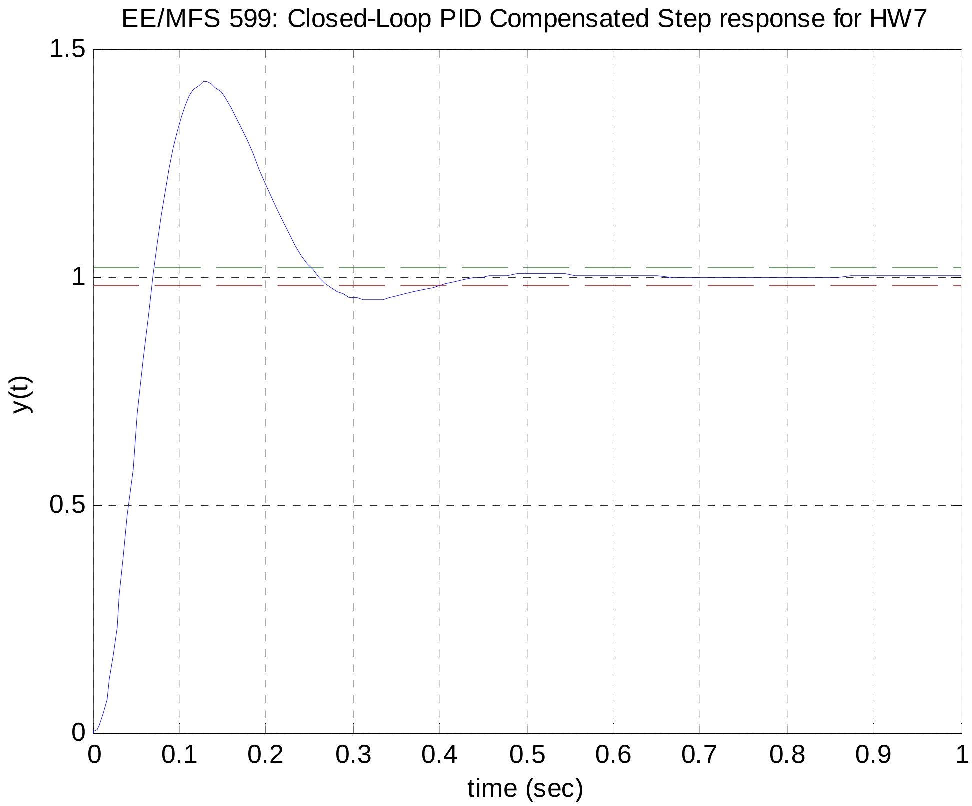
Note the period of oscillation is P = 0.17 seconds approximately. If we plug these values into the Zeigler-Nichols formula we have:

Controller	K_P	K_I	K_D
P	$0.5K_0$	∞	0
PI	$0.45K_0$	$P/1.2$	0
PID	$0.6K_0$	$0.5P$	$0.125P$

$$G_c(s) = K_P\left(1 + K_D s + \frac{1}{K_I s}\right)$$

so Kp = 0.6K0 = 100.2, Ki = 0.5*P = 0.85, and Kd =0.125*P = 0.2125. However, the form of the PID is slightly different (see above) so in actuality, Kd = Kp*Kd = 4.2585and Ki=Kp/Ki =1178.8

This produces the following step response:



b) Now find $G_{PID}(z)$ (using the approximation derived in class)

Ans: Using the same rectangular integration approximation for 1/s we find:

Gpid(z) = Kp + Ki (Tsz/(z-1)) + Kd(z-1)/(Tsz) = (Kp + KiTs +Kd/Ts)z^2 +(-Kp-2Kd/Ts)z +(Kd/Ts)]/(z^2-z) = (537.83z^2 - 951.9z + 425.85)/(z^2 – z) = (537.83 - 951.9z^{-1} + 425.85z^{-2})/(1 – z^{-1})

c) Write a flow chart similar to the one derived in class to implement your PID digital control

Ans:

If we have a gen'l 2nd order digital filter: Y(z)/W(z) = = b0+b1z^{-1} + b2z^{-2})/(a0+a1z^{-1}+ a2z^{-2}) then the update equation for yk is

yk = 1/a0*(b0*wk+b1*wk-1+b2*wk-2 – a1*yk-1-a2*yk-2)

Thus, the flow chart would be

Initialize yk=yk-1=yk-2=wk=wk-1=wk-2=0

set a0=1; a1=-1; a2=0; b0=537.83; b1=- 951.9; b2 = 425.85

start endless loop

temp = 1/a0*(b1*wk-1+b2*wk-2 – a1*yk-1-a2*yk-2) ← form partial sum

wk=atod ← get current value of input

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yk=temp +1/a0*(b0*wk) ← form rest of yk
dtoa(yk)     ← output yk
wk-2=wk-1  ← update old values
wk-1=wk
yk-2=yk-1
yk-1=yk
wait total of Ts seconds
loop back
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