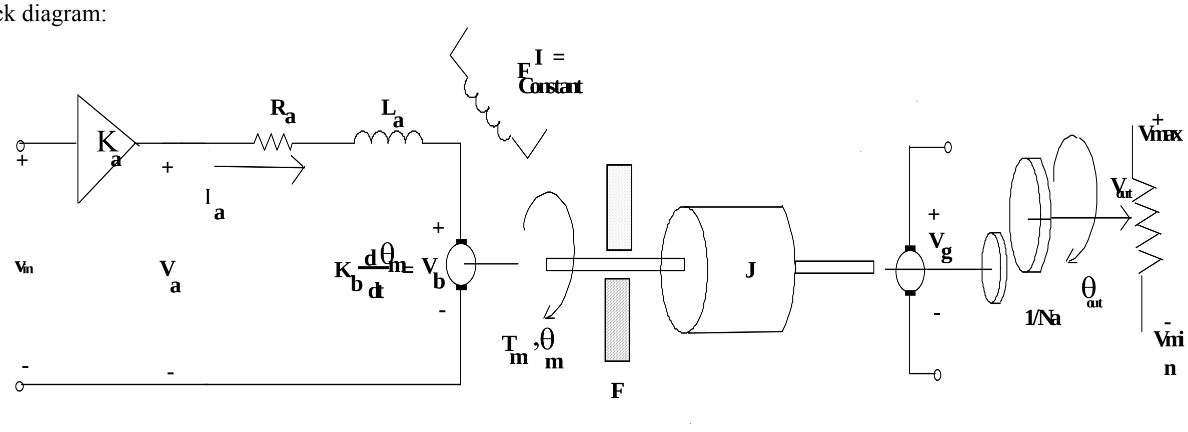
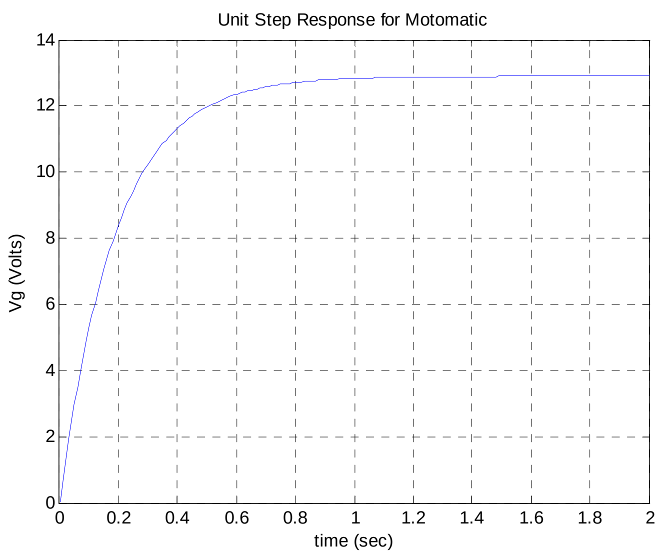
In our controls system lab, we have a DC Servo motor called the Motomatic®. We would like to design a controller for this DC servo using the following block diagram:



We can operate the Motomatic in either velocity mode or position mode. If we operate in velocity mode, the Motomatic is approximately a first order system. The open-loop unit step response of the Motomatic in velocity mode is given below




1.a) Find the transfer function, G(s), then find the value of Kp, Kv, and ess due to a step and ess due to a ramp

Ans: From the step response, G(s) = K/(Ts+1) where K = 13 and T = 1.2 seconds. Thus, G(s) = 13/(1.2s+1). Kp = 13/1.2 = 10.833 and Kv = 0. Therefore, ess due to a step = 1/(1+Kp) = 0.0845 and ess due to a ramp is infinite.

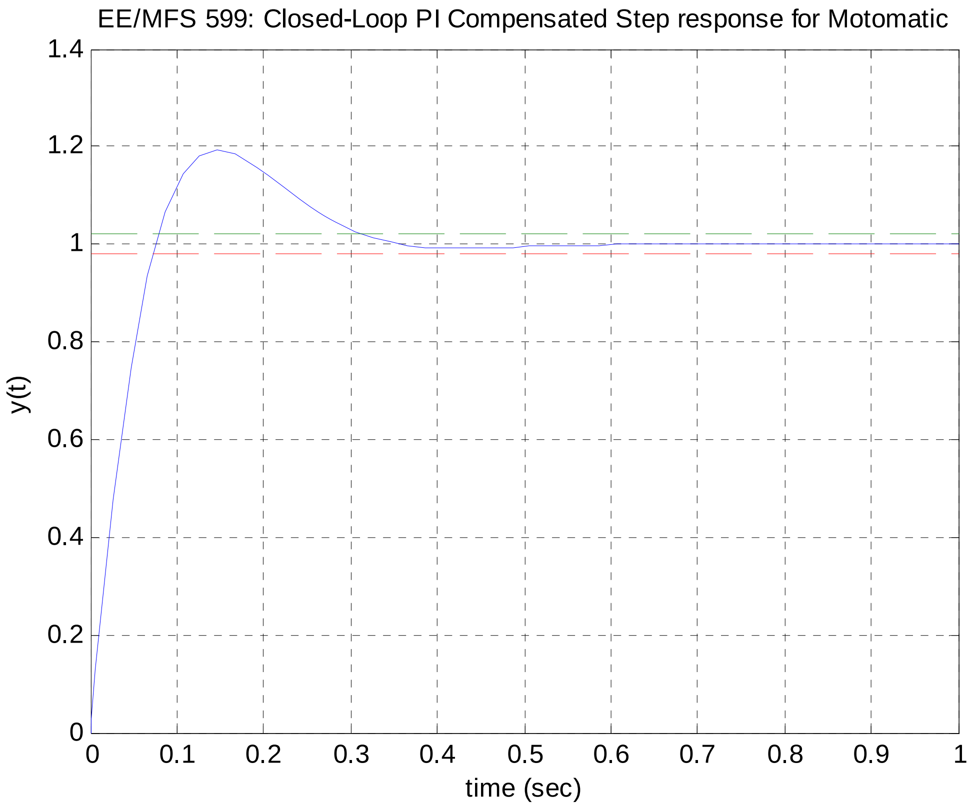
b) Design a PI compensator, $G_{PI}(s)$ to meet the following specs:

1) $t_s < 0.4$ seconds
2) $M_p < 5\%$
3) $e_{ss}$ due to a step = 0

Ans: We need a PI compensator to meet the 3rd criterion (i.e., we need a Type 1 system). The first specification means that the dominant poles have to be to the left of the line σ = -0.4/4 = -10. The second means that the damping ratio should be less than 0.707 (which produces 4.32% overshoot according to Mohannad). Let's pick s1=-11+j11. Gpi = Kp + Ki/s = K(s+z)/s where K=Kp and z= Ki/Kp. To design the PI, we lump the 1/s term in with G(s) and solve K(s1+z) = -1/[G(s1)/s1] = 0.8462 +j21.4923. Thus, K = 21.49/11 = 1.9538 and z = 0.8462/K +11 = 11.4331. So Kp = K = 1.954 and Ki = z*Kp = 22.339

c) Simulate your closed-loop design on Matlab

Ans:




The overshoot is much greater than expected (19% versus 4.32%) and the settling time is actually less than expected (0.31 seconds vers 4/11 = 0.364 seconds).

d) If your step response does not meet all the specifications, comment on why

Ans: Again, the presence of the zero at -11.431 causes extra overshoot to occur just as in HW#5

2. a) Design a PID compensator to meet the following specs:

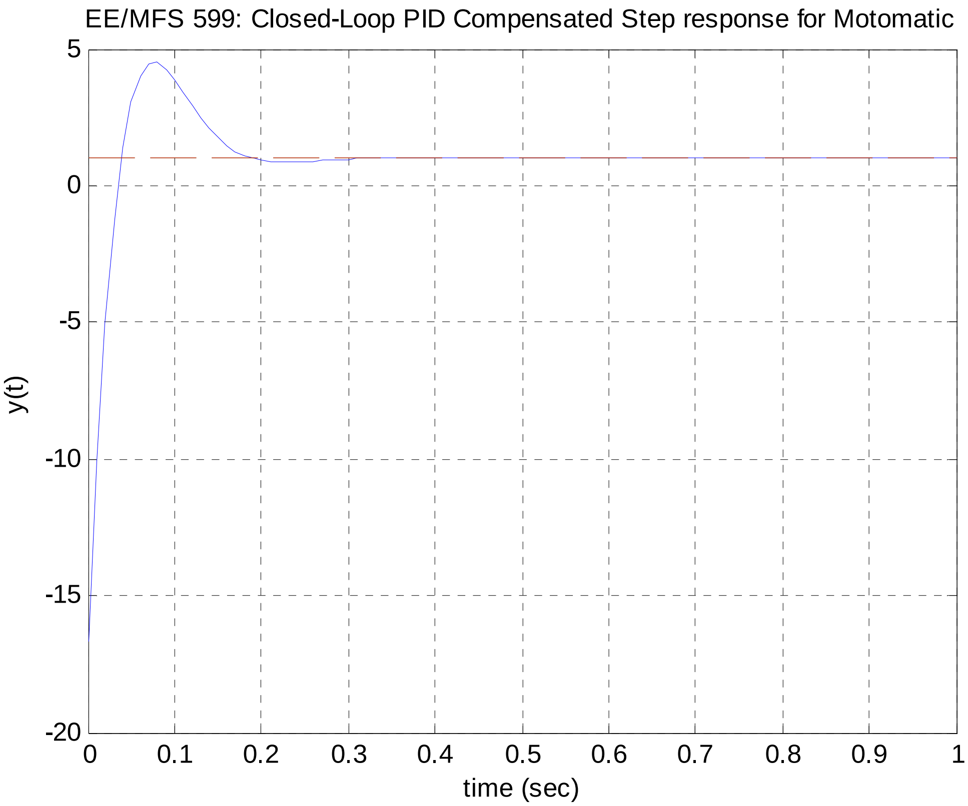
1) $t_s < 0.2$ seconds
2) $M_p < 5\%$
3) $e_{ss}$ due to a ramp = 1/50

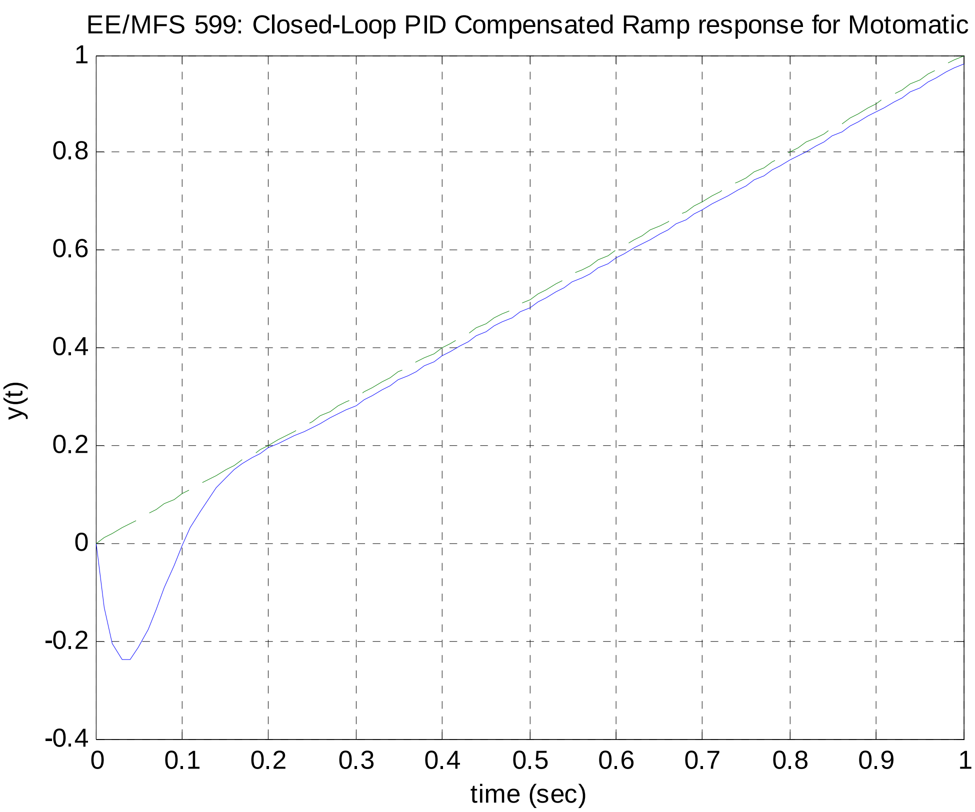
Ans:

The specification we address first in the PID design is the steady-state error spec (number 3). Recall, Gpid = Kp + Ki/s + Kds. Our desired Kv = 50 = Ki*13/1.2 . Thus, the necessary Ki = 50*1.2.13 = 4.6154. Next, we can address the transient specifications. The first specification means that the dominant poles have to be to the left of the line σ = -0.2/4 = -20. The second means that the damping ratio should be less than 0.707 (which produces 4.32% overshoot according to Mohannad). Let's pick s1=-21+j21. Thus, Gpid(s1) = Kp + Ki/s1 +Kds1 = -1/G(s1). When we lump the Ki/s1 term on the right hand side we have Kp+Kds1 = -Ki/s1 -1/G(s1) = 1.9714 - 1.8286. Equating real and imaginary parts we find that Kd = -1.8286/21 = -0.0871 and Kp = 1.9714 – Kd(-21) = 0.1429. Thus, the entire PID compensator is given by: Gpid(s) = Kp + Ki/s +Kds = 0.1429 + 4.6154/s + -0.0871s

b) Simulate your design on Matlab

Ans:




c) If your step response does not meet all the specifications, comment on why

Ans: While the closed loop poles are definitely at -21+j21, the PID has added two zeroes to the system. If we take the roots of $Kds^2+Kps+Ki$ for our values we find that we have added a zero at 8.1468 and one at -6.5062. The zero in the RHP at 8.1468 is causing the initial instability that we see. There is significant undershoot and the overshoot is about 400%. The settling time is very close to 0.2 seconds as expected.

In terms of the ramp response, the steady-state error is about 0.017 which is very close to our theoretical value of 1/50 = 0.02

The Bode Plot for the Motomatic in Position mode is given on the next page.

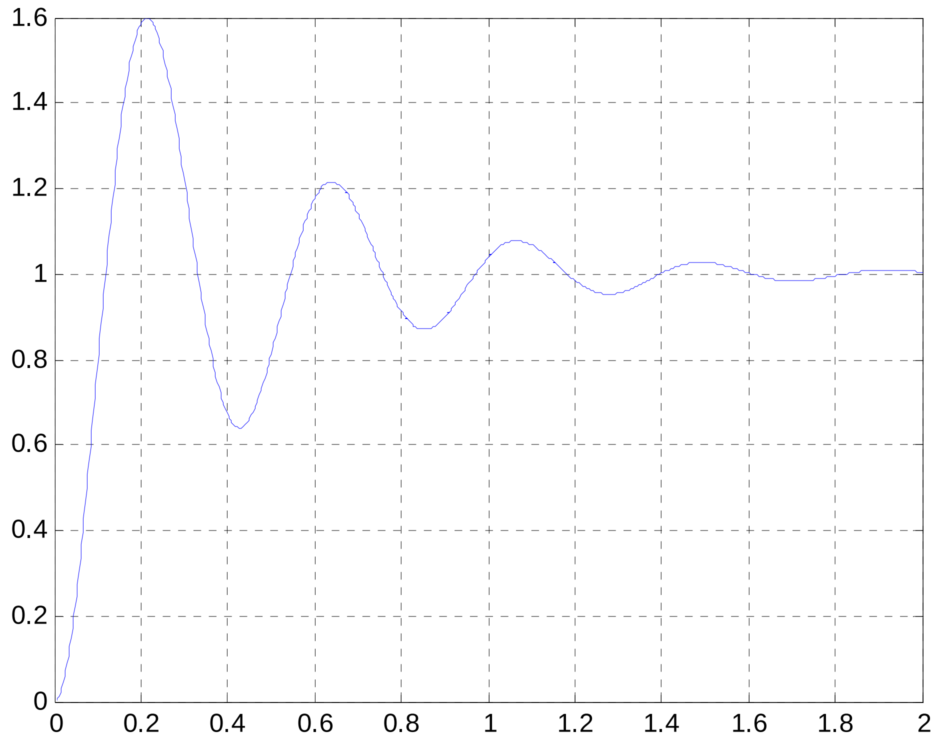
d) Find the new transfer function, G(s), then find the value of Kp, Kv, and ess due to a step and ess due to a ramp

Ans: $G(s) = K/[s^q(s/\omega_1+1)(s/\omega_2+1)]$. From the intersection of the asymptotes, we see that $\omega_1=5$ and $\omega_2=1100$. We also see that the type number is q=1 because the slope at low frequencies is -20 dB/decade. Finally, at frequencies below the breakpoints, G(s) = K/s. If we pick $\omega=1$, then the magnitude is about 33 dB or 44.67. Thus K/(1.0) = 44.67 or K = 44.67. Thus, in position mode, our transfer function for the Motomatic is G(s) = 44.67/[s(s/5+1) (s/1100+1)].

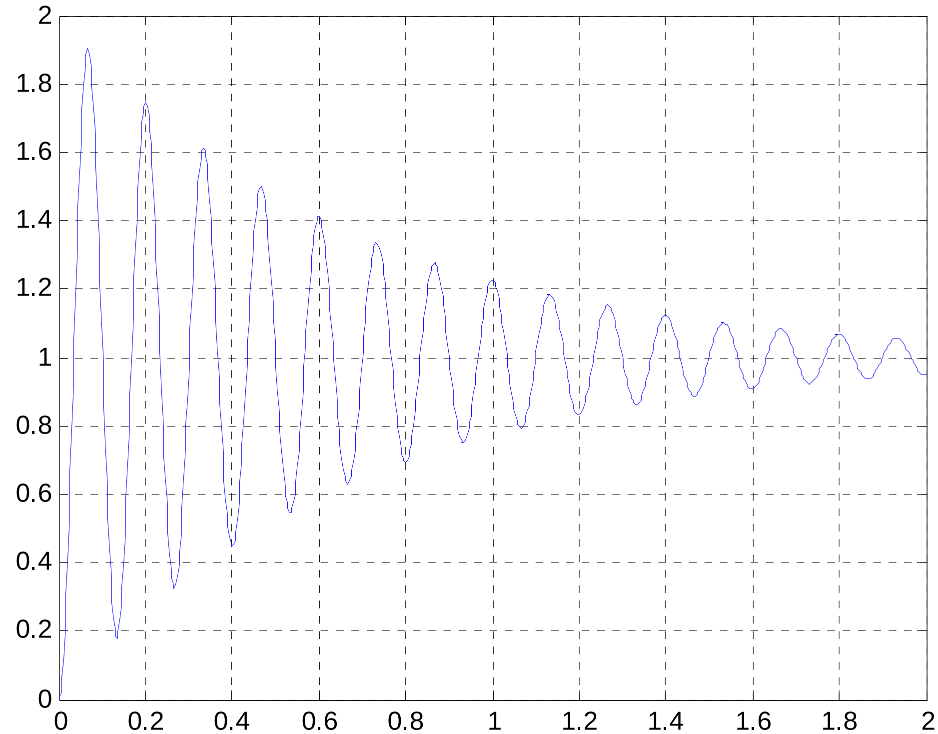
Kp is infinite and Kv = 44.67. Thus, $e_{ss}$ for a step is zero and $e_{ss}$ for a ramp is 1/44.67 = 0.0224

e) Use the 2nd Ziegler-Nichols method described in class to tune a PID controller for this system (i.e., find the gain K0 that makes the closed-loop system marginally stable then record this gain and the period of oscillation)

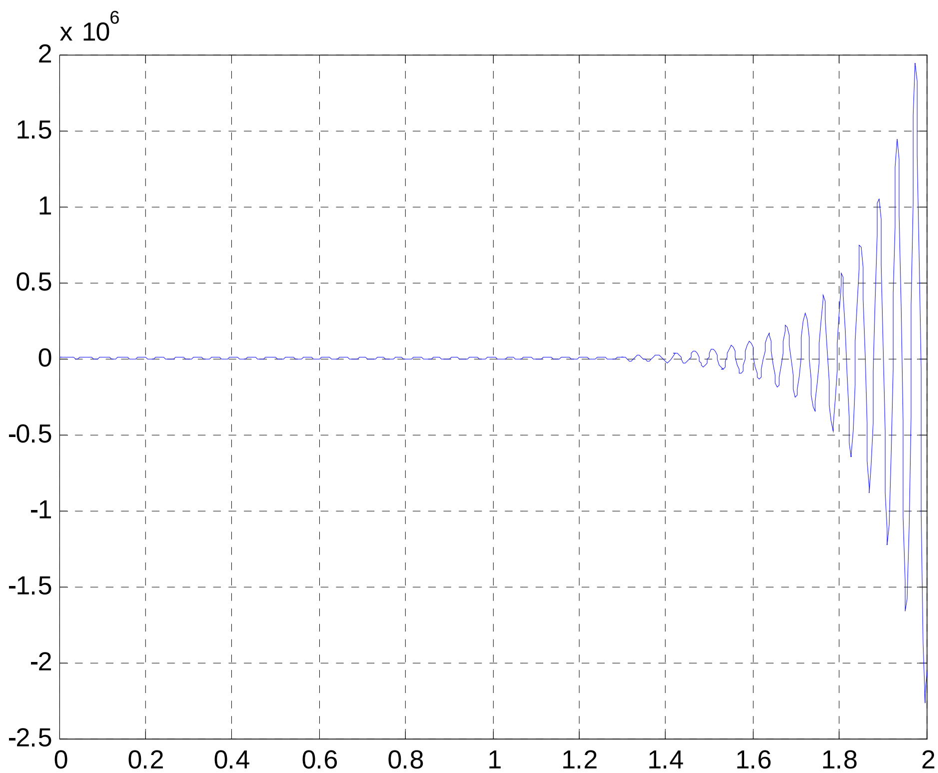
Ans: For a K0 =1 we obtain the following closed-loop unity-feedback response:



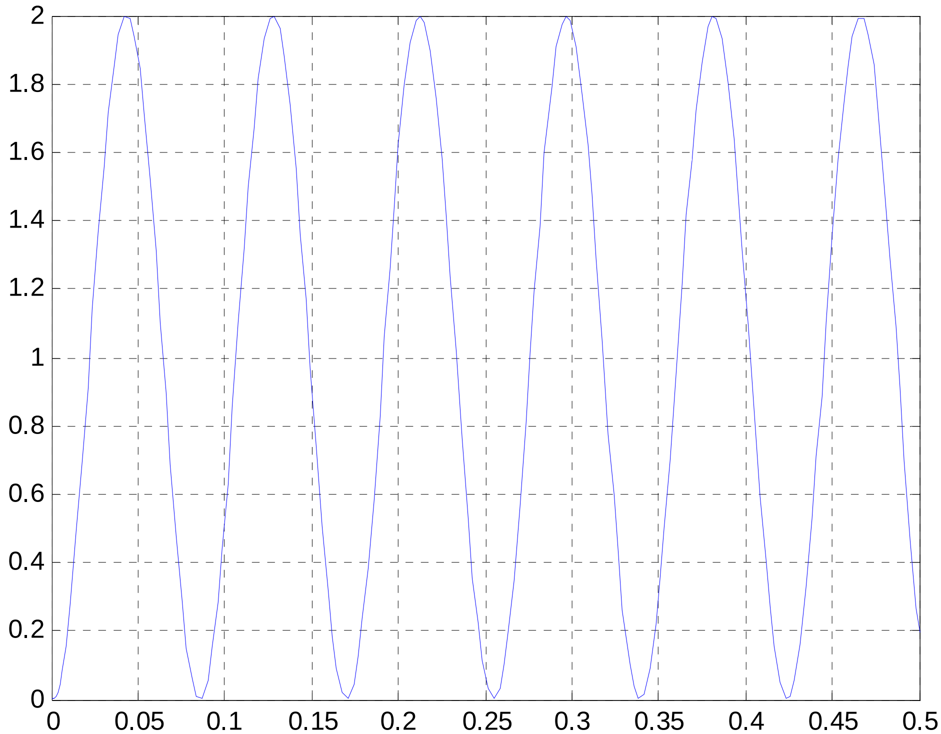

For a K0=10 we obtain:




For a K0 of 100 we obtain the following unstable response:




After a few iterations, a K0 of 24.8 produced the following marginally stable system:




So, K0=24.8 and the period of oscillation is about P=0.084 seconds. If we plug these values into the Zeigler-Nichols formula we have:

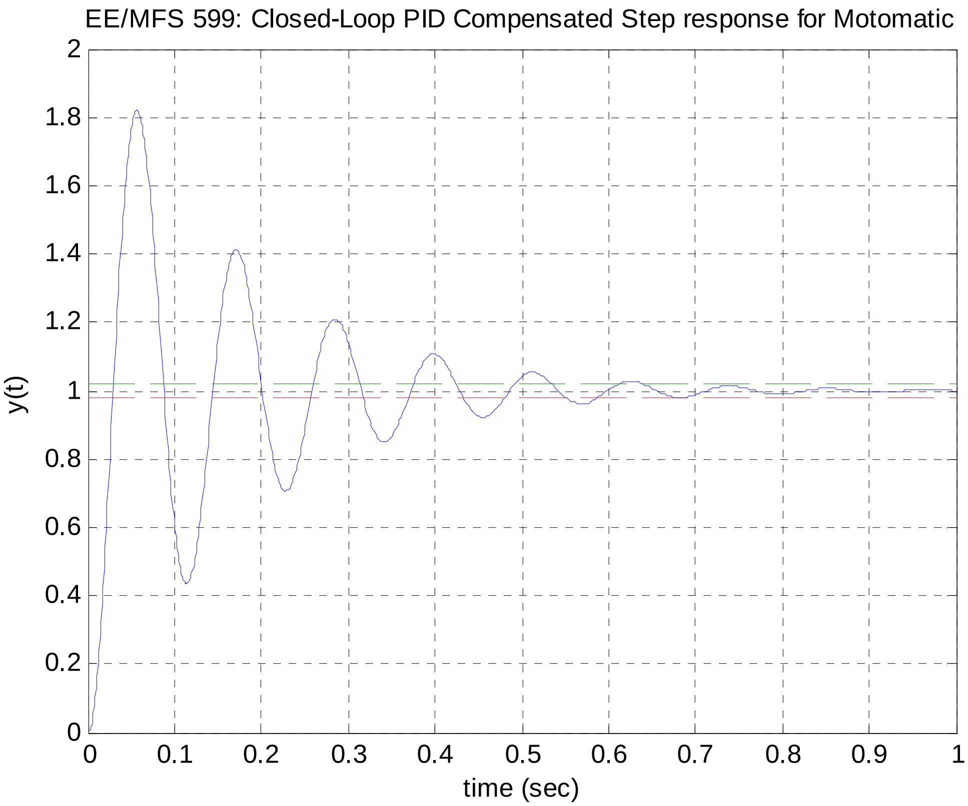
| Controller | $K_P$ | $K_I$ | $K_D$ |
|---|---|---|---|
| P | $0.5K_0$ | $\infty$ | 0 |
| PI | $0.45K_0$ | $P/1.2$ | 0 |
| PID | $0.6K_0$ | $0.5P$ | $0.125P$ |

$$G_c(s) = K_P\left(1 + K_D s + \frac{1}{K_I s}\right)$$

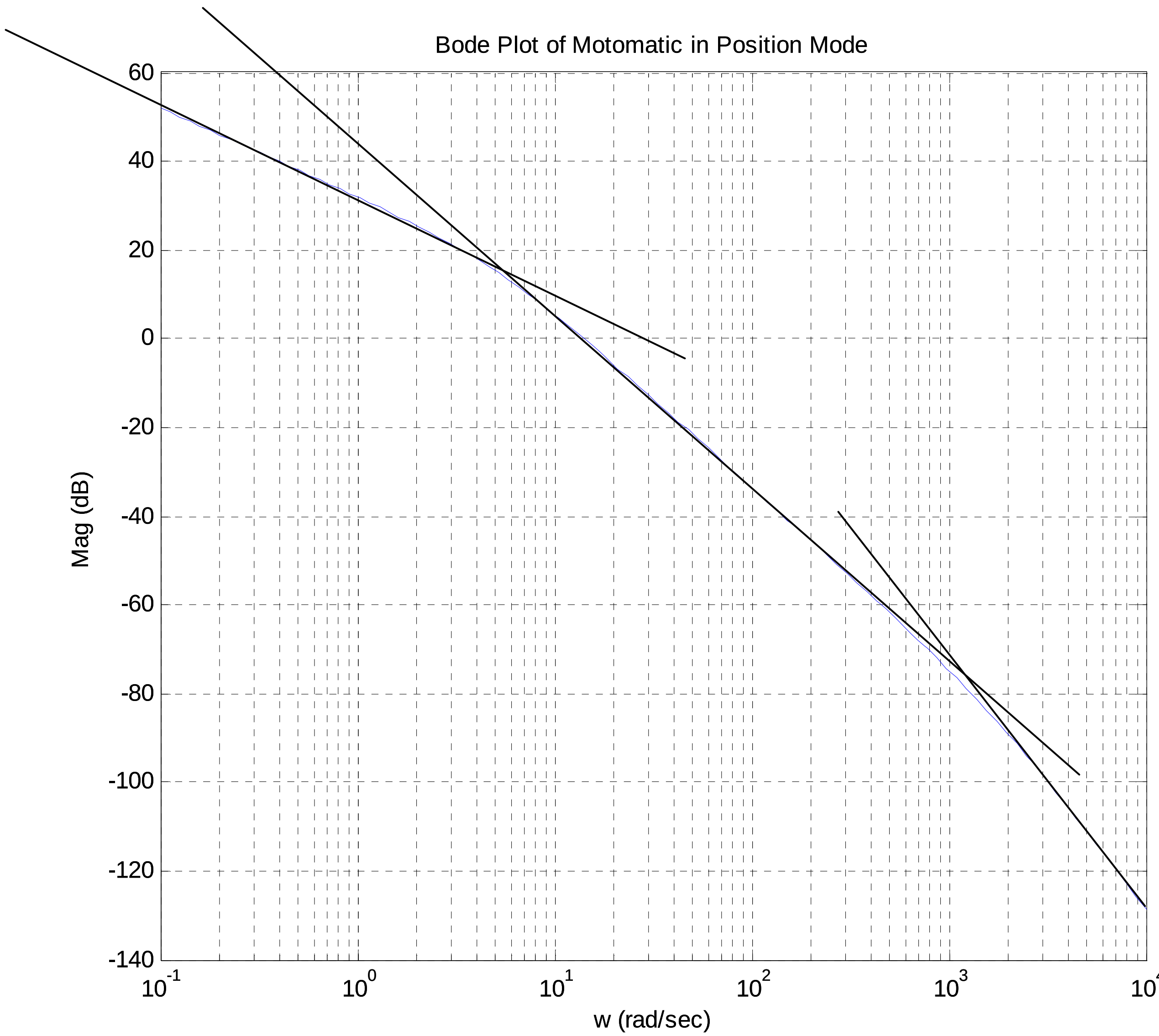
so Kp = 0.6K0 = 14.88, Ki = 0.5*P = 0.042, and Kd =0.125*P = 0.0105. However, the form of the PID is slightly different (see above) so in actuality, Kd = Kp*Kd = 0.156 and Ki=Kp/Ki =353.9297

e) Simulate your design on Matlab

Ans: Using these values in Matlab we obtain the following step response:




We can see the overshoot is about 82% and the settling time is 0.63 seconds


Bode Plot of Motomatic in Position Mode
Mag (dB)
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
10^-1
10^0
10^1
10^2
10^3
10^4
w (rad/sec)