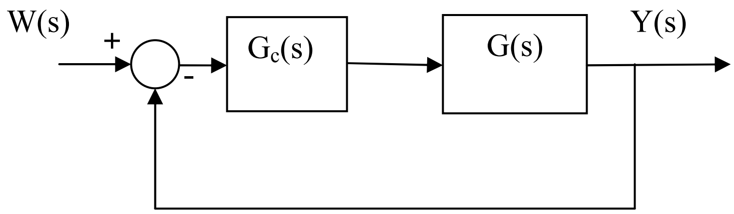
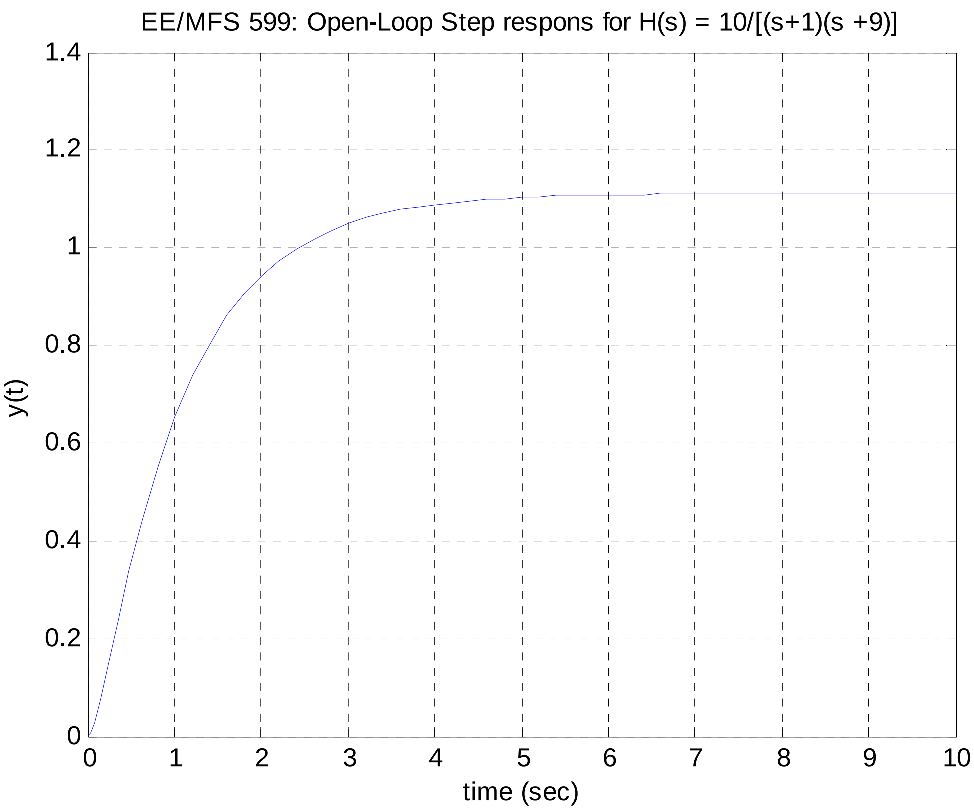
Due Wednesday, February 6th **EE/MFS 599** HW #5

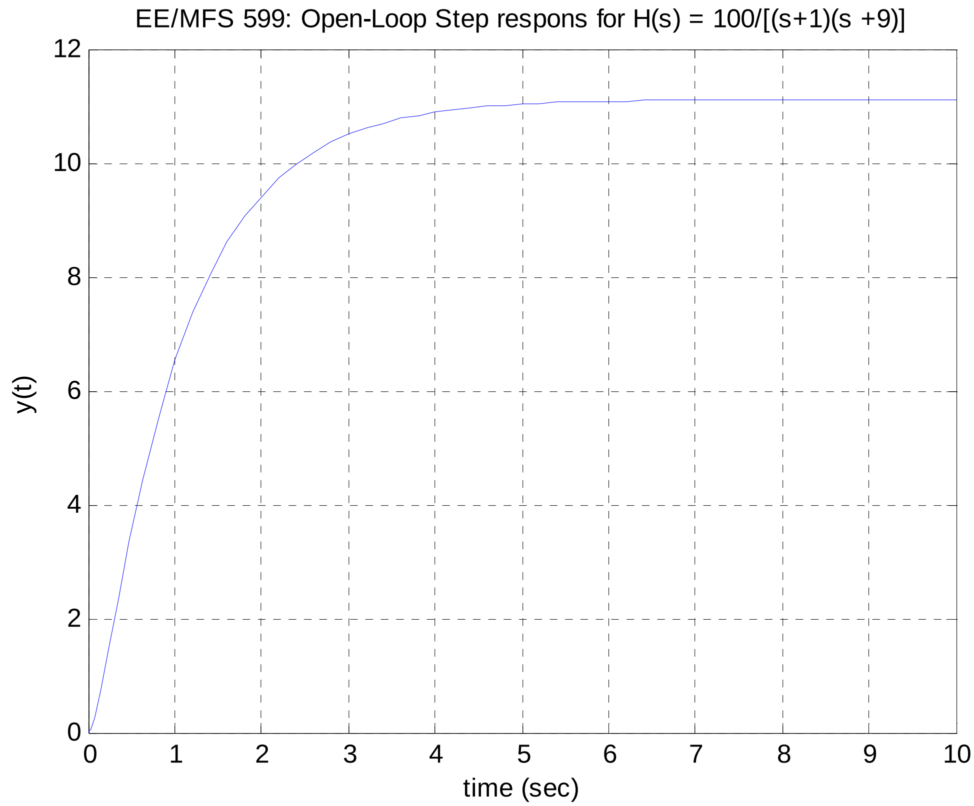
You may use Matlab/Simulink wherever applicable

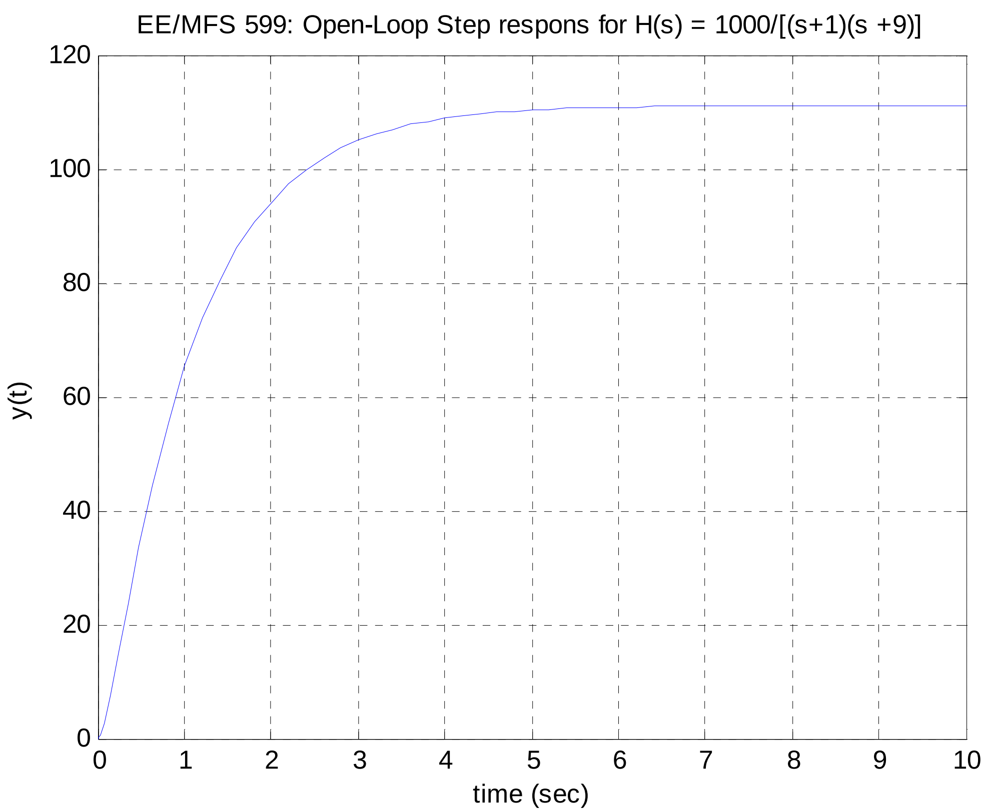
1. Consider the standard, unity-feedback closed loop control system shown below where $G(s) = 10/[s^q(s+1)(s+9)]$



a) Let q=0 (type zero system) and plot the OPEN-LOOP step response for K=1, 10 and 100.
Ans:





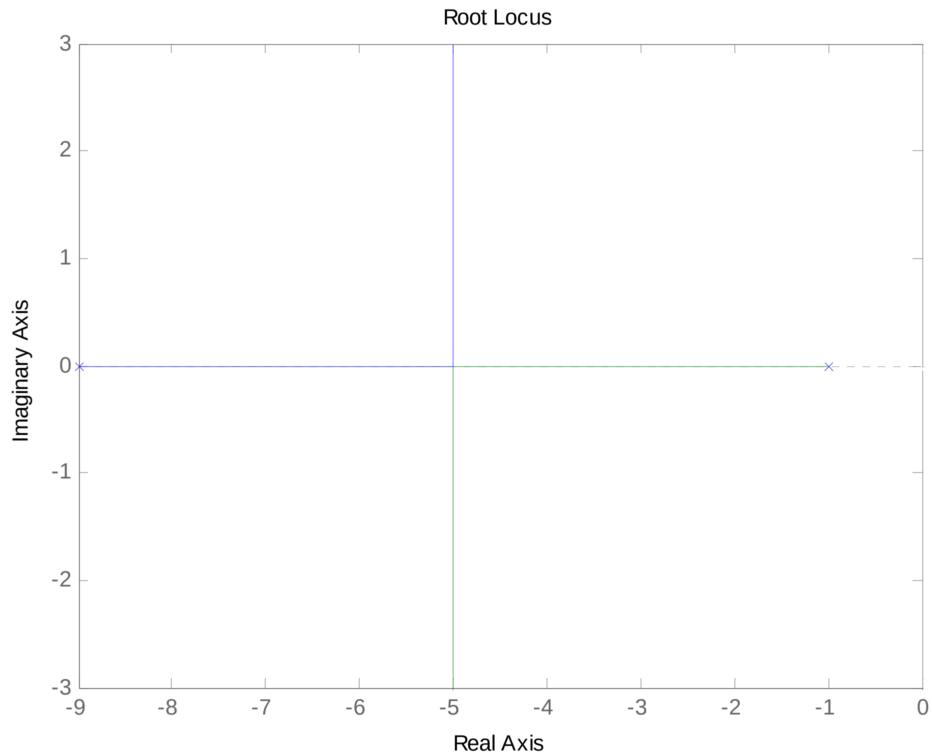


b) Measure the settling time and overshoot. Does changing the value of the gain alter the dynamics of the system (i.e., is the settling time faster, does the overshoot change)?

Ans: The settling time (2%) is about 4 seconds and is the same for all three responses. The overshoot is zero. No, changing the gain of the open-loop system does not alter the dynamics

c) Now, consider the closed-loop system and let our controller Gc(s) be a simple proportional gain K. Use the matlab rlocus() command to plot the closed-loop poles of this system as we vary K from 0.1 to 1000. Do these poles ever go unstable (cross into the RHP)?

Ans: The following is the result of the rlocus() command in Matlab



As we can see, the poles never cross the jω-axis and the closed-loop system is stable for all values of K

d) Look at the root locus. Let s1 be the pole that corresponds to a settling time of 0.8 seconds and a damping ratio of 0.707 (i.e., about 5% overshoot). Is the pole s1 on your root locus? If so, find the value of the gain K which produced these poles by solving the equation, 1+KG(s1)=0

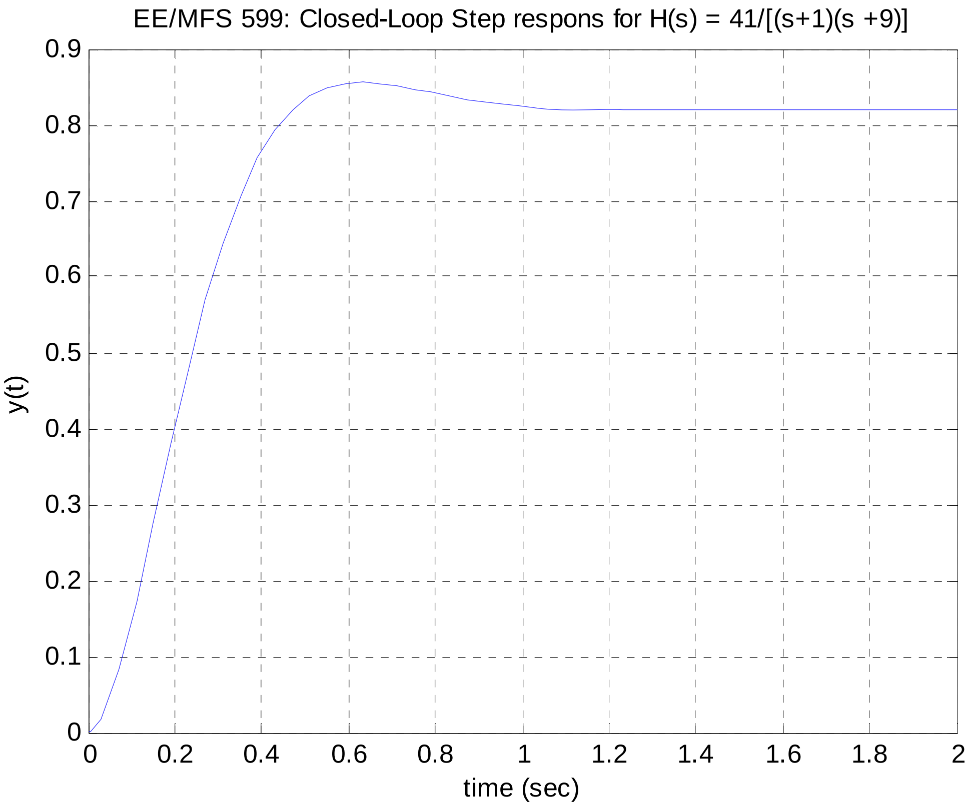
Ans: The pole s1 is -5+j5 and is on the root locus. The value of K which produces this pole is K = -1/G(s1) = 4.1

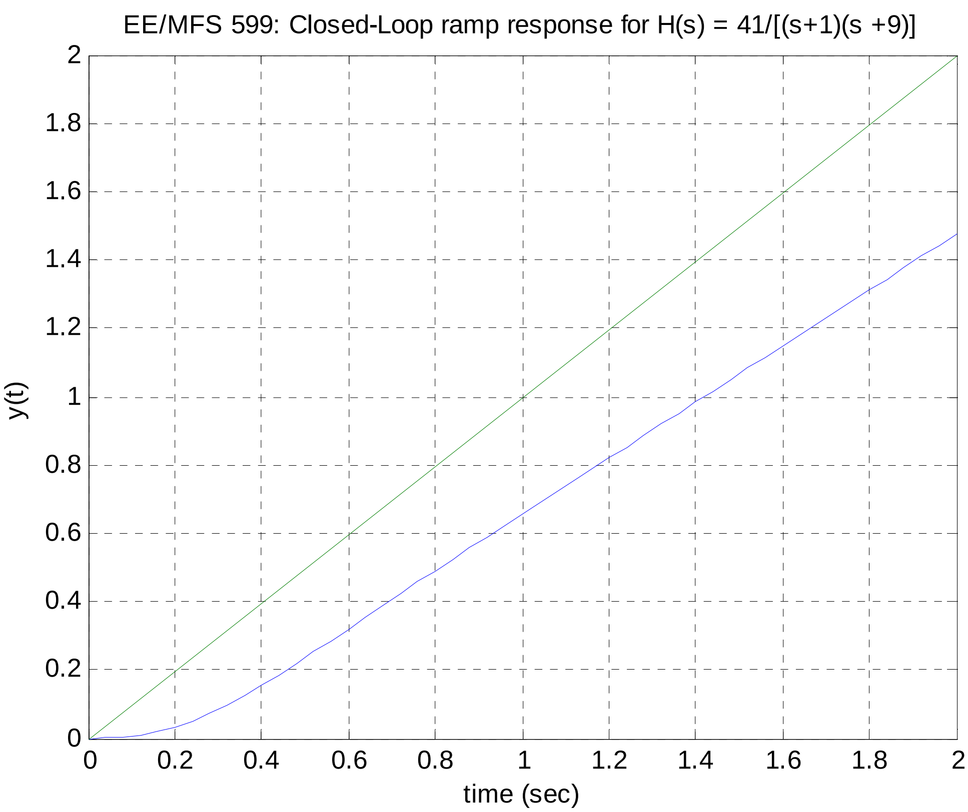
e) Find the static error and velocity coefficients (Kp and Kv) and calculate what the theorectical steady-state error should be for a step and ramp input

Ans: Kp = 41/9 and Kv = zero. Therefore, the steady-state error for a step is 1/(1+41/9) = 9/50 = 0.18 and e_{ss} for a ramp is ∞.

f) Plot the closed-loop step and ramp response for Gc(s) = K in part d). Does the settling time and overshoot for the step response agree with what you expected?

Ans: Yes (see plot below)





g) Use your plots to find the steady-state error for each input and compare to the theoretical answers. Can your system follow a ramp?

Ans: for the closed-loop step response, $y_{ss} = 0.82$ thus the steady-state error is 1-0.82 = 0.18 which agrees with our theory. The ramp error is clearly headed toward infinity which also agrees with our theory.

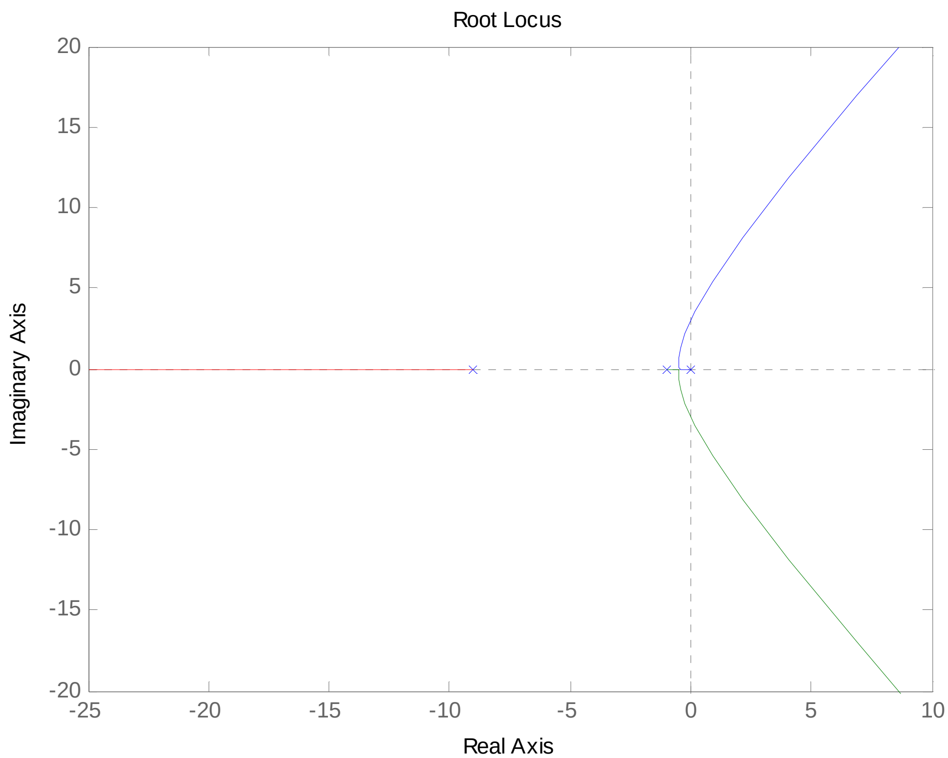
h) We can decrease the steady-state error due to a step by increasing the value of the gain K. Try using a K ten times greater and again find the step response and measure the steady-state error and compare to the new value of 1/(1+Kp). What happened to the overshoot when you increased the gain by a factor of 10?

Ans: As can bee seen from the plot below, the steady-state error due to a step is much smaller when we increase the gain by a factor of 10. Theoretically, the value is 1/(1 +410/9) = 9/419 = 0.0215

2. Next, consider the closed-loop system when q=1 (type 1 system)

a) Now, consider the closed-loop system and let our controller Gc(s) be a simple proportional gain K. Use the matlab rlocus() command to plot the closed-loop poles of this system as we vary K from 0.1 to 1000. Do these poles ever go unstable (cross into the RHP)?

Ans: Yes, the closed-loop system goes unstable as shown by the root locus plot below:



b) Look at the root locus. Let s1 be the pole that corresponds to a settling time of 1 seconds and a damping ratio of 0.707. Is the pole s1 on your root locus?

The desired dominant closed-loop pole is s1= -4+j4 which is not on the root locus

c) If no, we have to design a PD compensator to force the root locus to pass thru s1. Let Gpd(s) = Kp + Kds = K(s+z) where K=Kd and z=Kp/Kd. Find values of K and z to make s1 a closed-loop pole of the system

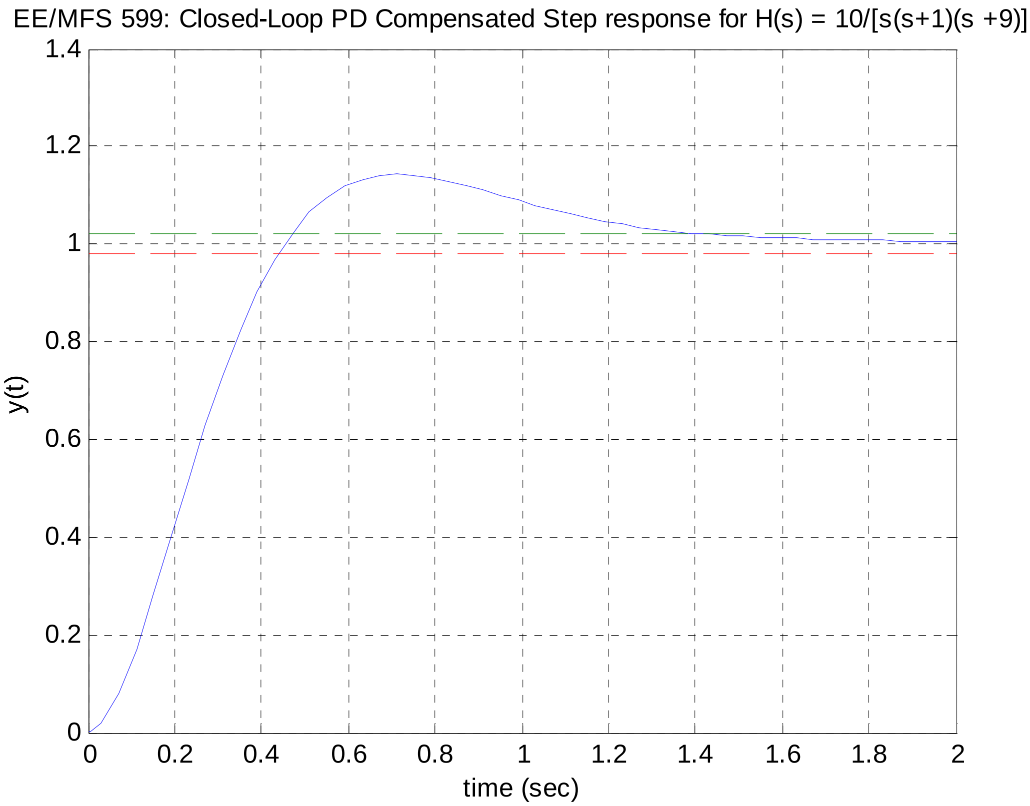
Ans: K(s1+z) = -1/G(s1) = -9.2+j15.6. Thus K = 15.6/4 = 3.9 and z = -9.2/K +4 = 1.641. Thus, Gpd(s) = 3.9(s+1.641)=3.9s+6.4. So, Kp =6.4 and Kd = 3.9

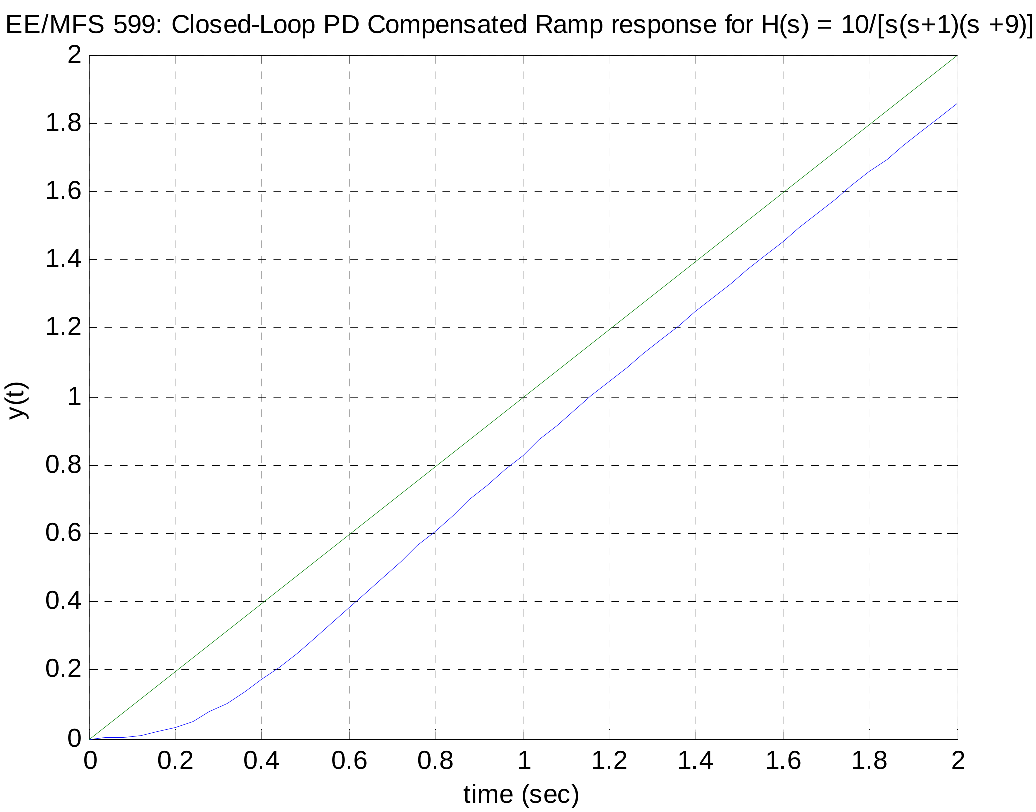
d) Find the static error and velocity coefficients (Kp and Kv) and calculate what the theorectical steady-state error should be for a step and ramp input

Ans: Kp is now infinite and Kv = 6.4*10/9=7.11. Thus, e_{ss} for a step is zero and e_{ss} for a ramp is 1/7.11 = 0.1406.

e) Plot the closed-loop step and ramp response for Gpd(s) = K(s+z) in part c). Does the settling time and overshoot for the step response agree with what you expected? If not, what have you inserted in the system that could be influencing these values?

Ans: Recall Mohannad stated in class that a $\zeta = 0.707$ yields an overshoot of 4.32%. The settling time should be $-4/\zeta\omega_n$ = 1 second. As can be seen from the step response, the overshoot is much greater than expected (18%) and the settling time is slightly larger (1.35 seconds). This is because we have inserted a zero at -1.641 which interferes with the dominant poles at -1+j1.





f) Use your plots to find the steady-state error for each input and compare to the theoretical answers. Can your system follow a ramp?

Ans: The ramp response now can follow a ramp and the steady-state error is 2-y(2)=2-1.8564=0.1436 (instead of 0.1406). The discrepancy is because we have only simulated for 2 seconds. If we allowed the simulation to run longer, we would obtain an identical answer. The steady-state error due to a ramp is zero as predicted.