Due Monday, February 4th **EE/MFS 599** HW #4

You may use Matlab/simulink wherever applicable

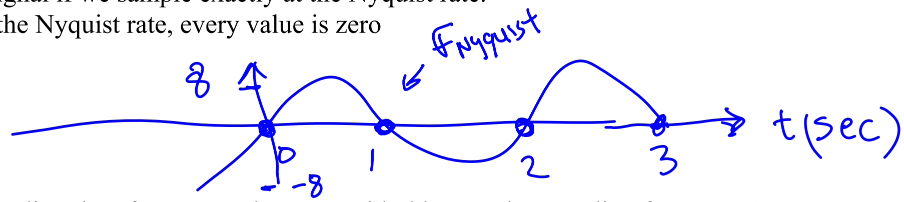
1. Consider the signal $x(t) = 8\sin\pi t$ volts and a 3-bit A-to-D converter (ADC)

a) According to the Nyquist Sampling Theorem, what is the largest sampling time, Ts, we could use on this signal and still recover the signal?

Ans: $\omega_{highest} = \pi$. Therefore, $f_{highest} = ½$ Hz. $F_{nyquist} = 2xf_{highest} = 1$Hz and Ts = 1 second

b) Sketch the result of sampling this particular signal at exactly the Nyquist samping rate. Look at the result and state whether we can recover this particular signal if we sample exactly at the Nyquist rate.

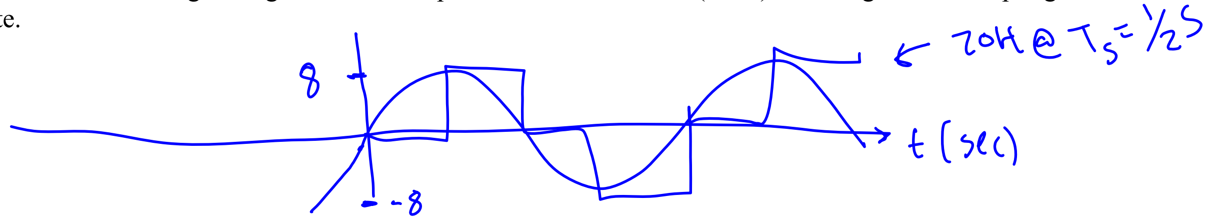
Ans: If we sample this signal at the Nyquist rate, every value is zero



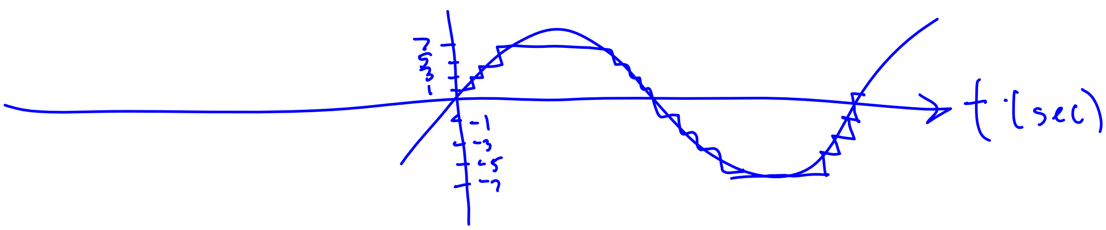
c) What would be a typical sampling time for a control system with this Nyquist sampling frequency?

Ans: Typically, control applications require 5-10 time the Nyquist rate or 5-10 Hz

d) In the **Time Domain**, sketch both the original signal and the output of a zero order hold (ZOH) of the signal at a sampling time of 2 times the Nyquist Rate.



e) In the time domain, sketch the original signal then sketch the results of quantizing the signal if we have an n=3 bit ADC



f) What is the mean-square-error (MSE) due to quantizing for our 3-bit A/D converter and our signal?

Ans: MSE = $q^2/12 = 2^2/12 = 1/3$

g) Devise the most logical encoding scheme to encodes the quantized signal into a 3-**bit** integer number (i.e., find an expression for $X_{encoded}$ as a function of the quantized value, $X_{quantixed}$)

Ans: Let 000 represent the quantized level -7 and 111 represent +7. Thus, $x_{encoded} = 0.5(x_{quantized} + 7)$

h) Use your answer to find both $X_{quantixed}$ and $X_{encoded}$ if the value of the sample of the original signal is 5.68 volts

Ans: $X_{quantixed}$ = 5 volts and $X_{encoded}$ = 0.5(5+7) = 6 = 110

i) Finally, using your encoding and quantizing scheme in reverse (i.e., D/A conversion), what voltage would the encoded number 011 produce?

Ans: $x_{quantized} = 2x_{encoded} - 7 = 2x3-7 = -1$

2. Consider the Bode plot on the following page of G_{ZOH} G(jω)

a) Use this Bode plot to predict what the open-loop steady-state response will be if the input is 100sin10t

Ans: The magnitude at ω = 10 r/s is about -20 dB = 0.1 and the phase is about -225 degrees. Thus, the steady-state output will be y(t) = 10sin(10t – 225°)

b) What type # is the system?

Ans: The slope at low frequencies is -20 db/dec therefore the type number is type 1

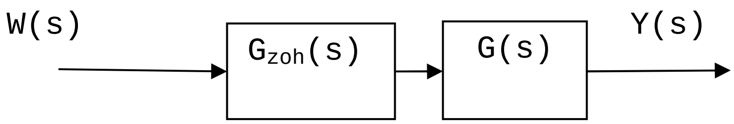
c) If the break point at jω = 10 rad/sec is due to a zero-order sample and hold circuit, find T_s

Ans: if $2/T_s = 10$ then $T_s = 1/5$ seconds

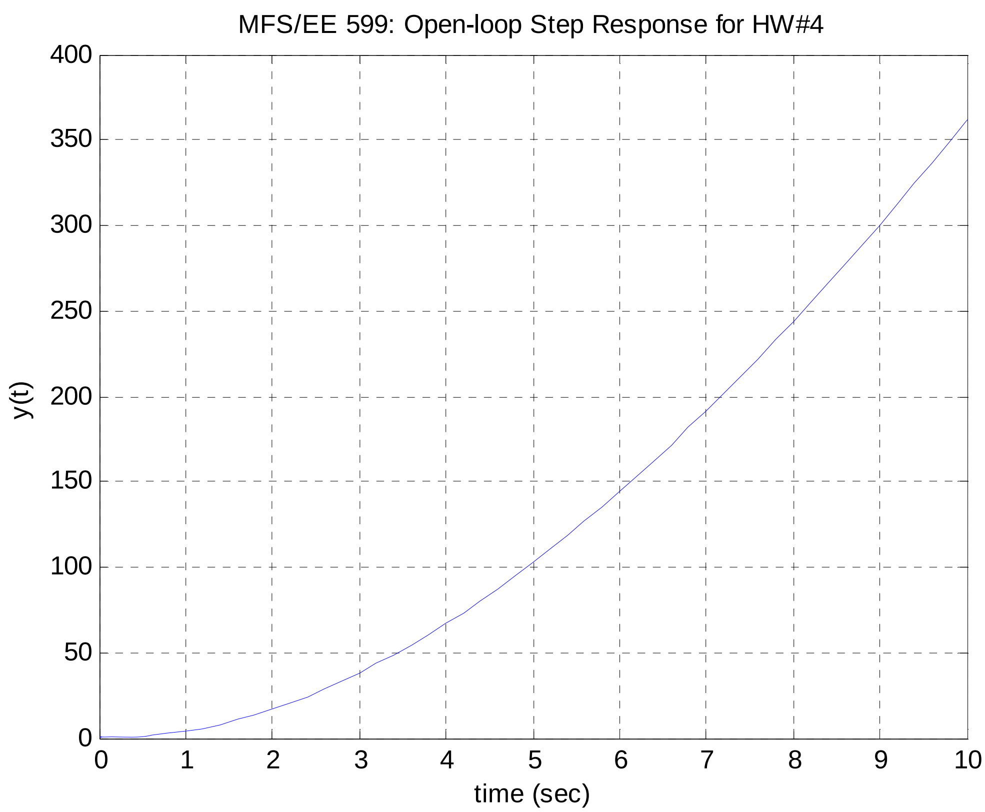
d) Now find G(s)

Ans: GzohG(s) = $K/[s^q(s/\omega_1+1)(s/\omega_2+1)]$. From the intersection of the asymptotes, we see that ω_1=0.1 and ω_2=10. We also see that the type number is q=1 as answered in part 2b). Finally, at frequencies below the breakpoints, GzohG(s) = K/s. If we pick ω=0.01, then the magnitude is 80 dB or 10^4. Thus $K/(0.01) = 10^4$ or K = 100. Thus, GzohG(s) = 100/[s(s/0.1+1) (s/10+1)]. Since Gzoh =1/(s/10+1) then G(s) = 100/[s(s/0.1+1)]

e) Input a unit step into the open-loop system shown below. Can the open-loop system follow a step? What is the steady-state error (i.e., the difference between the input and the output in steady-state)



Ans: Because of the integrating action, the steady-state response of the open-loop system will be a ramp (not a step). See the simulink output below:



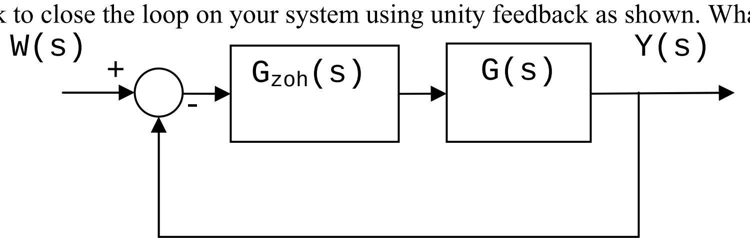
f) Now find the open-loop unit-impulse response. Using this as a starting point, think of an input that would make the open-loop output go to one (which is what we wanted the step input to go to)

Ans: The impulse response is the inverse Laplace transform of $H(s) = 100/[s(s/0.1+1)\ (s/10+1)] = 100[1/s + 10/1.01/(s+0.1) - 1.1/(s+10)]$. Thus, $h(t) = 100[u(t) + 10/1.01e^{-0.1t} - 1.101e^{-10t}]$. The steady-state value is 100. Thus, if we used an input of $1/100\ \delta(t)$, the output would go to 1 in steady-state.

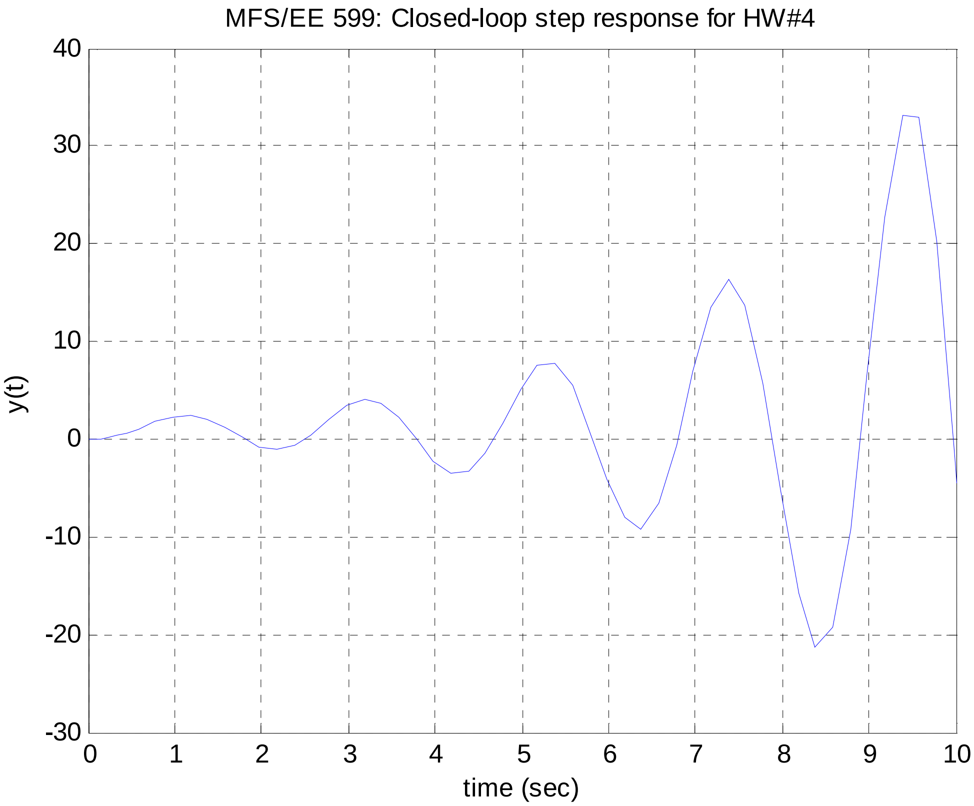
g) Does the input you specified above exist in the real world? What if we have a disturbance, is an open-loop design a robust design?

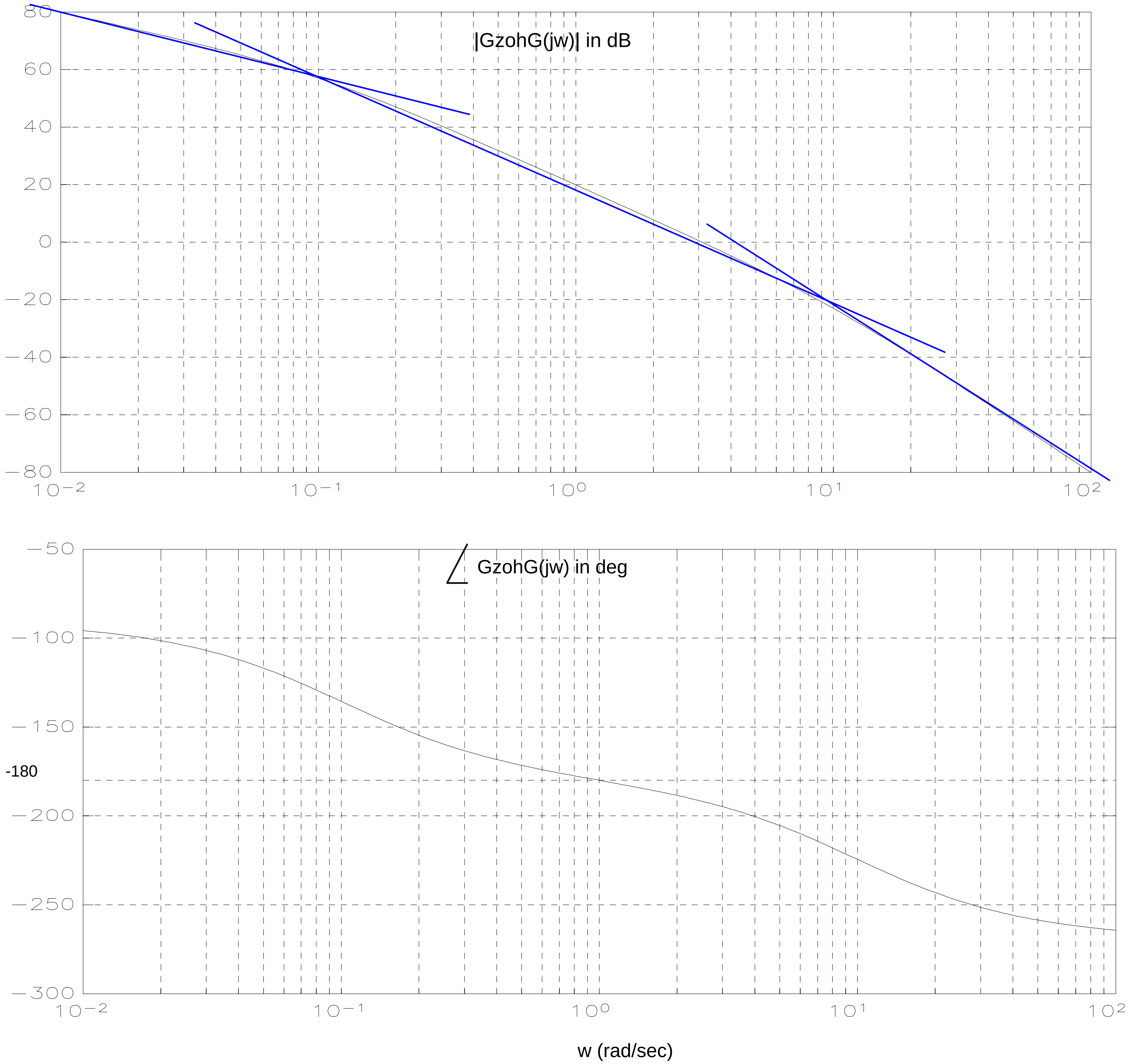
Ans: This is NOT a robust design. Plus, there is no such real-world input as an impulse function. If we had a disturbance, the output would be severely affected.

h) Now use simulink to close the loop on your system using unity feedback as shown. What is the steady-state error due to a step input now?



```
Ans: The steady-state error is infinite as the closed-loop system is unstable (see below)
```




|GzohG(jw)| in dB
80
60
40
20
0
−20
−40
−60
−80
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
GzohG(jw) in deg
−50
−100
−150
-180
−200
−250
−300
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
w (rad/sec)