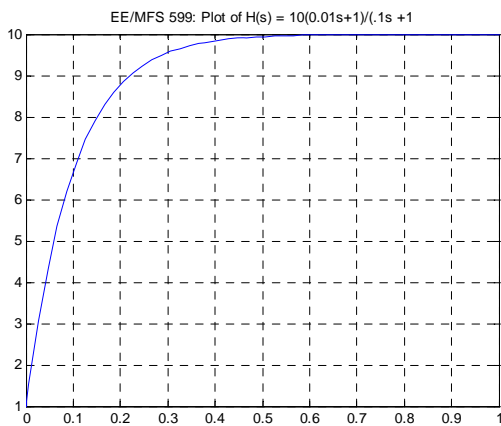
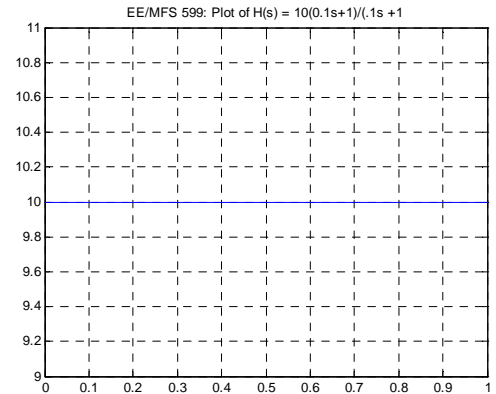
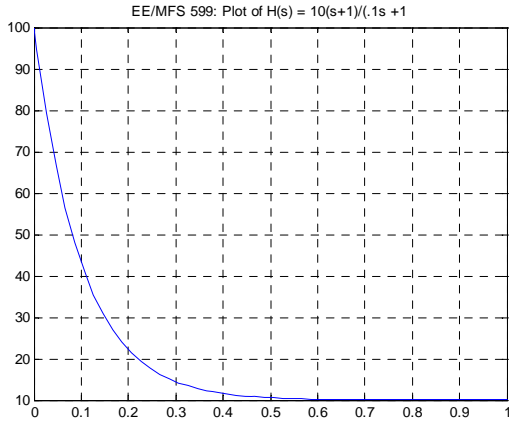


You may use Matlab/simulink wherever applicable

1. a) Plot the step responses of the following first order systems with a transmission zeroes. Can you see the effect of the zero?
 How about on part ii)?
 i) $H(s) = 10(s+1)/(.1s + 1)$ ii) $H(s) = 10(.1s+1)/(.1s + 1)$ iii) $H(s) = 10(.01s+1)/(.1s + 1)$

Sol'n:



- b) Go back and finish problem 2e) from HW#2 (hint:Think delay)

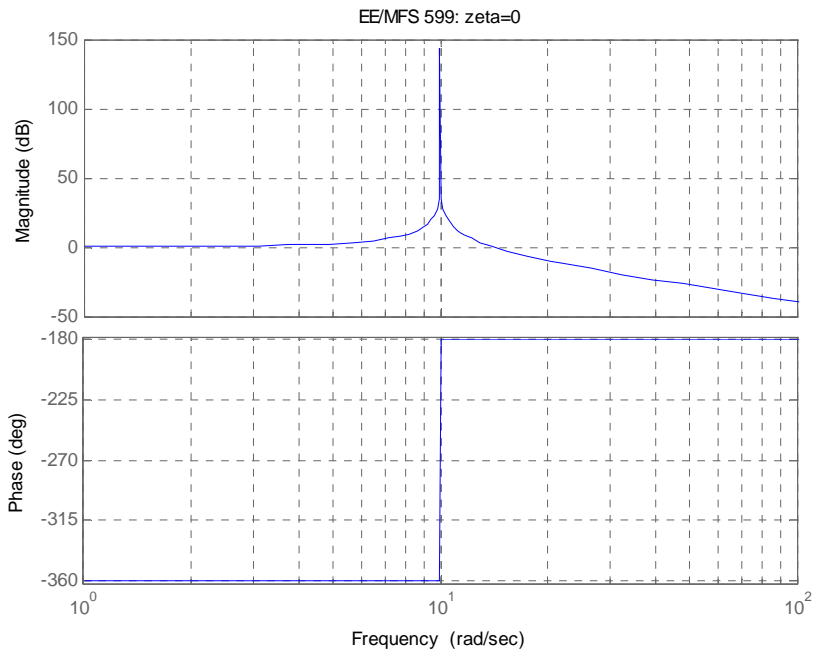
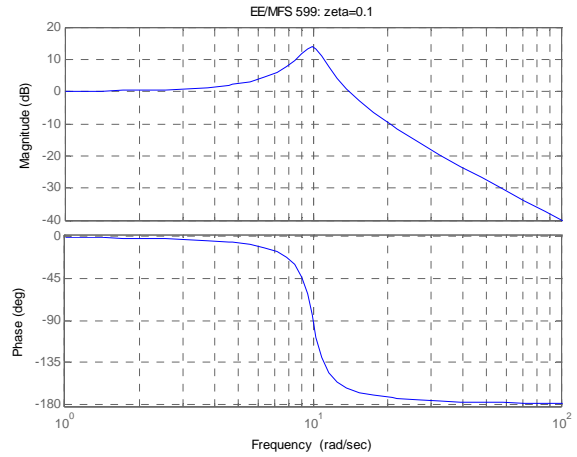
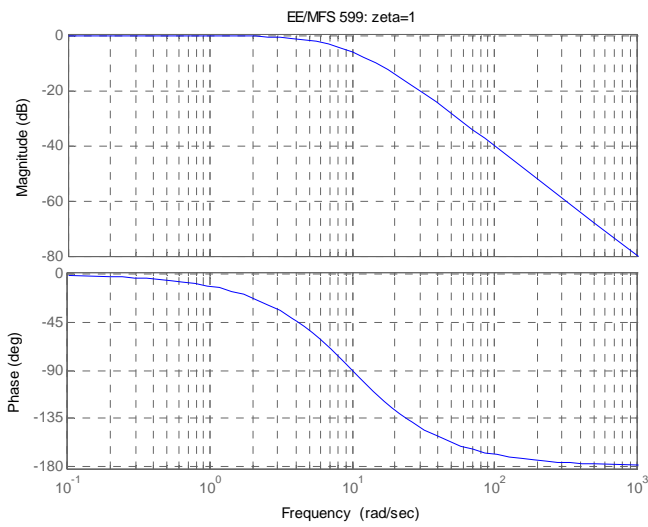
Sol'n : See HW2 solution

- c) Make a Bode plot of the following 2nd order system with the transfer function of:

$$\frac{Y(s)}{W(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

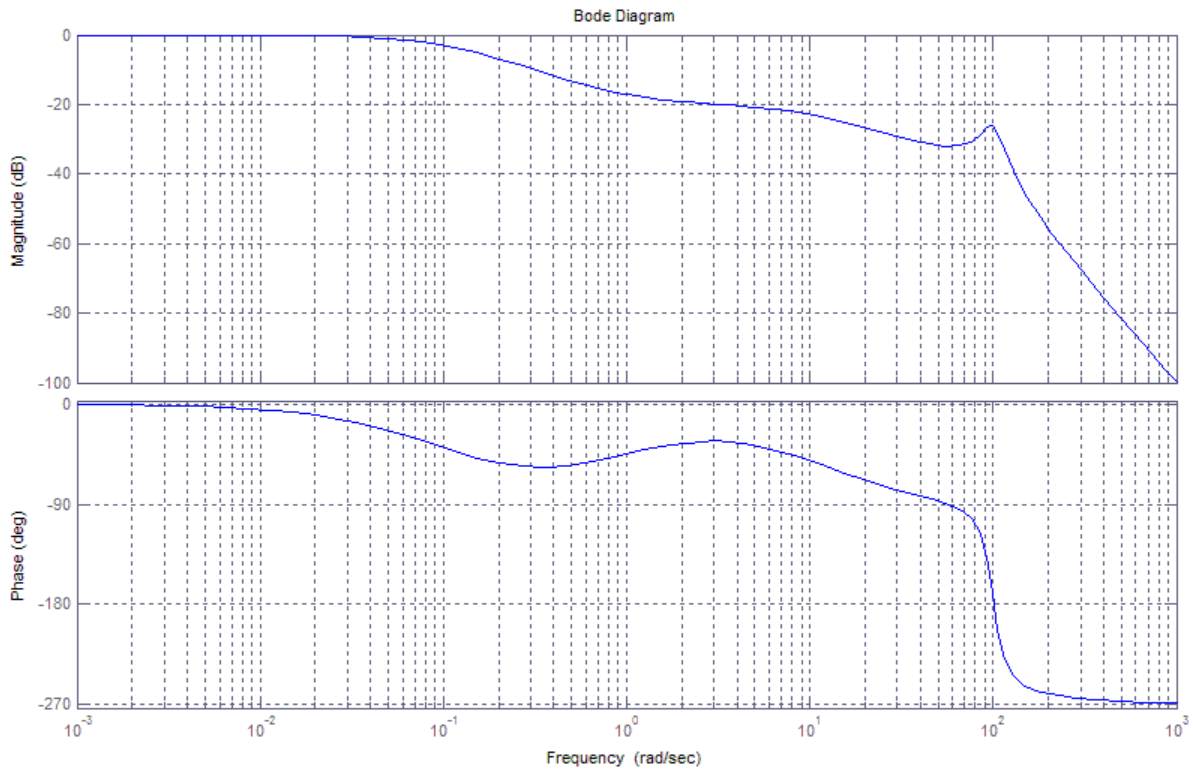
When ω_n is 10 rad/sec and the value of the damping ratio ζ is: i) 1 (critically damped) ii) 0.5 iii) 0.1 iv) 0 (no damping)

Sol'n:



d) Find the transfer function of the system with the following Bode plot (hint: compare the value of the resonant peak with your Bode plots above)

Sol'n: ω_n is 100 rad/sec and the value of the damping ratio ζ is about 0. Therefore, $H(s) = 100^2(1s+1)/(10s+1)(.1s+1)(s^2+20s+100^2)$



e) What is the type number of the model you found above?

Sol'n: Type 0 system

2. a) For the critically damped system modeled in problem 1c), what is the bandwidth?

Sol'n: The bandwidth (-6 dB frequency) is at 10 rad/sec

b) If we use the bandwidth as an estimate of the highest frequency present in the system, what is the Nyquist minimum sampling frequency we can use to recover all information?

Sol'n: Must sample at twice the highest frequency. Thus, the minimum frequency is 20 rad/sec

c) If we were going to control the system, what is a better choice of a sampling frequency (or period T_s)?

Sol'n: In controls, we should sample at 5 to 10 times the Nyquist rate (100 to 500 rad/sec)

d) What are the three parts of the analog-to-digital conversion process learned in class today?

Sol'n: sampling, quantizing, and encoding

e) For the transfer function, $H(s) = 10/(0.1s+1)$, pick a sampling time of 10 msec (is this reasonable?) and find an approximate discrete model $H(z)$ using i) the bilinear transform ii) step invariant design

Ans. Using the bilinear transformation, $H(z) = H(s)$ when $s = (2(z-1)/[Ts(z+1)]) = 25,600 [z-1]/[z+1] = [0.4762z-0.4762]/[z + 0.9048]$.

Using step invariant ($w(s) = 1/s$), then $Y(s) = H(s)/W(s) = 100/(s(s+10)) = 10/s - 10/(s+10)$. Therefore, $y(t) = 10(1-e^{-10t})u(t)$ and $y(kTs) = 10u(kTs) - 10e^{-0.1k} u(kTs)$. Thus, $Y(z) = 10z/(z-1) - 10z/(z-e^{-0.1})$. $W(z) = z/z-1$. Thus, $H(z) = Y(z)/W(z) = 10 - 10(z-1)/(z-e^{-0.1})$.

f) While most of the processes we will encounter in industry are stable, we should at least be familiar with what the tests are for stability. In the s-domain, a system modeled by $H(s)$ is asymptotically stable iff all of the poles of $H(s)$ are strictly in the Left Half Plane (LHP). Use the mapping between the s-plane and the z-plane you learned in class today to map the LHP into the z-plane and find an equivalent test for checking the asymptotic stability of $H(z)$.

Ans: The LHP maps inside the unit circle in the z-plane. Thus, $H(z)$ will be stable if it has poles strictly inside the Unit Circle.

Fun Fact: The bilinear transformation we learned about today preserves stability! That is, if we start with a stable $H(s)$ we will obtain a stable $H(z)$ using the bilinear transformation!

g) Finally, if the type number of $H(s)$ is the number of open-loop poles at the origin ($s=0$), speculate what the type number is for $H(z)$