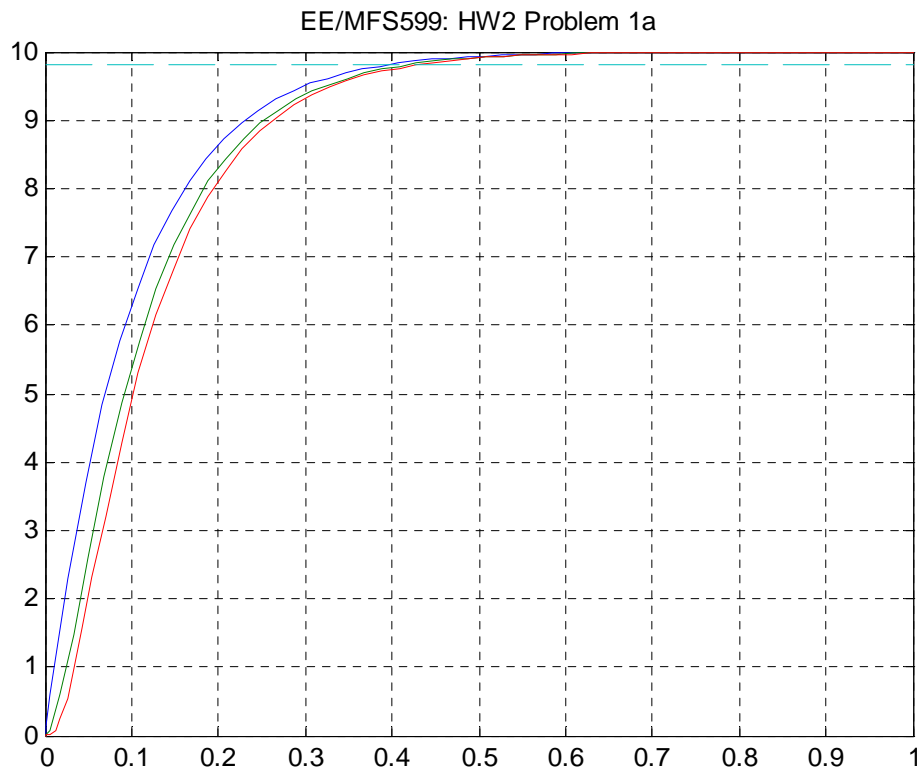


You may use Matlab/simulink wherever applicable

1. a) Plot the step responses of the following transfer functions on the same plot. Can you tell the difference between them?
 i) $H(s) = 10/(.1s + 1)$ ii) $H(s) = 10/[(.1s + 1)(0.05s + 1)]$ iii) $H(s) = 10/[(.1s + 1)(0.05s + 1)(0.01s + 1)]$

Sol'n: Here are the simulink plots for these 3 transfer functions:



The blue line is system i), the green line is system ii), the red line is system iii). As can be seen, all three step responses settle (2%) at roughly the same time (0.4 seconds) and the major differences occur close to $t=0$ as expected.

- b) Looking at your plots, find the dominant time constant (T_1) for each of the three systems

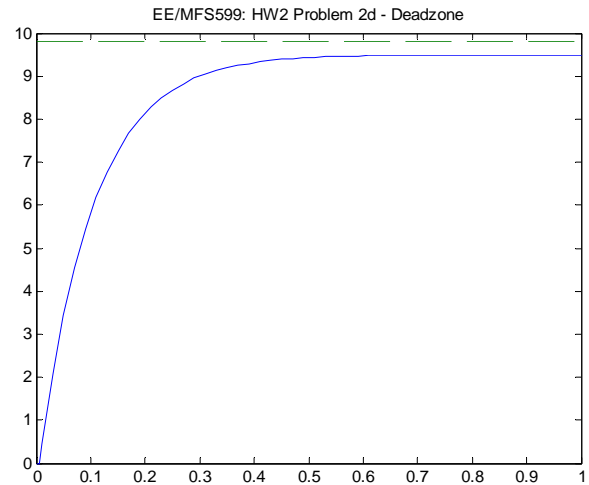
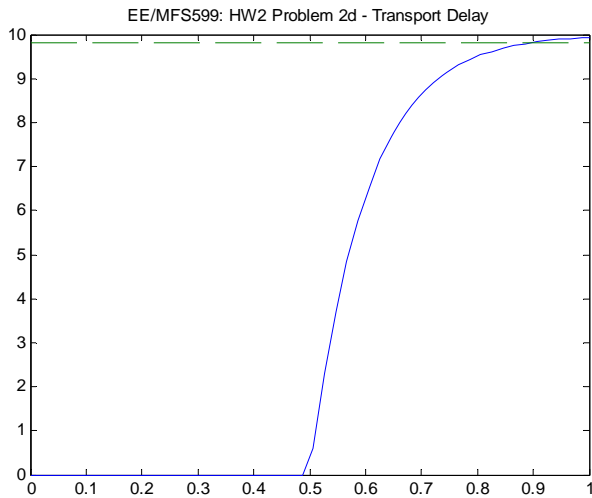
Sol'n: We can use the settling time for each systems (2%) and divide by 4 or we could draw a tangent to the step response at $t=0$ and see where it intersects the final value (10). If we use the settling time method, for system i) $T_1 = 0.4/4 = 0.1$ seconds, system ii) $T_1 = 0.42/4 = 0.1025$ which is about the same for system iii). If we use the tangent method, for system i) $T_1 = 0.1$ seconds, system ii) $T_1 = 0.14$ and $T_1 = 0.15$ for system iii). This shows why I prefer to use the settling time to determine T_1 .

- c) Suppose both the input and output are voltages for the 3 systems. If we wanted the output of each of the three systems to go to a value of 4 volts in steady-state, what open-loop control would we input?

From our class discussion, the value of the open-loop control should be $B/A = 4/10 = 0.4u(t)$

- d) Go back and plot the step response of $H(s) = 10/(.1s + 1)$ if
 i) We have a delay of .05 seconds
 ii) We have a deadband of 0.5 volts

Sol'n: In Simulink, the transport delay can be found in the continuous library and the deadband can be found in the discontinuities library. Here are the results of the simulink simulation:

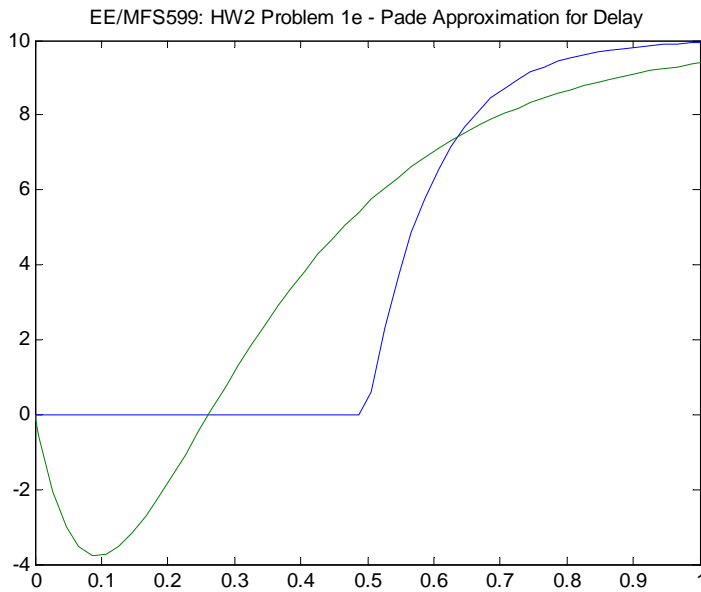


As can be seen, the delay causes the step response to start at $t=0.5$ and the deadzone causes the response to settle at a value less than the expected value of 10 volts.

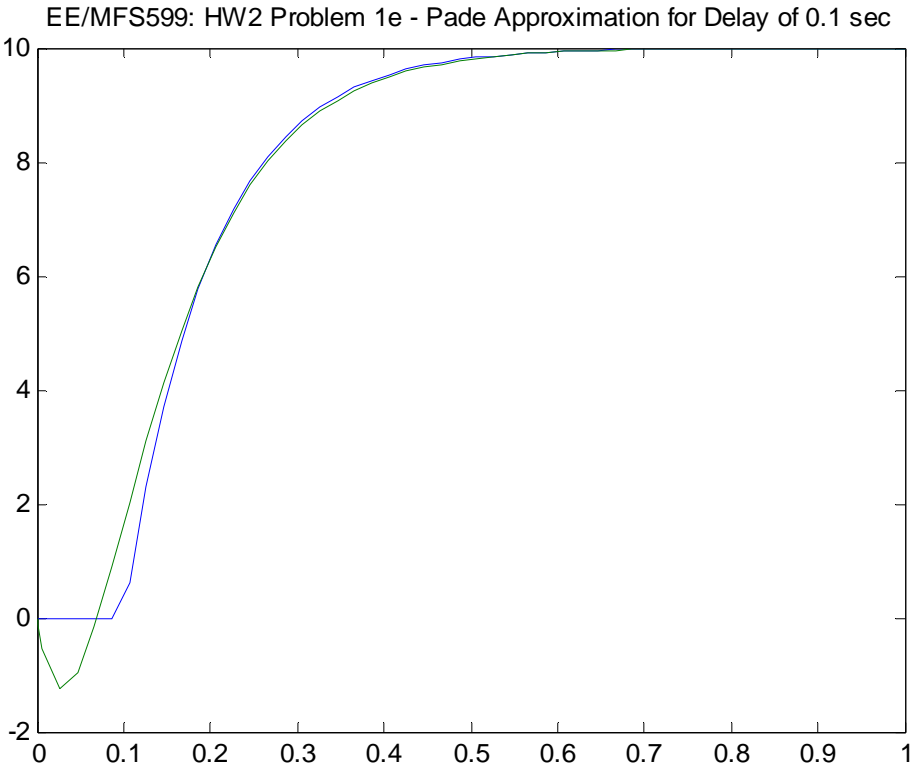
e) What use the Pade approximation for the delay above in part d) i) and again find the step response. Can you see a difference?

The Pade' approximation for the delay is $e^{-st_1} = \frac{e^{-s\frac{t_1}{2}}}{e^{s\frac{t_1}{2}}} \cong \frac{1 - s\frac{t_1}{2}}{1 + s\frac{t_1}{2}} = \frac{1 - s\frac{1}{4}}{1 + s\frac{1}{4}}$ for a t_1 of 0.5 seconds. Putting this term in series with

our $H(s)$ we find the following step response:



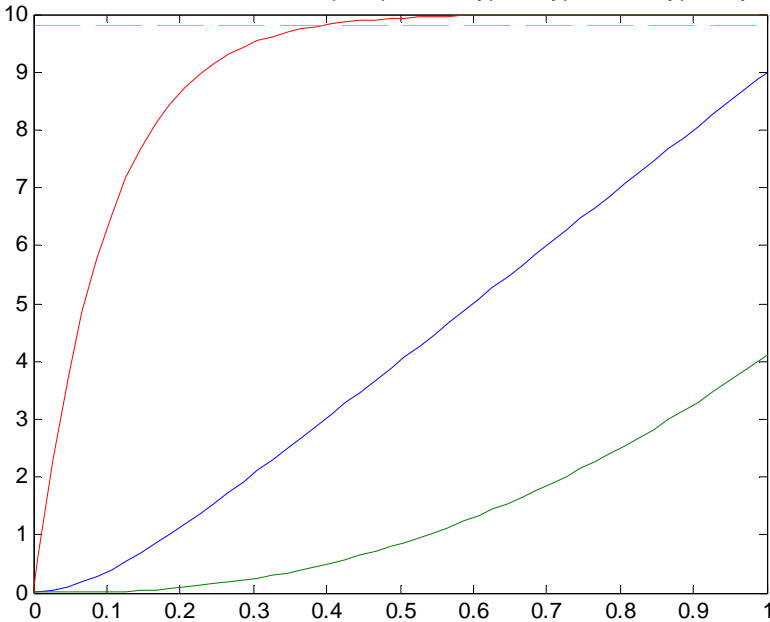
We can definitely see a difference. Recall the Pade approximation was only valid for small delays. A delay of $t_1 = 0.5$ seconds is not that small. If we repeat the problem using a delay of $t_1 = 0.1$ seconds, we find:



Note that for this much smaller delay, the Pade' approximation works very well.

- e) Plot the step responses of the following transfer functions on the same plot.
 i) $H(s) = 10/(.1s + 1)$ ii) $H(s) = 10/[s(0.1s+1)]$ iii) $H(s) = 10/[s^2(.1s + 1)]$

EE/MFS599: HW2 Problem 1f - Step response of type 0, type 1, and type 2 systems



For system ii), we obtain a ramp in steady state, for system iii) we obtain a parabola in steady state.

- f) What is the type number for the above transfer functions?
 i) Type 0, ii) Type 1, iii) Type 2

We learned in class that a classical second order system has the transfer function,

$$\frac{Y(s)}{W(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ζ is the damping coefficient and ω_n is the natural frequency. We also learned that the unit step response of such a system has the form:

$$y(t)|_{w(t)=u(t)} = 1 - e^{-\zeta\omega_n t} \left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_d t \right)$$

where $\omega_d = \omega_n(1-\zeta^2)^{1/2}$ is the damped natural frequency and we have assumed an underdamped system (i.e., $\zeta < 1$). Some of the important features that can be measured from a 2nd-order step response are the settling time (t_s), the percent overshoot (M_p), the peak time (t_p) and the period of damped oscillation (T_d). From the equation for the unit step response, we can derive the following expressions for these parameters:

$$t_s = 4 / \zeta\omega_n \quad M_p = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad T_d = 2\pi / \omega_d = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

2. a) From use the expressions for M_p and t_p to find the values of ζ and ω_n from the following unit step response:

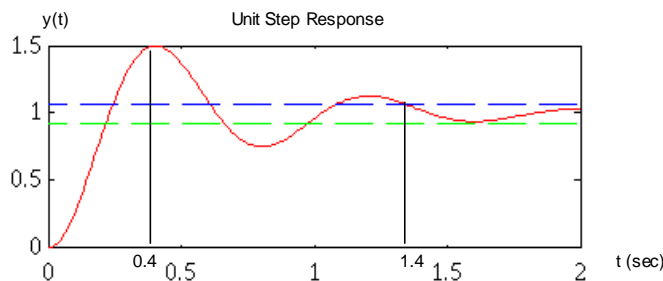


Figure 1. Unit Step Response of an approximate 2nd-order system

$t_p=0.4$ seconds and $M_p=0.5 \times 100\%=50\%$. Therefore, $\ln(M_p)=\ln(0.5)=-\zeta\pi/(1-\zeta)^{1/2} \Rightarrow \ln(0.5)^2=(\zeta\pi)^2/(1-\zeta)$
 $\Rightarrow \zeta = [\ln(0.5)^2/(\pi^2+\ln(0.5)^2)]^{1/2}=0.21545$

$\Rightarrow \omega_n = \pi/[t_p(1-\zeta^2)^{1/2}] = \pi/[0.4(1-(0.21545)^2)^{1/2}] = 8.04288$. Let's substitute these values to find the transfer function:

$$\frac{Y(s)}{W(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{64.6879}{s^2 + 3.46568s + 64.6879}$$

b) Note that $y(t_s)=1.02=1+e^{-\zeta\omega_n t_s}$. Therefore, $t_s=-1/\zeta\omega_n \times \ln(0.02) = 3.91/\zeta\omega_n$

c) From plot, $t_s = 1.4$ seconds $\Rightarrow 1.4 = 3.91/\zeta\omega_n \Rightarrow$ If $\zeta = 0.21545$ then $\omega_n = 12.96961$ which is close to the previous value of $\omega_n = 8.04288$

b) Find numerical value for t_s from the step response shown in part 2a). Then, use the value of ζ found in 2a) and your expression for t_s to find ω_n . Is this value close to your value ω_n found in part 2a)?

Note that $y(t_s)=1.02=1+e^{-\zeta\omega_n t_s}$. Therefore, $t_s=-1/\zeta\omega_n \times \ln(0.02) = 3.91/\zeta\omega_n$ which we usually approximate as $4/\zeta\omega_n$

From plot, $t_s = 1.4$ seconds $\Rightarrow 1.4 = 3.91/\zeta\omega_n \Rightarrow$ If $\zeta = 0.21545$ then $\omega_n = 12.96961$ which is close to the previous value of $\omega_n = 8.04288$

We learned in class today that a very important tool used in industry process modeling is frequency response (Bode plots). Instead of using a step input to our unknown process, we input sinusoids at a lot of different frequencies then and measure the steady-state input and output.

c) Given the following steady-state sinusoidal plot of $G(j\omega)=V_{out}(j\omega)/V_{in}(j\omega)$. find

- i) T (the period)
- ii) ω
- iii) $|G(j\omega)|$ (in dB)
- iv) $\angle G(j\omega)$

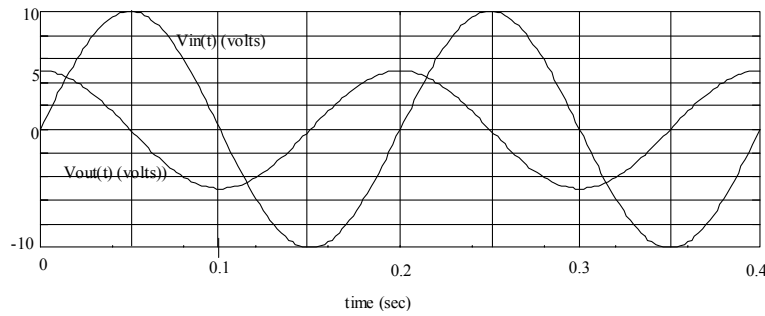


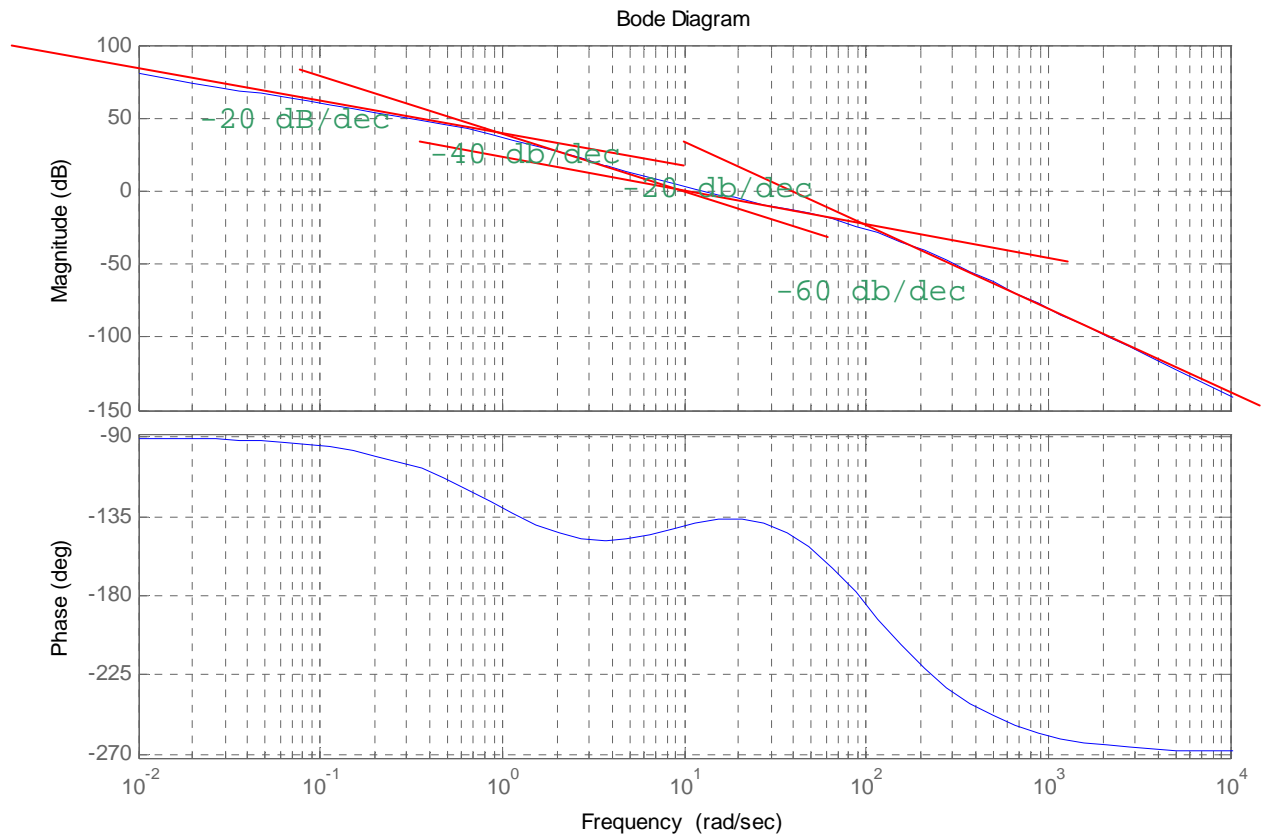
Figure 2.

Steady-state sinusoidal

response of an unknown process

Solution: i) The period is 0.2 seconds, ii) $\omega = 2\pi/T = 10\pi$ rad/sec iii) $|G(j\omega)|$ (in dB) = $|V_{out}/V_{in}| = 0.5 = -6$ dB iv) $\angle G(j\omega) = \angle V_{out} - \angle V_{in} = -270$ degrees (or $+90$ degrees)

- d) The above method would give us just one data point. By sweeping the frequency of the sinusoid that we input to the system, we can find an entire Bode plot. Given the Bode plot shown below, find $G(s)$



The form we use for $G(s)$ is $\frac{Y(s)}{W(s)} = \frac{K(s/\omega_a + 1)(s/\omega_b + 1)\dots}{s^q(s/\omega_1 + 1)(s/\omega_2 + 1)\dots}$ where q is the type number. As can be seen from the asymptotes

drawn on the Bode Plot, the system is a Type 1 system with a pole at 1 r/s, a zero at 10 r/s and two poles at 100 r/s. We can measure the value of K at 0.1 r/s to be: K (dB) - 0.1 (dB) = 60 dB. Or, $K = 60$ dB - 20 dB = 40 dB = 100.

Therefore, $G(s) = 100(0.1s+1)/[s(s+1)(0.01s+1)^2]$

- e) Suppose instead of leveling off at high frequencies, the above phase response continued to slope down at a rate of 0.01 degrees per radian/second. Find $G(s)$.

Sol'n: If the phase plot doesn't level out as expected and continues to slope down then we have transportation delay present. The value of the delay would be $T_d = 0.01 \times 180^\circ/\pi$ or $1.8/\pi$ seconds. Therefore, $G(s) = 100(0.1s+1)/[s(s+1)(0.01s+1)^2]e^{-s1.8/\pi}$