

You may use Matlab/simulink wherever applicable

1. a) Plot the step responses of the following transfer functions on the same plot. Can you tell the difference between them?
  - i)  $H(s) = 10/(.1s + 1)$
  - ii)  $H(s) = 10/[(.1s + 1)(0.05s + 1)]$
  - iii)  $H(s) = 10/[(.1s + 1)(0.05s + 1)(0.01s + 1)]$
- b) Looking at your plots, find the dominant time constant ( $T_1$ ) for each of the three systems
- c) Suppose both the input and output are voltages for the 3 systems. If we wanted the output of each of the three systems to go to a value of 4 volts in steady-state, what open-loop control would we input?
- d) Go back and plot the step response of  $H(s) = 10/(.1s + 1)$  if
  - i) We have a delay of .05 seconds
  - ii) We have a deadband of 0.5 volts
- e) What use the Pade approximation for the delay above in part d) i) and again find the step response. Can you see a difference?
- e) Plot the step responses of the following transfer functions on the same plot.
  - i)  $H(s) = 10/(.1s + 1)$
  - ii)  $H(s) = 10/[s(0.1s + 1)]$
  - iii)  $H(s) = 10/[s^2(.1s + 1)]$
- f) What is the type number for the above transfer functions?

We learned in class that a classical second order system has the transfer function,

$$\frac{Y(s)}{W(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where  $\zeta$  is the damping coefficient and  $\omega_n$  is the natural frequency. We also learned that the unit step response of such a system has the form:

$$y(t)|_{w(t)=u(t)} = 1 - e^{-\zeta\omega_n t} \left( \cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_d t \right)$$

where  $\omega_d = \omega_n(1-\zeta^2)^{1/2}$  is the damped natural frequency and we have assumed an underdamped system (i.e.,  $\zeta < 1$ ). Some of the important features that can be measured from a 2<sup>nd</sup>-order step response are the settling time ( $t_s$ ), the percent overshoot ( $M_p$ ), the peak time ( $t_p$ ) and the period of damped oscillation ( $T_d$ ). From the equation for the unit step response, we can derive the following expressions for these parameters:

$$t_s = 4 / \zeta\omega_n \quad M_p = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad T_d = 2\pi / \omega_d = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

2. a) From use the expressions for  $M_p$  and  $t_p$  to find the values of  $\zeta$  and  $\omega_n$  from the following unit step response:

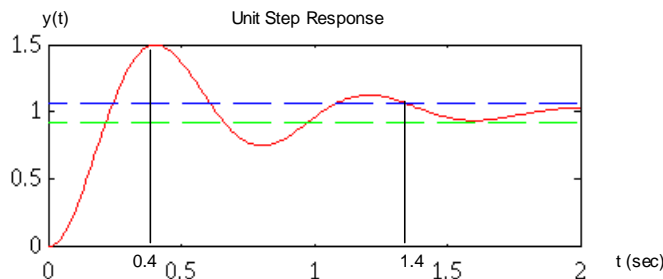
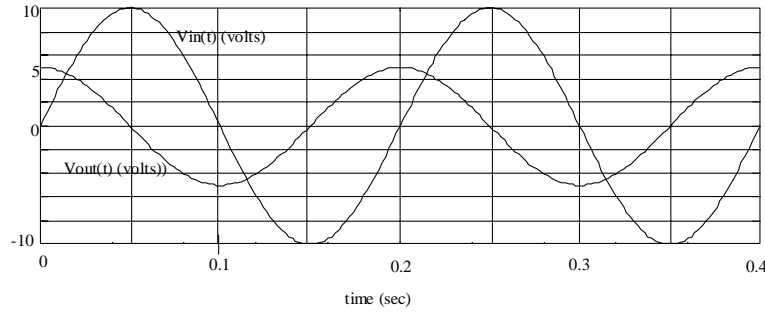


Figure 1. Unit Step Response of an approximate 2nd-order system

- b) Find numerical value for  $t_s$  from the step response shown in part 2a). Then, use the value of  $\zeta$  found in 2a) and your expression for  $t_s$  to find  $\omega_n$ . Is this value close to your value  $\omega_n$  found in part 2a)?

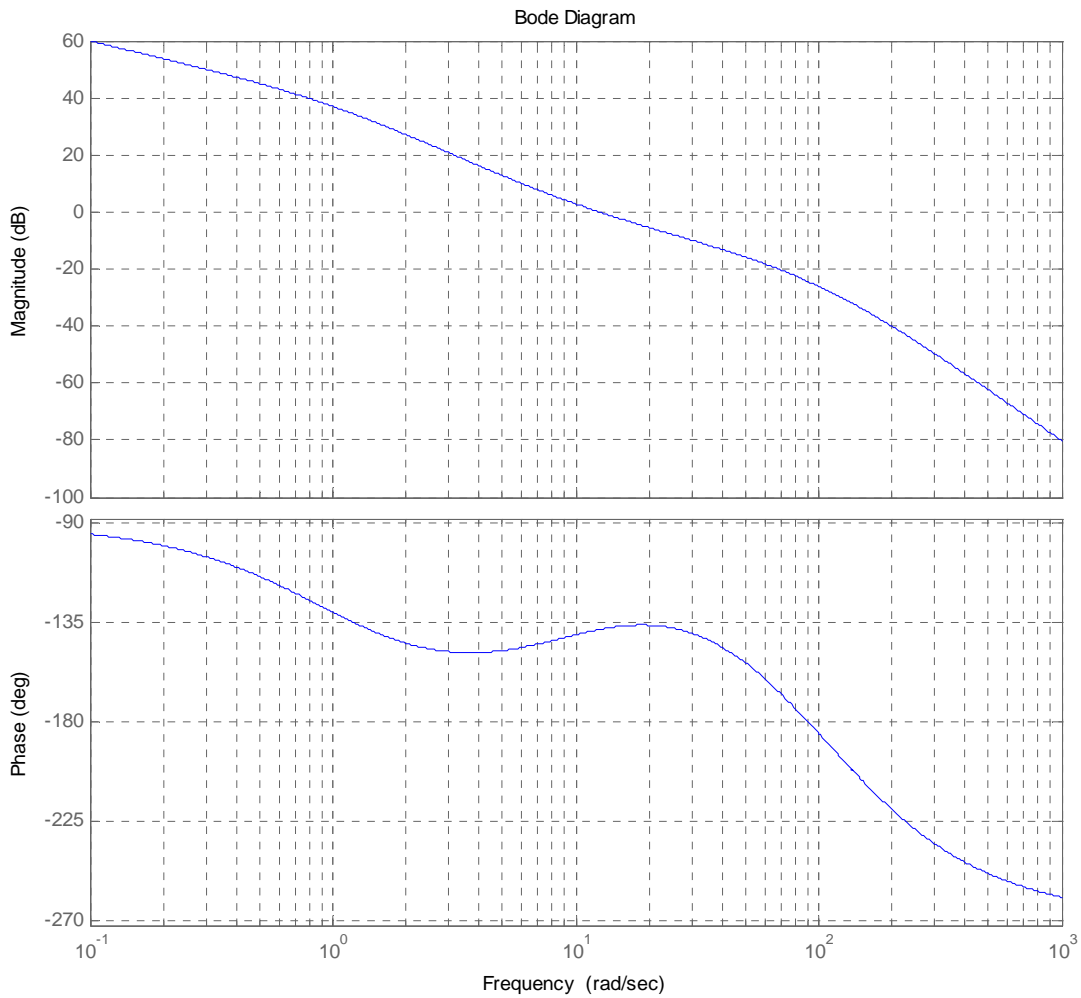
We learned in class today that a very important tool used in industry process modeling is frequency response (Bode plots). Instead of using a step input to our unknown process, we input sinusoids at a lot of different frequencies then and measure the steady-state input and output.

- c) Given the following steady-state sinusoidal plot of  $G(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$ . find  
 i) T (the period)    ii)  $\omega$     iii)  $|G(j\omega)|$  (in dB)    iv)  $\angle G(j\omega)$



**Figure 2.** Steady-state sinusoidal response of an unknown process

- d) The above method would give us just one data point. By sweeping the frequency of the sinusoid that we input to the system, we can find an entire Bode plot. Given the Bode plot shown below, find  $G(s)$



- e) **Don't do this part!** Suppose instead of leveling off at high frequencies, the above phase response continued to slope down at a rate of 0.01 degrees per radian/second. Find  $G(s)$ .