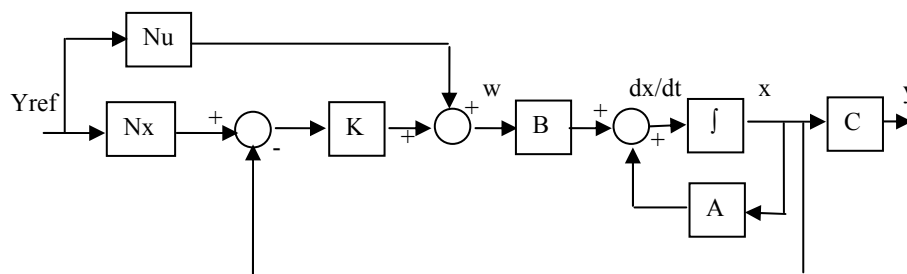


Using methods learned in class, we found the open-loop transfer function of the Motomatic to be:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{210.9}{s^2 + 5.27s}$$

1. In HW#9, we used Matlab's `tf2ss()` command to find a continuous-time state space representation of the form  $\dot{x} = Ax + Bw$ ,  $x(0^+)$  for this system. We also verified that the  $y = Cx + Dw$  resulting system was completely controllable. Now, we want to use optimal control theory (LQR), to find an optimal feedback gain,  $K(\infty)$ , for this system.
  - a) Use Matlab's `lqr()` command in the form  $[K,S,E] = \text{LQR}(A,B,Q,R)$  to find the optimal value of the feed back control,  $w = -Kx$ , that will minimize the linear quadratic cost function,  $J = \int_0^{\infty} 1/2 x^T Q x + 1/2 w^T R w dt$  using a 2x2 identity matrix for Q and R=1.
  - b) What are the closed-loop eigenvalues (E) for your LQR system? Is the closed-loop system stable?
  - c) Go back and repeat part a) but this time substitute  $A + \alpha I$  for A where  $\alpha = 16$ . Now check the eigenvalues of the closed-loop system. Are the real parts less than -16?
  - d) Simulate your LQR closed-loop state-space system from part c) using Simulink using an initial state of  $x(0) = [10 \ 0]^T$  (hint: you will need to connect a 2x1 gain block -K to the output of your Simulink model and feed the output of this block it back to the input )
2. The 'R' in LQR stands for regulator (dRoop). Most industrial control applications require that we track a reference signal, not just regulate (droop). To this end, consider the tracking architecture:



- a) For your servo system state space model, what are the dimensions of  $y_{\text{ref}}$ ,  $N_x$ , and  $N_u$ ?
- b) Use the equations given today in class to find the values of  $y_{\text{ref}}$ ,  $N_x$ , and  $N_u$
- c) Simulate your tracker architecture in Simulink using an initial state of  $x(0) = [0 \ 0]^T$  and a  $y_{\text{ref}} = 5.0$  volts. How long does it take  $y = x_1 = V_{\text{out}}$  to settle within 2% of its final value? What is the overshoot on  $y$ ?
- d) Now, let  $y_{\text{ref}} = r(t) =$  a unit ramp. Does your tracker follow a ramp? What is the steady-state error for a ramp?