MFSEE599 Cheat Sheet

1^{st} Order Open-Loop System $H(s) = K/[(T)s+1]$

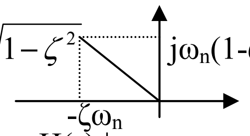
Zero-order Hold: $Gzoh(s) = \dfrac{1-e^{-sT_s}}{s} = \dfrac{T_s}{\frac{T_s}{2}s+1} \cong \dfrac{1}{\frac{T_s}{2}s+1}$

2^{nd} Order-Systems:

Quantizing: MSE = $q^2/12$ where q is width of quantizing zone

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n{}^2 y = \omega_n{}^2 w \Rightarrow \frac{Y(s)}{W(s)} = \frac{\omega_n{}^2}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

$$s_1 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$



$$t_s = 4/\zeta\omega_n = -4/\mathrm{Re}(s)_{\max} \quad M_p = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Bilinear: $H(z) = H(s)\,|_{s = 2/Ts*(z-1)/(z+1)}$

$$y|_{u(t)} = 1 - e^{-\zeta\omega_n t}\left[\cos(\omega_d t + \left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\sin(\omega_d t)\right] \; where \; \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Mapping $z = e^{sT_s}$

Type Number (q): $G(s) = \dfrac{K(s/\omega_1+1)(s/\omega_2+1)...}{s^q(s/\omega_a+1)(s/\omega_b+1)...} \cong \dfrac{K}{s^q}$ at low frequencies

$G_{PID} = K_i/s + K_p + K_d s$

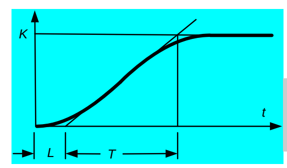
Error Analysis: (Valid for unity-feedback with closed loop system stable):

$$K_p = \lim_{s\to 0} G(s) \qquad K_v = \lim_{s\to 0} sG(s) \qquad ess|_{step} = 1/(1+K_p) \qquad ess|_{ramp} = 1/K_v$$

Delay: $H_{delay}(s) = e^{-s\tau} = \dfrac{e^{-s\tau/2}}{e^{s\tau/2}} \cong \dfrac{1-s\tau/2}{1+s\tau/2}$ (Pade' approximation)

Eigenvalues: det[sI-A] = 0

Ziegler-Nichols Method I



$$G_c(s) = K_P\left(1 + K_D s + \frac{1}{K_I s}\right)$$

Controller	K_P	K_I	K_D
P	T/L	∞	0
PI	$0.9T/L$	$L/0.3$	0
PID	$1.2T/L$	$2L$	$0.5L$

Method II – Tune K_0 until CL is marginally stable. Record period P

Controller	K_P	K_I	K_D
P	$0.5K_0$	∞	0
PI	$0.45K_0$	$P/1.2$	0
PID	$0.6K_0$	$0.5P$	$0.125P$

State Variables:

$$J = \frac{1}{2}\int_0^\infty x^T Q x + w^T R w dt$$

State Model: $\begin{aligned}\dot{x} &= Ax + Bw, x(0)\\ y &= Cx + Dw\end{aligned}$ LQR: $0 = A^T S(\infty) + S(\infty)A - S(\infty)BR^{-1}B^T S(\infty) + Q$

$$K(\infty) = R^{-1}B^T S(\infty)$$

Solution (zero-input): $x(t) = e^{At}x(0) = P_1 e^{s_1 t} + P_2 e^{s_2 t} + ... + P_n e^{s_n t}$

$$e^{At} = Pe^{St}P^{-1} \; where \; S = \begin{bmatrix} s_1 & 0 & 0 & \cdots \\ 0 & s_2 & 0 & \cdots \\ 0 & 0 & \ddots & 0 \\ \vdots & \vdots & 0 & s_n \end{bmatrix}, P = [P_1\, P_2 ... P_n], \; e^{St} = \begin{bmatrix} e^{s_1 t} & 0 & 0 & \cdots \\ 0 & e^{s_2 t} & 0 & \cdots \\ 0 & 0 & \ddots & 0 \\ \vdots & \vdots & 0 & e^{s_n t} \end{bmatrix}$$

Transfer Function:

$\dfrac{Y(s)}{W(s)} = C[sI - A]^{-1}B + D$ Regulator: w = -Kx CL equation: $\dot{x} = (A - BK)x$ Design K to set the eigenvalues

$M = [B \; AB \; A^2B \; \ldots \; A^{n-1}B]$ rank(M) = # of controllable eigenvalues

Sensitivity: $S_y^x = \dfrac{\partial x}{\partial y}\dfrac{Y}{X}|_{Xnom,Ynom}$ Disturbance Rejection: $\dfrac{\Delta Y(s)}{\Delta D(s)} = \dfrac{GzohG(s)}{1 + GcGzohG(s)} \cong \dfrac{1}{Gc(s)}$ if GcGzohG(s) >> 1

Observer Equation: $\dot{\hat{x}} = A\hat{x} + Bw + K_o(y - C\hat{x})$