Due Monday , January 26 <u>EE613</u> HW #2

1.a) Determine the controllability of the discrete state variable model: $\begin{aligned} x_{k+1} &= \hat{A}x_k + \hat{B}w_k \\ y_k &= \hat{C}x_k \end{aligned}$

if:

i) $\hat{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \hat{B} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 2 \end{bmatrix}, \hat{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ ii) $\hat{A} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}, \hat{B} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}, \hat{C} = \begin{bmatrix} -1 & 0 \end{bmatrix}$

b) Find the feedback control, w=-Kx, which sets the eigenvalues of the continuous time state variable model: $\begin{aligned} \dot{x} &= Ax + Bw \\ y &= Cx \end{aligned}$ to {-5,-6,-7} if:

$$A = \begin{bmatrix} -4 & 3 & 3 \\ 0 & -1 & 0 \\ -3 & 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ 5/2 & 1 \\ 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & -1 \\ -1.5 & 1 & 0.5 \end{bmatrix}$$

c) For the above problem, design an observer which sets the eigenvalues of $[A-K_0C]$ to {-4,-15,-16}.

d) Draw a block diagram of the combined observer/regulator for problems 1b) and 1c)

2.a) Find the gradient of $f(x) = \cos(x_1)x_3 + x_1e^{x_2} + x_1x_2x_3^2$

b) Find the Hessian of f(x) in part 2a).

c) Find the Jacobian of $g(x) = \begin{bmatrix} \sin^2(x_1x_2^2) & x_3x_1^2 & x_1x_3e^{x_2} \end{bmatrix}^T$

d) What would the dimension of the Hessian be in part 2c)?

e) Prove that the transformation, $T_{pv} = MM_{pv}^{-1}$ will transform a controllable single-input system into phase-variable form

f) When will this matrix be a valid similarity transformation matrix (i.e., when will T_{pv} be nonsingular)?