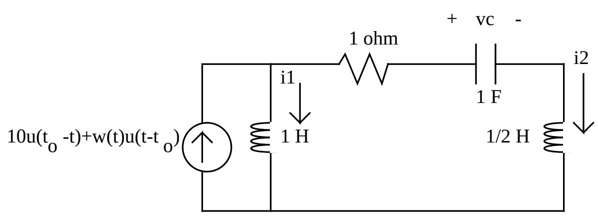
1. a) Find the state variable model of the form $\dot{x} = Ax + Bw,\ x(0^+)$ for the following electrical network:



(make sure to include your initial conditions)

b) Find the state transition matrix, $\Phi(t,\tau) = e^{A(t-\tau)}$, for the following state variable model:

$$\dot{x} = \begin{bmatrix} -10 & -2 \\ 18 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w,\ \ x(t_0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} x$$

c) Find y(t) for problem 1b) if the input w(t) is: i) w(t)= e^{-4t} u(t) (assume that $t_0 < 0$)
ii) w(t)=$e^{-4(t-to)}$ u(t-t_0)

2a) Determine the observability and controllability of the state variable model, $\dot{x} = Ax + Bw,\ y = Cx$, if:

i) $A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ B = \begin{bmatrix} 12 & 8 \\ -3 & -4 \\ -2 & 0 \end{bmatrix},\ C = \begin{bmatrix} -1 & 4 & 0 \\ 0 & 6 & 3 \end{bmatrix}$

ii) $A = \begin{bmatrix} -6 & 0 & 0 \\ 10 & -4 & 0 \\ 20 & 30 & -2 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{bmatrix}$

b) For the continuous state variable models in problem 2a), find the corresponding discrete models by making the approximation that $\dot{x}(t) = \dfrac{x((k+1)T) - x(T)}{T}$ where T is the sampling time. (i.e., find a next state equation of the form $x((k+1)T) = \hat{A}x(kT) + \hat{B}w(kT)$).

c) Repeat part b) but now use the approximation that the input w(t) is constant over a sampling period (i.e., that w(t)=w(kT) over kT < t < (k+1)T). Hint: use the solution to the continuous-time state variable model,

$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bw(\tau)d\tau$, and evaluate at t=(k+1)T and t_0 = kT)

d) Find the discrete state transition matrix, $\hat{\Phi}(k,j) = \hat{A}^{k-j}$, for the next state model given by:

$$x((k+1)T) = \begin{bmatrix} -11/2 & -2 \\ 15 & 11/2 \end{bmatrix} x(kT) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(kT),\ \ x_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

e) Find the zero-input solution for problem 2d)