The URL for the Virtual Lab is csllab1.ee.engr.uky.edu. Please use my numbers for the lead and the PID and be prepared to enter the coefficients of the numerator and denominator for the filter (i.e., b0,b1,... and a0,a1,...). If you try to run the program from off campus, you will probably have to use VPN (available at download.uky.edu). Lastly, to see the live lab cam, you might have to install a suggested ActiveX plug-in.

EE572

Sol'n to Pre LAB # 4 (HW # 19)

1a) Open loop: $\frac{Y(s)}{W(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s} \Rightarrow W(s)=\frac{1}{s} \Rightarrow Y(s) = \frac{\omega_n^2}{s^2(s+2\zeta\omega_n)} = \frac{\omega_n/2\zeta}{s^2} + \frac{1/4\zeta^2}{s+2\zeta\omega_n} - \frac{1/4\zeta^2}{s}$

$\Rightarrow y(t) = \omega_n/2\zeta\, t + 1/4\zeta^2\left(e^{-2\zeta\omega_n t} - 1\right)$

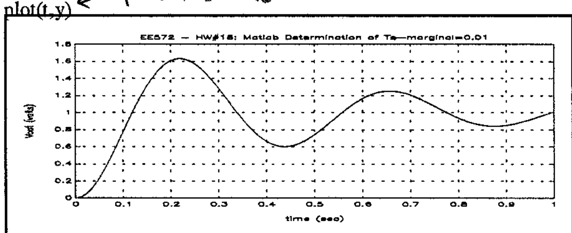
b) Find where $y(t) = 2 = \omega_n/2\zeta\, t + 1/4\zeta^2\left(e^{-2\zeta\omega_n t} - 1\right)$ ← this is a transcendental equation, and cannot be solved explicitly for t. ∴ assume system has reached steady-state by $T_s \Rightarrow e^{-2\zeta\omega_n T_s} \cong 0 \Rightarrow 2 \cong \omega_n/2\zeta\, T_s - 1/4\zeta^2 \Rightarrow T_s \cong \frac{2\zeta}{\omega_n}\left(2 + \frac{1}{4\zeta^2}\right)$

c) For the motomatic, $\omega_n = \sqrt{211} = 14.53$, $\zeta = \frac{5.27}{2\omega_n} = 0.1814 \Rightarrow T_s \cong 0.2397$ seconds

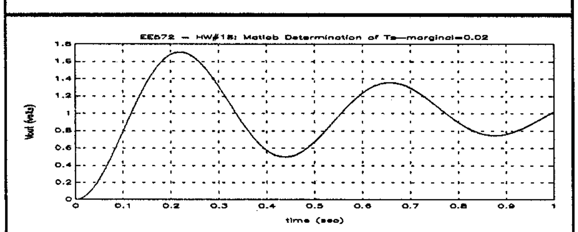
Note that $e^{-2\zeta\omega_n T_s} = 0.2827 \cong 0$ (the exact numerical sol'n is $T_s = 0.156552$)

d) Matlab "m" file:

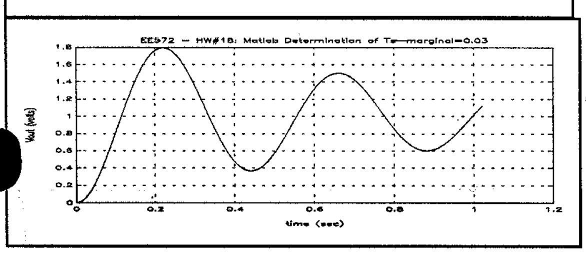
```
function [y,t,u]=ts_marg(zeta,wn,n,Ns)
%          ts_marg(zeta,wn,n,Ns) plots the step response of a second-order
%          system in closed loop with a sampling time of n msec:
%          Ns is the number of points and must be a multiple of n.

num=wn^2;            <- num = wn^2        den = s^2+2ζwn s
den=[1 2*zeta*wn 0]; 
[a,b,c,d]=tf2ss(num,den);   <- convert to state-space
y=0;
x0=[0;0];       <- initial state is zero
=n/1000;        <- sampling time (Ts_marginal)
 1/1000*[0:Ns];  <- t
for j=1:Ns/n    <- form error value which remains constant
e=1-y((j-1)*(n)+1);
[ytemp,x]=lsim(a,b,c,d,e*ones(1,n+1),1/1000*[0:n],x0);   <- simulate open-loop response over one sampling period
x0=x(n+1,:)';
y((j-1)*(n)+1:j*n+1)=ytemp;      <- tack ytemp onto y
u((j-1)*(n)+1:j*n+1)=e*ones(1,n+1);   <- form actuating signal
end
end
plot(t,y)   <- plot results
```



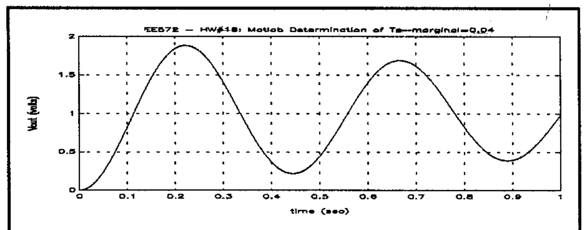
← $T_s = 0.01$



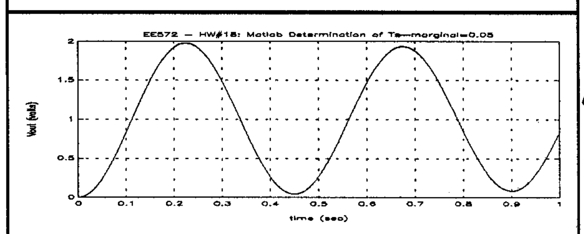
$T_s = 0.02$



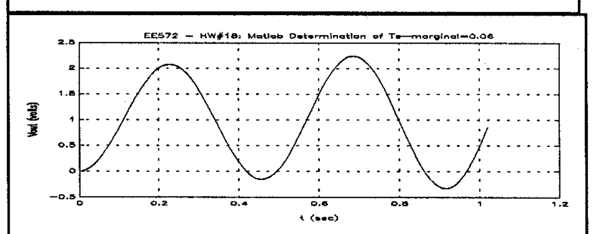
$T_s = 0.03$



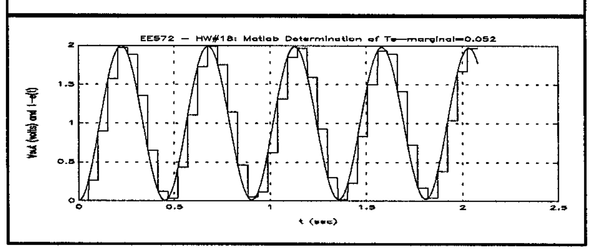
$T_s = 0.04$



$T_s = 0.05$



$T_s = 0.06$



$T_s = 0.052$ ← $T_{s\_marginal}$

Gc(z) = (1.014-0.856z^-1) / (1 - 0.772z^-1)

Sol'n to PreLab 4 (cont)

2a)

```
» wn=sqrt(211)
wn =
   14.5258
» zeta=5.27/(2*wn)
zeta =
   0.1814
» num=wn^2
num =
  211.0000
» den=[1 2*zeta*wn 0]
den =
   1.0000   5.2700   0
» Ts=.01
Ts =
   0.0100
» num_zoh=2/Ts
num_zoh =
  200
» den_zoh=[1 2/Ts]
den_zoh =
   1   200
» numz=conv(num,num_zoh)
numz =
  4.2200e+004
» denz=conv(den,den_zoh)
denz =
  1.0e+003 *
   0.0010   0.2053   1.0540   0
» ts=.35
ts =
   0.3500
» zwn_minimum=4/ts
zwn_minimum =
   11.4286
» Mp=.02
Mp =
   0.0200
»
zeta_minimum=sqrt((log(Mp)^2)/(pi^2+log(Mp)^2))
zeta_minimum =
   0.7797
» zwn_desired=12
zwn_desired =
   12
» zeta_desired=.8
zeta_desired =
   0.8000
» wn_desired=12/.8
wn_desired =
   15
»
s1=-zwn_desired+j*wn_desired*sqrt(1-zeta_desired^2)
s1 =
 -12.0000 + 9.0000i
»
```

(open-loop numerator; open-loop denominator; add in ZOH dynamics!; desired poles)

```
180/pi*angle(polyval(numz,s1)/polyval(denz,s1))
ans =
   87.3408
» phi=180-ans
phi =
   92.6592
» z=11.5
z =
   11.5000
» 180/pi*angle(s1+z)-phi
ans =
   0.5206
» p=imag(s1)/tan(pi/180*ans)-real(s1)
p =
  1.0025e+003
»
kc=abs((s1+p)*polyval(denz,s1)/((s1+z)*polyval(numz,s1)))
kc =
   82.6165
» num_cl=conv(kc*[1 z],numz)
num_cl =
  1.0e+007 *
   0.3486   4.0094
» den_cl=conv([1 p],denz)+[0 0 0 num_cl]
den_cl =
  1.0e+007 *
   0.0000   0.0001   0.0207   0.4543
4.0094
» t=Ts*[0:100];
» y=step(num_cl,den_cl,t);
» plot(t,y)
» grid
» one=ones(1,101);
» plot(t,y,t,.98*one,'--',t,1.02*one,'--')
» grid
» title('EE572 - HW#18: 1st attempt at Lead design, kc=82.6, z=11.5, p=1K')
» ylabel('Vout (volts)');xlabel('t(sec)')
» meta hw18a
» z=5.5
z =
   5.5000
» 180/pi*angle(s1+z)-phi
ans =
   33.1784
» p=imag(s1)/tan(pi/180*ans)-real(s1)
p =
   25.7647
»
kc=abs((s1+p)*polyval(denz,s1)/((s1+z)*polyval(numz,s1)))
kc =
   1.1138
» num_cl=conv(kc*[1 z],numz)
num_cl =
  1.0e+005 *
   0.4700   2.5850
» den_cl=conv([1 p],denz)+[0 0 0 num_cl]
den_cl =
  1.0e+005 *
   0.0000   0.0023   0.0634   0.7416
2.5850
```

(Angle of deficiency)

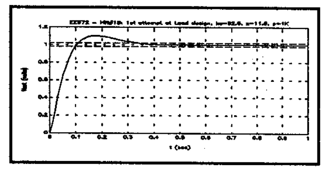
$G_c(s) = 1.114 \frac{s+5.5}{s+25.8}$

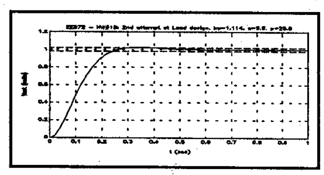
```
» y=step(num_cl,den_cl,t);
» plot(t,y,t,.98*one,'--',t,1.02*one,'--')
» title('EE572 - HW#18: 2nd attempt at Lead design, kc=1.114, z=5.5, p=25.8')
» ylabel('Vout (volts)');xlabel('t(sec)')
» grid
» meta hw18b
» [numc,denc]=bilinear(kc*[1 z],[1 p],1/Ts)
numc =
   1.0138   -0.9595
denc =
   1.0000   -0.7718
» Ki=0.1
Ki =
   0.1000
» -polyval(denz,s1)/polyval(numz,s1)-Ki/s1
ans =
  -0.0295 + 0.7550i
» y=step(num_pid,den_pid,t);
» plot(t,y)
» inv([0 imag(s1); 1 real(s1)])*[imag(ans);real(ans)]
ans =
   0.9772
   0.0839
» Kp=ans(1)
Kp =
   0.9772
» Kd=ans(2)
Kd =
   0.0839
» num_pid=conv([Kd Kp Ki],numz)
num_pid =
  1.0e+004 *
   0.3540   4.1236   0.4220
» den_pid=conv([1 0],denz)+[0 0 num_pid]
den_pid =
  1.0e+004 *
   0.0001   0.0205   0.4594   4.1236
0.4220
» y=step(num_pid,den_pid,t);
» plot(t,y)
» grid
» plot(t,y,t,.98*one,'--',t,1.02*one,'--')
» grid
» title('EE572 - HW#18: 1st attempt at PID design, Ki=0.1, Kp=0.978, Kd=0.08')
» ylabel('Vout (volts)');xlabel('t(sec)')
» meta hw18c
» s1=-8+j*.2
s1 =
  -8.0000 + 0.2000i
» Ki=.01
Ki =
   0.0100
» -polyval(denz,s1)/polyval(numz,s1)-Ki/s1
ans =
  -0.0979 + 0.0097i
» inv([0 imag(s1); 1 real(s1)])*[imag(ans);real(ans)]
ans =
   0.2897
   0.0485
» Kp=ans(1)
Kp =
```

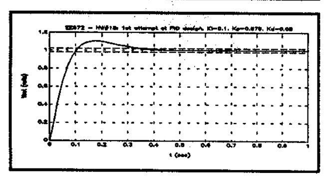
$G_c(z) = \frac{1.014 - 0.956z^{-1}}{1 - 0.772z^{-1}}$

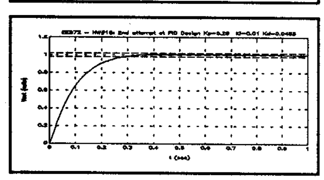
```
   0.2897
» Kd=ans(2)
Kd =
   0.0485
» num_pid=conv([Kd Kp Ki],numz)
num_pid =
  1.0e+004 *
   0.2045   1.2226   0.0422
» den_pid=conv([1 0],denz)+[0 0 num_pid]
den_pid =
  1.0e+004 *
   0.0001   0.0205   0.3099   1.2226
0.0422
» y=step(num_pid,den_pid,t);
» plot(t,y,t,1.02*one,'--')
» plot(t,y,t,1.02*one,'--',t,.98*one,'--')
» grid
» title('EE572 - HW#18: 2nd attempt at PID Design Kp=0.29 Ki=0.01 Kd=0.0485')
» ylabel('Vout (volts)');xlabel('t(sec)')
» meta hw18d
» save
Saving to: matlab.mat
» quit
 225610 flops.
```

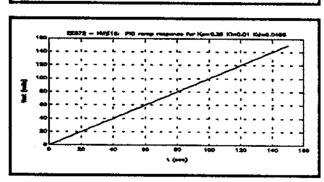
(0.2897 = Kp)

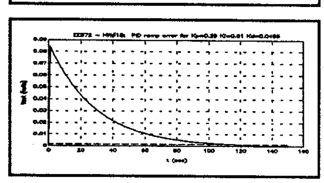












$G_{PID}(z) = 0.2897 + 0.01 \frac{T_s z}{z-1} + 0.0485 \frac{(z-1)}{T_s z}$

Gpid(z) = ((Kp+KiTs +Kd/Ts)z^0+(-2Kd/Ts-Kp-KiTs)z^-1 - (Kd/Ts)z^-2) / (1 - z^-1) = (5.1398 - 9.9898z^-1 + 4.85z^-2) / (1 - z^-1)