

1. Consider the following discrete-time multi-input state variable model:

$$x_{k+1} = \begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix} x_k + \begin{bmatrix} 2 & -I \\ -I & I \end{bmatrix} w_k$$

- Find the eigenvalues of the open-loop system.
- Is the system stable?
- Is the system controllable?
- If yes, design a feedback regulator law, $w_k = -Kx_k$ such that the closed-loop system has eigenvalues $\{-.5, .25\}$.
- What is the settling time of the closed-loop system if $T_s = 10\text{msec}$?
- If $x_{k+1} = \hat{A}x_k + \hat{B}w_k$ determine which systems are stabilizable:

$$\text{i) } \hat{A} = \begin{bmatrix} -1 & -3 \\ 3/2 & 7/2 \end{bmatrix} \hat{B} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \text{ii) } \hat{A} = \begin{bmatrix} 11 & 10 \\ -5 & -4 \end{bmatrix} \hat{B} = \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix} \quad \text{iii) } \hat{A} = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix} \hat{B} = \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$$

2. Consider the following discrete-time state variable model:

$$x_{k+1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 2 \end{bmatrix} x_k + \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 0 & -2 \end{bmatrix} w_k \quad x_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- Find the eigenvalues of the open-loop system.
- Is the system stable?
- Is the system controllable? Is it stabilizable?
- If yes, design a feedback regulator law, $w_k = -Kx_k$ such that all the controllable eigenvalues are $\{1/4\}$.
- What is the settling time of the closed-loop system if $T_s = 10 \text{ msec}$?