EE572
HW #8
Due Monday, February 15

1. Consider the following discrete-time state variable model (don't use Matlab):

$$x_{k+1} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 2 \end{bmatrix} w_k$$

a) Find a similarity transformation, $x(kT_s)=T_{pv}\, z_{pv}\,(kT_s)$ such that z_{pv} is in phase variable form

b) Now find values for A_{pv} and B_{pv} where $z_{pv}((k+1)T_s)= A_{pv}\, z_{pv}(kT_s)+ B_{pv}\, w(kT_s)$

c) Design a feedback control law, $w_k = -\underline{k}x_k$ such that the closed-loop eigenvalues are {0.4 0.5}.

d) What is settling time if T_s = 10 msec?

2. Consider the following discrete-time state variable multi-input model (use Matlab):

$$x_{k+1} = \begin{bmatrix} 3 & 0.5 & 1 \\ -1 & 0.75 & -0.5 \\ 0.5 & 0.625 & 2.25 \end{bmatrix} x_k + \begin{bmatrix} 0.5 & 0 \\ -0.25 & 1 \\ 0.625 & 0 \end{bmatrix} w_k$$

a) Is the system stable?

b) Find the controllability matrix for the system. Is the system controllable?

c) Now show how we can reduce the system to a controllable single input system and then write the new state equation for the single-input system.

d) Now, transform the single-input system so that it is in phase-variable form

e) Then, find the single-input $w_{sk} =-K_{pv}z_{pvk}$ such that the closed-loop eigenvalues are {0.5 0.6 0.7}

f) Finally, go back and find an expression for the original multi-input feedback control, w_k

g) How quickly will the decoupled modes decay to 2% of their initial values (i.e., what is settling time) if T_s = 10 msec?