EE572
HW #7
Due Wednesday, February 10

1. a) If $x_{k+1} = \hat{A} x_k + \hat{B} w_k$, find the eigenvalues and eigenvectors then use the similarity transformation, $x_k = Pz_k$, to determine which of the eigenvalues are controllable (you may use Matlab if you wish):

i) $\hat{A} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \hat{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ii) $\hat{A} = \begin{bmatrix} 1/2 & -2 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}, \hat{B} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

b) For the above problem, find the rank of the controllability matrices and determine if the systems are completely controllable. Do your answers agree with the answers in part a) ?

c) For problem 1a), state which of the eigenvalues are stable and which are unstable.

d) Find the region in the s-plane which corresponds to the following design specifications:
A settling time (2%) of 0.2 sec and no overshoot/oscillations (recall $Re[s_i]_{max} = -4/t_s$ defines the border of the settling-time region in the LHP of the S-Plane.)

e) Repeat part d) in the Z-plane if T_s = 10 msec. (map the above region using $z=e^{sT}$)

2. Consider the following discrete-time state variable model:

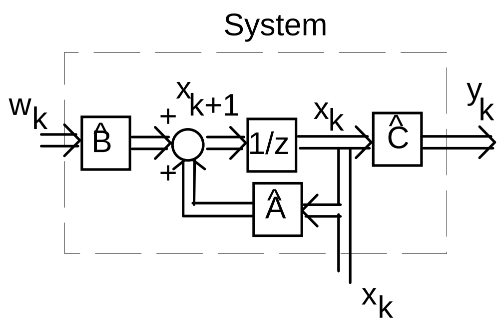
$$x_{k+1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w_k$$

a) Is the system completely controllable (use Matlab to find $x_k = Tz_k$ to decouple) ?

b) Is the system stable?

c) If the system is completely controllable (which it is), design a feedback control law, $w_k = -\underline{k}x_k$ such that the closed-loop eigenvalues are {.4,.4,.4}.

d) Complete the block diagram of your new closed-loop system (we drew this in class):



e) What is settling time of the new closed-loop system if T_s = 10 msec (hint: $|z|_{max}$ = 0.4 after we set all the closed-loop eigenvalues. Solve $|z|_{max} = e^{-4T/ts}$ for the settling-time, t_s.