EE572
RISHW #5*
Due Wednesday, February 2nd (Happy Ground Hog's Day)

0. Please complete Lab 1 that we took data for after class today. It's due on Wednesday, too! On days when you have a lab due, I usually give you a short, one-problem HW just to keep you out of trouble.

Given the continuous state variable model, $\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} w, \; x(0) = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$, $y = \begin{bmatrix} 3 & 4 \end{bmatrix} x$ and the corresponding discrete next-state approximation to this model, $x_{k+1} = \hat{A}x_k + \hat{B}w_k, \; x_0$, $y_k = \hat{C}x_k$ with sampling period T_s

a) Calculate the zero input response for t=1,2, and 3 seconds (hint: $x_{zero\text{-}input}(t) = e^{At}x(0)$)

b) Find values for $\hat{A}, \hat{B}, \hat{C}$ and x_0 if we use the approximation that $\hat{A} = I + TA$ and T_s =1 second and calculate the first three values of the zero-input solution for k=1,2, and 3. Compare to your answer to part a)

c) Repeat part b) using the approximation that, $\hat{A} = e^{AT}$. What conclusions can you make about the two methods of finding a discrete state model from a continuous time state model?

d) Now that we know how to convert a continuous state model into a discrete state model, we have another way to find a digital filter from an analog model. Given the SCALAR system, dx/dt=ax+bw, with output, y=cx and sampling time, T_s. Convert to a discrete state model using the approximation scheme given below. From this next-state equation, find the approximate digital transfer function, G(z)=Y(z)/W(z):

   i) Convert using the approximation that $\hat{A} = e^{AT}$
   ii) Convert using the approximation that $\hat{A} = I + TA$

We also know, from Pre-Lab 1, three other ways to find H(z). Starting from the SCALAR system, dx/dt=ax+bw, with output, y=cx, the continuous transfer function is $H(s)=c(s-a)^{-1}b$ = cb/(s+a). Starting with this H(s), find the digital filter H(z) using:

   iii) The bilinear transformation
   iv) The correct impulse invariant design method you learned in class today
   v) The correct step invariant design method you learned in class today

e) Are any of your answers for G(z) the same? What if T_s=1 sec?

*RISHW = reduced instruction set HW