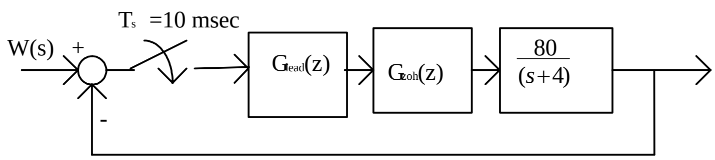
EE572 Solution to HW#22

0. Keep working on your project!!

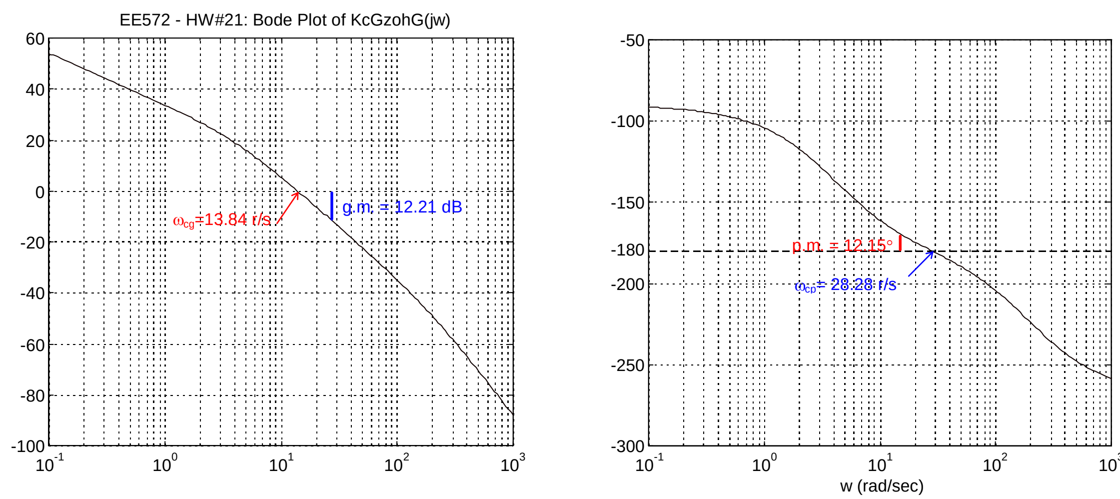
1. Consider the following system:



a) Design a lead compensator, $G_{lead}(z)$, which meets the following specs:

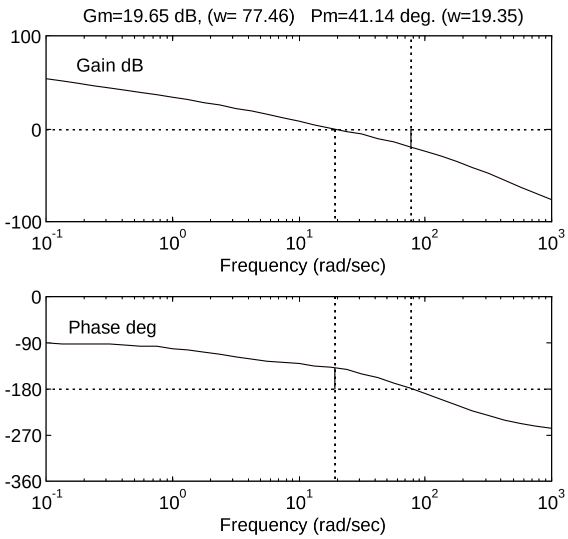
$ess|_{ramp} = 1/50$
gain margin > 5 dB
phase margin > 30^o

Solution: If we let $G_{lead}(s) = K_c \dfrac{Ts+1}{\alpha T_s + 1}$, we see that we need a type 1 system to meet the steady-state error specifications. Since we only have a type one system, we must increase the system type. This is accomplished by letting K_c=K/s. We can find the appropriate value of K from $K_{v_{desired}} = 50 = \lim_{s \to 0} s G_{lead} G_{zoh} G(s) = K(80)/4 = 20K$. Thus, K=50/20=2.5 and K_c=2.5/s. Next, we must make a Bode plot of $K_c G_{zoh} G(s) = \dfrac{200}{s(s+4)(1+s/200)}$. The Bode plot is shown below:



Next, we can determine how much more phase angle we need to meet specifications from:
ϕ_m = p.m.$_{desired}$ - p.m.$_{current}$ + fudge factor = 35^o-12.15^o+10^o = 32.85^o. We can use this value to solve for α :
$\sin(\phi_m) = \dfrac{1-\alpha}{1+\alpha}$ or $\alpha = \dfrac{1-\sin(\phi_m)}{1+\sin(\phi_m)} = 0.2712$. Recall that the lead compensator adds a gain of $1/\sqrt{\alpha}$ = 1.92=5.66 dB. Thus, the gain crossover frequency will shift to the right where $|K_c G_{zoh} G|$=-5.66 dB. From the Bode plot, we see that this occurs at $\omega_{cg_{new}}$ = 19.35 rad/sec. We can find the last parameter, T, from $\omega_{cg_{new}} = \dfrac{1}{\sqrt{\alpha}\, T} = 19.35$ or T=0.992.

Hence, $G_{lead}(s) = K_c \dfrac{Ts+1}{\alpha T_s + 1} = \dfrac{2.5}{s}\dfrac{0.0992s+1}{0.0269s+1}$. We can check our answer by making a Bode plot of $G_{lead} G_{zoh} G(s)$:



As we can see, the compensated system has a phase margin of 41.14° at a new gain cross-over frequency of 19.65 rad/sec and a gain margin of 19.65 dB at a phase cross-over frequency of 77.46 rad/sec (well within our specs!!!)

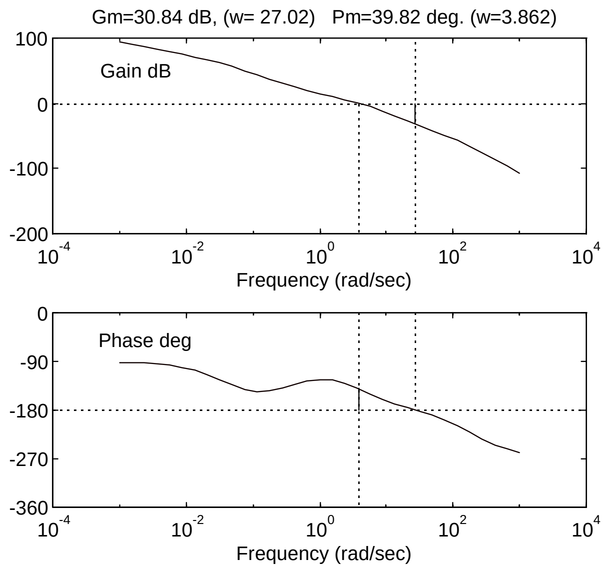
Finally, using the Bilinear transformation, we find $G_{lead}(z)$ to be $G_{lead}(z) = 125.18\dfrac{1+1.996z^{-1}+0.0996z^{-2}}{1-0.0011z^{-1}-0.9989z^{-2}}$.

2. a) Repeat problem 1 using lag compensation techniques

Solution: If we let $G_{lag}(s) = K_c \dfrac{Ts+1}{\beta T_s + 1}$, again we see that we need a type 1 system to meet the steady-state error specifications. Since we only have a type one system, we must increase the system type. This is accomplished by letting K_c=K/s. We can find the appropriate value of K from $K_{v_{desired}} = 50 = \lim_{s\to 0} sG_{lag}G_{zoh}G(s) = K(80)/4 = 20K$.

Thus, K=50/20=2.5 and K_c=2.5/s. Next, we must make a Bode plot of $K_cG_{zoh}G(s) = \dfrac{200}{s(s+4)(1+s/200)}$. The Bode plot is the same as shown in problem 1.

Next, we need to find a new gain cross-over frequency where the system has sufficient phase angle to meet our phase margin specifications. That is, we need to find $\omega_{cg_{new}}$ where $\angle K_cG_{zoh}G(j\omega_{cg_{new}}) = p.m._{desired} - 180° + fudge\ \ factor = 35° - 180° + 5° = -140°$. From the Bode plot, we can see that this occurs at $\omega_{cg_{new}}$ = 3.85 rad/sec. At this frequency, the magnitude of $|K_cG_{zoh}G|$=19.42 dB = 9.36. Since our lag compensator will introduce an attenuation of $1/\beta$ at high frequencies, we set $\beta = |K_cG_{zoh}G(j\omega_{cg_{new}})| = 9.36$. Finally, to insure that we get the desired attenuation of $1/\beta$, we must set the zero of our lag compensator (located at s=-1/T) a decade or so below $\omega_{cg_{new}}$. In other words, $T = 10/\omega_{cg_{new}} = 2.5981$. Hence, $G_{lag}(s) = K_c \dfrac{Ts+1}{\beta T_s + 1} = \dfrac{2.5}{s}\dfrac{2.5981s+1}{24.3149s+1}$. We can check our answer by making a Bode plot of $G_{lag}G_{zoh}G(s)$:

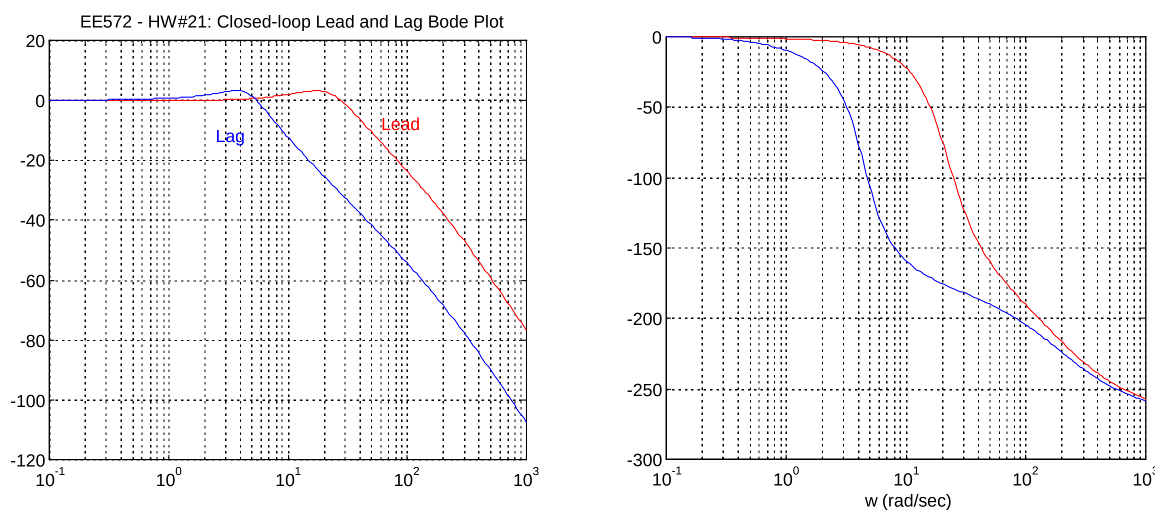


As we can see, the compensated system has a phase margin of 39.82° at a new gain cross-over frequency of 3.862 rad/sec and a gain margin of 30.84 dB at a phase cross-over frequency of 27.02 rad/sec (well within our specs!!!)

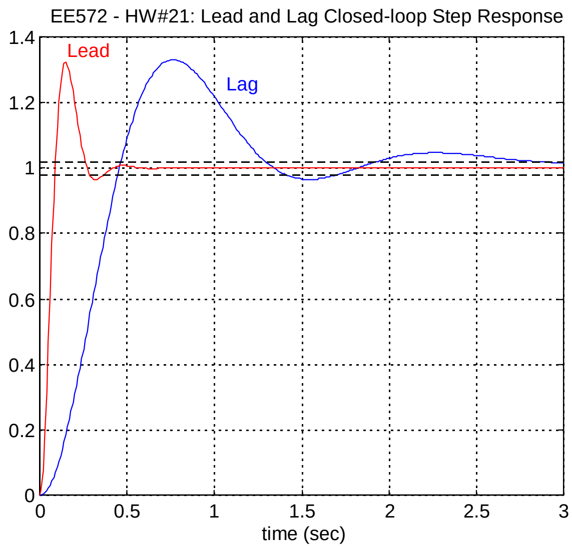
Finally, using the Bilinear transformation, we find $G_{lag}(z)$ to be $G_{lag}(z) = 88.47\dfrac{1+1.9012z^{-1}+0.9012z^{-2}}{1-0.6544z^{-1}-0.3456z^{-2}}$.

b) Find the step response and the closed-loop Bode plot for both of your designs. What is the bandwidth of each closed-loop compensated system?

<u>Solution:</u> The following is a closed-loop Bode plot of both the lead and lag compensated systems:



The bandwidth of the lag compensated system is about 6 rad/sec while the lead compensated system has a bandwidth of about 30 rad/sec. We can also find the closed-loop step response for both systems:



Note that the settling time for the lead step response is about 0.4 seconds while the lag settling time is nearly 3 seconds!

c) Suppose our input is corrupted by periodic additive noise such that the actual input is w(t)+2.5cos20t. For both closed-loop systems, find the magnitude of the noise at the output (i.e., the magnitude of $y|_{2.5\cos 20t}$). (Hint:use your closed-loop Bode plot from part b)). Which system does a better job at attenuating the effects of the noise?

Solution: From the closed-loop lead Bode plot, at a frequency of 20 rad/sec the magnitude of the lead system is about 3.2 dB = 1.44 while the phase is about -75°. Thus, the steady-state output of the lead system due to the noise will be about 3.6cos(20t -75°). From the closed-loop lag Bode plot, at a frequency of 20 rad/sec the magnitude of the lag system is about -25 dB = 0.0562 while the phase is about -175°. Thus, the steady-state output of the lead system due to the noise will be about 0.1406cos(20t -175°). Obviously, the lag system does a far better job of attenuating the additive input noise!

APPENDIX - Matlab Code for HW#21

```
» num=200;
» den=conv([1 4 0],[1/200 1])

den =

   0.0050    1.0200    4.0000         0

» w=logspace(-1,3,200);
» [mg,ph]=bode(num,den,w);
» semilogx(w,20*log10(mg))
» grid;title('EE572 - HW#21: Bode Plot of KcGzohG(jw)')
» margin(num,den)
» semilogx(w,ph)
» grid;xlabel('w (rad/sec)')
» semilogx(w,ph,[.1 1000],[-180 -180],'--')
» grid;xlabel('w (rad/sec)')
» phim=35-12.15+10

phim =
```

```
   32.8500

» phim*pi/180

ans =

    0.5733

» alpha = (1-ans)/(1+ans)

alpha =

    0.2712

» 1/sqrt(alpha)

ans =

    1.9203

» 20*log10(ans)

ans =

    5.6674

» margin(1/sqrt(alpha)*num,den)
» T=1/(sqrt(alpha)*19.35)

T =

    0.0992

» alpha*T

ans =

    0.0269

» numc=2.5*[T 1]

numc =

    0.2481    2.5000

» denc=[alpha*T 1 0]

denc =

    0.0269    1.0000         0

» [numd,dend]=bilinear(numc,denc,1/100)

numd =
```

```
  125.1807  249.8655  124.6848


dend =

    1.0000   -0.0011   -0.9989

» numd/numd(1)

ans =

    1.0000    1.9960    0.9960


» denlead=conv([alpha*T 1],den)

denlead =

    0.0001    0.0325    1.1276    4.0000         0

» numlead=conv([T 1],num)

numlead =

   19.8481  200.0000

» margin(numlead,denlead)
» [g,p,w,w2]=margin(mg,ph-45,w)

g =

    0.1069


p =

  -32.8424


w =

    3.8489


w2 =

   13.8447

» beta=1/g

beta =

    9.3586
```

```
» 20*log10(beta)

ans =

   19.4242

» T=10/w

T =

    2.5981

» beta*T

ans =

   24.3149

» numl=conv([T 1],num)

numl =

  519.6277  200.0000

» denl=conv([beta*T 1],den)

denl =

    0.1216   24.8062   98.2798    4.0000         0

» margin(numl,denl)

Warning: Divide by zero
» numc=2.5*[T 1]

numc =

    6.4953    2.5000

» denc=[beta*T 1 0]

denc =

   24.3149    1.0000         0

» [numd,dend]=bilinear(numc,denc,1/100)

numd =

   88.4717  168.2031   79.7314


dend =
```

```
    1.0000   -0.6544   -0.3456

» numd/numd(1)

ans =

    1.0000    1.9012    0.9012

» numclag=2.5*[T 1]

numclag =

    6.4953    2.5000

» denclag=[beta*T 1 0]

denclag =

   24.3149    1.0000         0

» T=1/(sqrt(alpha)*19.35)

T =

    0.0992

» numclead=2.5*[T 1]

numclead =

    0.2481    2.5000

» [numldcl,denldcl]=cloop(numlead,denlead)

numldcl =

         0         0         0   19.8481  200.0000


denldcl =

    0.0001    0.0325    1.1276   23.8481  200.0000

» [numlgcl,denlgcl]=cloop(numl,denl)

numlgcl =

         0         0         0  519.6277  200.0000


denlgcl =

    0.1216   24.8062   98.2798  523.6277  200.0000
```

```
» [mgld,phld]=bode(numldcl,denldcl,w);
» [mglg,phlg]=bode(numlgcl,denlgcl,w);
» semilogx(w,20*log10(mgld),w,20*log10(mglg))
» w=logspace(-1,3,200);
» [mgld,phld]=bode(numldcl,denldcl,w);
» [mglg,phlg]=bode(numlgcl,denlgcl,w);
» semilogx(w,20*log10(mgld),w,20*log10(mglg))
» grid;title(EE572 - HW#21: Closed-loop Lead and Lag Bode Plot')
??? d;title(EE572 - HW#
                    |
Improper function reference. A "," or ")" is expected.

» grid;title('EE572 - HW#21: Closed-loop Lead and Lag Bode Plot')
» semilogx(w,phld,w,phlg)
» grid;xlabel('w (rad/sec)')
» ylg=step(numldcl,denldcl);
» yld=step(numldcl,denldcl);
» ylg=step(numlgcl,denlgcl);
» plot(yld)
» t=[0:.01:2];
» yld=step(numldcl,denldcl,t);
» ylg=step(numlgcl,denlgcl,t);
» plot(t,ylg,t,yld)
» t=[0:.01:3];
» yld=step(numldcl,denldcl,t);
» ylg=step(numlgcl,denlgcl,t);
» plot(t,ylg,t,yld,[0 3],[.98 .98],'--',[0 3],[1.02 1.02],'--')
» grid;title('EE572 - HW#21: Lead and Lag Closed-loop Step Response')
» xlabel('time (sec)')
» semilogx(w,20*log10(mgld),w,20*log10(mglg))
» grid
» max(mgld)

ans =

    1.4453

» 20*log10(ans)

ans =

    3.1995

» semilogx(w,phld,w,phlg)
» grid
» semilogx(w,20*log10(mgld),w,20*log10(mglg))
» grid
» 10^(-25/20)

ans =

    0.0562
```

```
» 1.44*2.5

ans =

    3.6000

» 10^(-25/20)

ans =

    0.0562

» ans*2.5

ans =

    0.1406
```