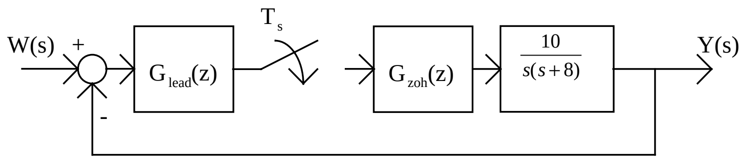
2. Given the system:




*a) Find an s-domain model for the open-loop system including the ZOH if $T_s$ = 10 msec.*

Solution: From class we learned that the approximate transfer function for $G_{ZOH}(s)$ is $2/T_s/(2/T_s+s)$. Substituting $T_s$ = 10 msec. into this expression yields $G_{ZOH}(s) = 200/(s+200)$. Thus, the entire open-loop model is $G_{ZOH}G(s) = \dfrac{2000}{s(s+8)(s+200)}$

*b) What type number is your S-domain model?*

Solution: The system is a type 1 system (i.e., one open-loop pole at the origin)

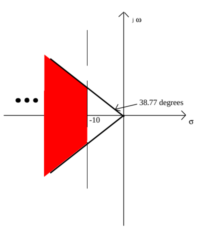
*c) Find $e_{ss}$ due to a unit step, $e_{ss}$ due to a unit ramp, and $e_{ss}$ due to a unit parabola for your uncompensated model.*

Solution: $ess|_{\text{unit step}} = 1/(1+K_p) = 0$, $ess|_{\text{unit ramp}} = 1/K_v = (200)(8)/(2000) = 4/5$, and $ess|_{\text{unit ramp}} = 1/K_a = \infty$

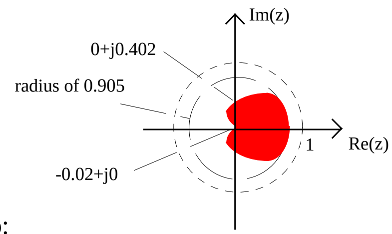
*d) Given the following transient specifications: $t_s < 0.4$ sec and $M_p < 2\%$. Illustrate the region of the s-plane and then the z-plane where we must place our dominant poles to satisfy these specs.*

Solution: from the first spec, we find that $t_s < 0.4 \Rightarrow \zeta\omega_n > 10$. From the second spec we find that $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$. Or, solving this relationship for the damping coefficient we obtain, $\zeta = \sqrt{\dfrac{\ln(m)^2}{\pi^2 + \ln(m)^2}}$ where m=$M_p$/100% = 0.02. Thus, $\zeta = \sqrt{\dfrac{\ln(0.02)^2}{\pi^2 + \ln(0.02)^2}} = 0.7797 = \cos\theta$. Hence, $\theta = \cos^{-1}(\zeta) = \cos^{-1}(0.7797) = 38.77^\circ$.

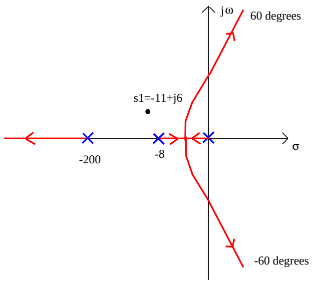
Since we must have $M_p$ < 2%, our constraint becomes $\theta \le 38.77^\circ$. In the s-plane, we obtain the following region:



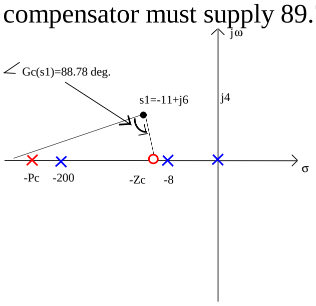

In the z-plane, this region maps to:




*e) Sketch the root locus. Design $G_{lead}(s)$ then find $G_{lead}(z)$ (using the bilinear transformation) to meet the above specs.*

Solution: Choose the desired dominant poles to be $s_1$=-11+j6. The root locus of $G_{ZOH}G(s) = \dfrac{2000}{s(s+8)(s+200)}$ looks like: 

. Obviously, the uncompensated root locus does not pass thru the desired poles. So, we need to insert a lead compensator of the form $G_{lead}(s_1) = K_c \frac{s+z_c}{s+p_c}$ Our first step in designing this compensator is to find the angle of deficiency (i.e., the amount of lead angle needed to bend the root locus thru $s_s$). Find the angle of deficiency $\angle G_{lead}(s_1) = 180° \times odd\# - \angle G_{zoh}G(s_1)|_{s_1=-11+j6}$ = 180° x odd# - 90.23° = 89.77°. Therefore, our lead compensator must supply 89.77 degrees.




Let's pick $z_c$ to be as far to the left as possible. Let $z_c$ be 10.8 (by geometry, $z_c$ must be less than or equal to 11). Then $\angle G_{lead}(s_1) = 89.77° = \angle(s_1 + 10.8) - \angle(s_1 + p_c) = 91.9° - \angle(s_1 + p_c)\ or\ \angle(s_1 + p_c) = 2.13°$. Solving for $p_c$: $IM(s_1)/RE(s_1 + p_c) = \tan(2.13°)\ or\ p_c = IM(s_1)/\tan(2.13°) - RE(s_1) = 171.85$. The last step is to find $K_c$ from the magnitude condition: $K_c = 1/|G_{zoh}G(s_1)(s_1 + z_c)/(s_1 + p_c)| = 213.08$. Thus, the lead compensator is $G_{lead}(s)$=213.08(s+10.8)/(s+171.85). Finally, the digital filter design can be found using the bilinear transformation: $G_{lead}(z) = G_{lead}(s)|_{s=\frac{2(z-1)}{T_s(z+1)}}$ = 153.4(z-0.8975)/(z-0.0757)

*e) Use Matlab to determine where all the closed-loop compensated poles and zeros are (Hint: Find the closed-loop compensated transfer function, Y(s)/W(s)=$G_{lead}G_{zoh}G(s)/(1+ G_{lead}G_{zoh}GH(s))$ then use the Matlab roots() function to find the poles and zeroes)*

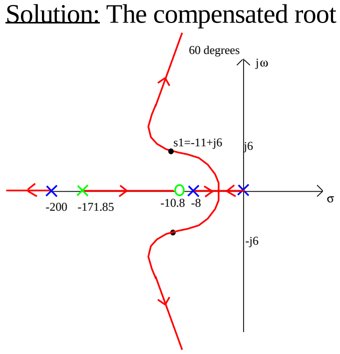
<u>Solution:</u> the following commands in Matlab can be used to find the closed-loop transfer function, $G_{lead}G_{zoh}G(s)/(1+ G_{lead}G_{zoh}GH(s))$:

```
» num=2000*kc*[1 z]

num =

  1.0e+006 *

    0.4262    4.6024

» den=poly([0,-8,-200,-p])

den =
```

```
  1.0e+005 *

   0.0000    0.0038    0.3734    2.7496         0

» [num_closed_loop,den_closed_loop]=cloop(num,den)

num_closed_loop =

  1.0e+006 *

        0         0         0    0.4262    4.6024


den_closed_loop =

  1.0e+006 *

   0.0000    0.0004    0.0373    0.7011    4.6024

» roots(den_closed_loop)

ans =

  1.0e+002 *

  -2.3088
  -1.2697
  -0.1100 + 0.0600i
  -0.1100 - 0.0600i
```

*f) Sketch the compensated root locus in the S-plane.*

Solution: The compensated root locus looks like:




*g) Does your compensated system have closed-loop dominant poles which meet the specifications?*

Solution: as can be seen from the Matlab output, the closed-loop poles are {-11+j6, -11-j6, -126.97, -230.88}. So, yes the dominant poles are at exactly $s_1$=-11+j6.