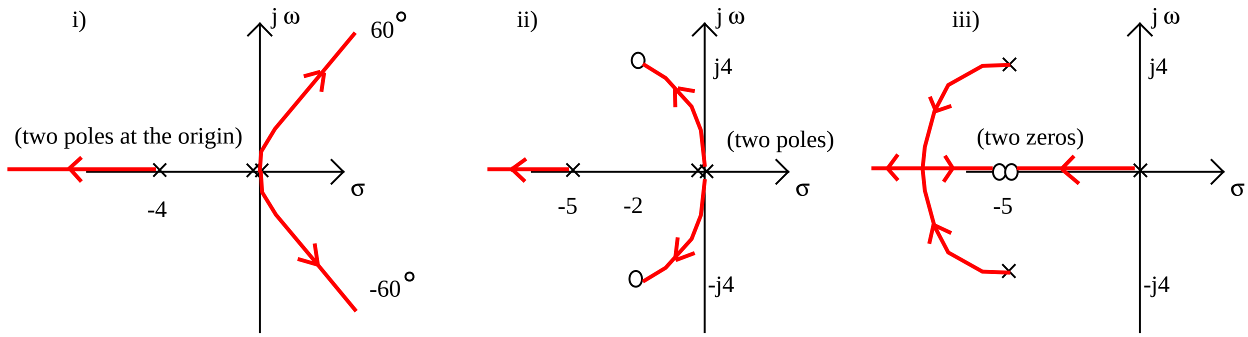
0. Check your scores under the grades option and make sure all records are correct!

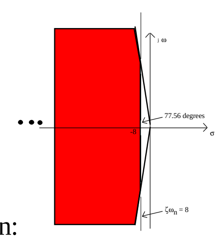
1.a) Sketch the root locus of the following s-plane open-loop pole zero. Solution:



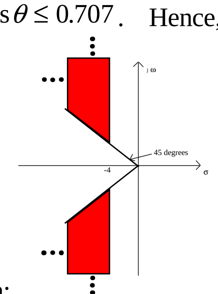
1b) Show the region of the s-plane where we must place our dominant poles to satisfy the following specifications:

i) $t_s \leq .5$ sec and $M_p \leq 50$ % ii) $t_s \leq 1$ sec and z ≤ 0.707

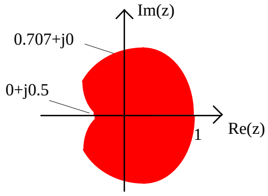
i) Solution: from the first spec, we find that $t_s \leq 0.5 \Rightarrow \zeta\omega_n \geq 8$. From the second spec we find that $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$. Or, solving this relationship for the damping coefficient we obtain, $\zeta = \sqrt{\frac{\ln(m)^2}{\pi^2 + \ln(m)^2}}$ where m=M_p/100%. Thus, $\zeta = \sqrt{\frac{\ln(0.5)^2}{\pi^2 + \ln(0.5)^2}} = 0.2155 = \cos\theta$. Hence, $\theta = \cos^{-1}(\zeta) = \cos^{-1}(0.2155) = 77.56^\circ$. Since we must have M_p < 50%, our constraint becomes $\theta \leq 77.56^\circ$. Putting both of these constraints together, we find the region in the s-plane shown:



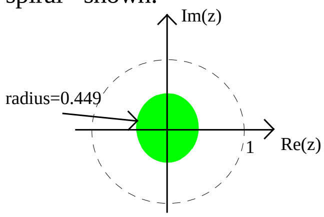
ii) Solution: from the first spec, we find that $t_s \leq 1.0 \Rightarrow \zeta\omega_n \geq 4$. From the second spec we find that $\zeta = \cos\theta \leq 0.707$. Hence, $\theta = \cos^{-1}(\zeta) \geq \cos^{-1}(0.707) = 45^\circ$. Putting both of these constraints together, we find the region in the s-plane shown:



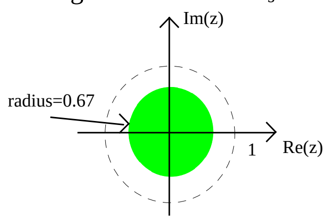
1c) Solution: plugging in the region $-\sigma = \zeta\omega_n \geq 8$ into our mapping z=e^{sT} produces the following circle within the unit circle with a radius of $z_{max} = e^{-8x0.1} = 0.449$. Mapping the region corresponding to $s = r\angle\theta \text{ where } \theta \ge 180^\circ - 87.34^\circ$ produces the logarithmic



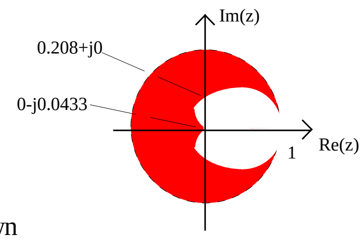
spiral shown: By intersecting these two areas, we find the complete solution shown:



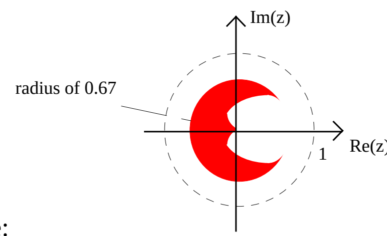
ii) For part ii), the settling time condition $t_s \le 1.0 \Rightarrow \zeta\omega_n \ge 4$ produces the circle within the unit circle shown with a radius of



$z_{max} = e^{-4x0.1} = 0.67$:



The region defined $\theta = \cos^{-1}(\zeta) \ge \cos^{-1}(0.707) = 45^\circ$ maps into the region shown



Intersecting these two regions produces the following region in the z-plane:

2a) i) <u>Solution:</u> $e_{ss}|_{u(t)}= 1/(1+K_p)$. Thus, $e_{ss}|_{10u(t)}= 10/(1+K_p) = 10/(1+12/5)= 50/17$
ii) <u>Solution:</u> $e_{ss}|_{r(t)}= 1/K_v$. Thus, $e_{ss}|_{10r(t)}= 10/K_v = \infty$
iii) <u>Solution:</u> $e_{ss}|_{p(t)}= 1/K_a$. Thus, $e_{ss}|_{10p(t)}= 10/K_a = \infty$ (where $p(t) = 0.5t^2u(t)$ = a unit parabola)

2b) i) <u>Solution:</u> $e_{ss}|_{u(t)}= 1/(1+K_p)$. Thus, $e_{ss}|_{10u(t)}= 10/(1+K_p) = 0$
ii) <u>Solution:</u> $e_{ss}|_{r(t)}= 1/K_v$. Thus, $e_{ss}|_{10r(t)}= 10/K_v = 50/12 = 25/6$
iii) <u>Solution:</u> $e_{ss}|_{p(t)}= 1/K_a$. Thus, $e_{ss}|_{10p(t)}= 10/K_a = \infty$ (where $p(t) = 0.5t^2u(t)$ = a unit parabola)

2c) Matlab output:
```
» ahat=[1 0.0302;0 0.9487];
» bhat=[0.0104;0.6627];
```

```
» chat=[1 0];
» Ko=[1.9487   29.8024]';
» Kc=[2.5697    0.5764];
» %coupled 2nth order observer-controller matrix:
» obscon=[ahat -bhat*Kc;Ko*chat ahat-Ko*chat-bhat*Kc]

obscon =

    1.0000    0.0302   -0.0267   -0.0060
         0    0.9487   -1.7029   -0.3820
    1.9487         0   -0.9754    0.0242
   29.8024         0  -31.5053    0.5667

» eig(obscon)

ans =

   0.0000 + 0.0009i
   0.0000 - 0.0009i
   0.7599
   0.7801
```

According to the separation principle, the eigenvalues should $eig(\hat{A}-\hat{B}K_c)\cup eig(\hat{A}-K_o\hat{C})$ which are {0.78 0.76} U {0 0}. As can been seen from the Matlab output, this is the case.