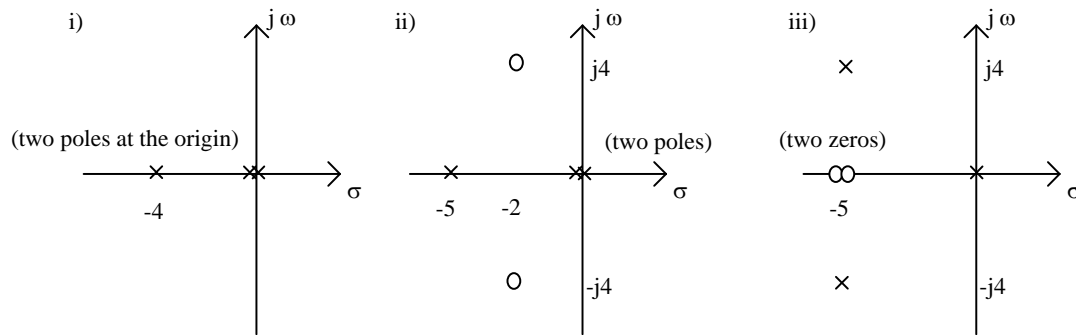


1.a) Sketch the root locus of the following s-plane open-loop pole zero configurations (Use Matlab's rlocus() command to check your answers) :

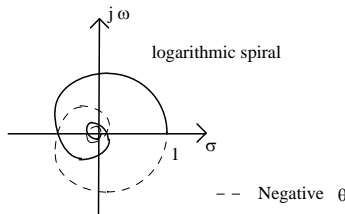


b) In class we learned about the concept of dominant closed-loop poles and that we want to design a filter (compensator) to produce closed-loop poles that obey the equation  $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$  or  $s_{1,2} = -\zeta\omega_n \pm j\omega_n(1-\zeta^2)^{1/2}$ . Show the region of the s-plane where we must place our dominant poles to satisfy the following specifications (hint: see cheat sheet II for the transient specification equations relating settling time and overshoot to  $\zeta$  and  $\omega_n$ ) :

- i)  $t_s \leq .5$  sec and  $M_p \leq 50\%$
- ii)  $t_s \leq 1$  sec and  $\zeta \leq 0.707$

Fun Fact: Did you know that constant  $\zeta$  (damping) lines in the s-plane map into logarithmical spirals in the z-plane? A point on a constant line can be described by the equation

$s = -\zeta\omega_n + j\omega_n \sin\theta$  where  $\cos\theta = \zeta = \text{constant}$  and the radius  $r$  varies from 0 to  $\infty$  (the  $-r$  indicates that the point is in the LHP). If we use the mapping  $z = e^{sT_s}$ , then these damping lines map to  $z = e^{sT_s} = e^{-\zeta\omega_n T_s + j\omega_n \sin\theta T_s} = e^{-\zeta\omega_n T_s} e^{j\omega_n \sin\theta T_s}$  which in polar coordinates has a radius of  $e^{-\zeta\omega_n T_s}$  and an angle of  $\omega_n \sin\theta T_s$  (recall both  $\cos\theta T_s$  and  $\sin\theta T_s$  are constant). Thus, as we vary  $r$  from 0 to  $\infty$  in the s-plane, the point  $z$  will start at (1,0) in the z-plane and its radius will decrease exponentially while its angle will increase linearly. Such a plot is called a logarithmic spiral and looks like:



c) Using this fun fact, repeat part b) in the z-plane if our sampling time is  $T_s = 100$  msec.  
 2a) Given a unity feedback system with the open-loop transfer function  $G(s) = \frac{6(s+2)}{s^m(s+5)}$  1, where the system type number  $m=0$ , find (hint: remember the transfer function,  $E(s)/W(s)$ , is LINEAR so if we multiply  $w(t)$  by a scalar,  $e(t)$  is multiplied, too!):

- i)  $e_{ss}$  due to  $10u(t)$
- ii)  $e_{ss}$  due to  $10r(t)$
- iii)  $e_{ss}$  due to  $5t^2$

b) Repeat part b) for a type 1 system.  
 c) Go back to your prelab #3, and check the eigenvalues of your combined full-order observer/controller design. That is, find the eigenvalues of the  $2n^{\text{th}}$  order system:

$$\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} \hat{A} & -\hat{B}K \\ K_o\hat{C} & \hat{A} - K_o\hat{C} - \hat{B}K \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix}$$

Does your answer agree with the separation principle?