

Did you know that discrete-time systems occur everywhere (even outside of engineering!)? The world of biology has many examples of discrete-time systems as does business and economics as does...

1a) A certain closed ecosystem is comprised solely of UK Blue Wildcats (predators) and Tennessee Orange Volunteer Jackrabbits (food for the wildcats). The initial population of jackrabbits is P_0 and these jackrabbits reproduces (as rabbits tend to do!) at a rate of “ r ” per month. Of course, without a “control” this population would increase with out bound. Our “control” is the number of rabbits consumed by the wildcats. Assuming our wildcats eat w_k jackrabbits in the k th month, write an equation which relates the remaining population at the $k+1$ month (P_{k+1}) in terms of the birth rate (r), the previous population (P_k), and the number of rabbits eaten (w_k). (Note: this equation will be a discrete version of the state variable equation you studied in EE422. We call such an equation a *Next State* equation because it relates the value of the next state (at time $k+1$) to the previous state and previous input (at time k). In this problem, P_k is the state and w_k is the input).

b) The next state equation has the form, $x_{k+1} = Ax_k + Bw_k$ with some initial state, x_0 . In EE572, we will derive the solution of this equation to be $x_k = A^k x_0 + \sum_{j=1}^k A^{k-j} B w_{j-1}$. Also, recall from calculus that when a is

a scalar, $\sum_{j=1}^k a^{k-j} = a^{k-1} \sum_{j=0}^{k-1} \left(\frac{1}{a}\right)^j = a^{k-1} \left[\frac{\left(\frac{1}{a}\right)^k - 1}{\frac{1}{a} - 1} \right] = \frac{a^k - 1}{a - 1}$. Use these two facts to solve for the

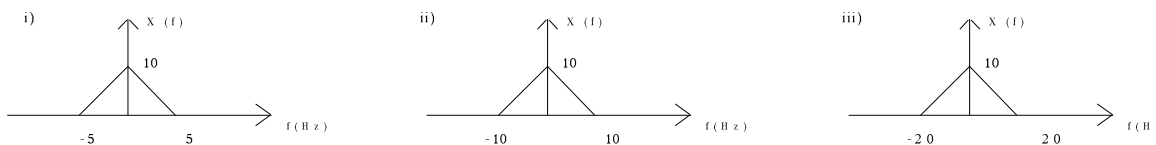
constant number of eaten jackrabbits that must be eaten per month (i.e., $w_k = \text{constant}$) when if we want to make an initial population of $P_0=100$ orange jackrabbits extinct in two years with a birth rate of $r=10\%$ per month) (hint: find the constant value of w_k such that the population is zero after 24 months).



- c) In the above problem, what is the value of the constant monthly payment necessary to maintain the principle at a constant $P_k=P_0=100$ rabbits.
- d) If we view the above biology problem as a system, identify the states of the system and the input to the system.
- e) Is the system stable? (i.e., will it blow-up when no rabbits are eaten ($w_k=0$)?)
- f) Would you say the system is controllable (i.e., by eating rabbits, can we force the population to zero)?

2. a) Let $x(t)$ be a signal. Suppose we sample this signal using ideal impulse sampling at a sampling period T_s to obtain the sampled version of $x(t)$, $x_s(t)$. Show that the Fourier transform $X_s(f)$ is $X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s)$ where $X()$ is the Fourier transform of $x(t)$ and $f_s=1/T_s$. (we did this in class today!)

b) Use the result of problem 2a) to sketch the magnitude of the spectrum of the sampled signal (i.e., sketch $X_s(f)$) given the following fourier transforms of the original signal (assume the sampling period is 50 msec):



c) Use your answer to part b) to explain the Nyquist Sampling Theorem. Then, find the minimum Nyquist sampling frequency to avoid aliasing for each signal given in part 2b). (Note: in EE572, you will prove that the Nyquist frequency is not necessarily sufficient for closed-loop stability!!!!!!!!!!!!)