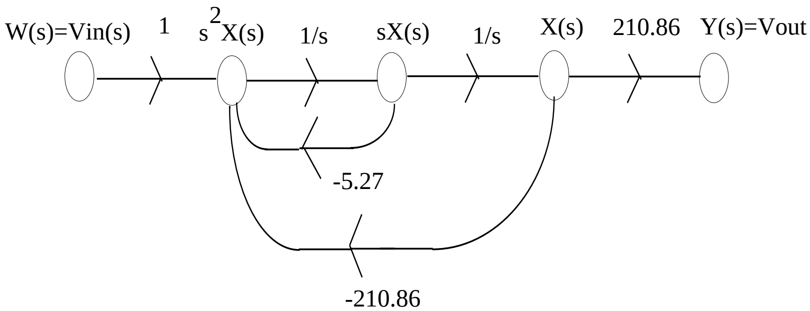
EE571 Solution to Pre-Lab 2 (HW#9)

1a) From Lab 1, our transfer function is $\dfrac{Y(s)}{W(s)}=\dfrac{V_{out}(s)}{V_{in}(s)}=\dfrac{210.86}{s^2+5.2711s+210.86}$. Using phase variables, we obtain the following SFG:



$$\Rightarrow \dot{x}=Ax+Bw=\begin{bmatrix}0 & 1\\ -210.86 & -5.27\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}w$$

$$y=V_{out}=Cx+Dw=\begin{bmatrix}210.86 & 0\end{bmatrix}x$$

1b) Note that y=V_{out} =210.86x_1 and $V_g=K_g\dot{\theta}_m=K_gN_a\dot{\theta}_{out}=K_g(9)\dot{\theta}_{out}=K_g(9)\dot{V}_{out}/K_p=\dfrac{K_g(9)}{K_p}210.86\dot{x}_1$

But from our state variable model, $\dot{x}_1=x_2$ and from Lab 1, K_g = 0.1842 vs/r and K_p=5.1394 v/r. Hence, V_g=(0.1842)(9)(210.86)(x_2)/(5.1394) = 68.0164x_2. Thus, we can write the following similarity transformation which relates our state variables in part a) to the (measurable) physical state variables, $[V_{out}\ V_g]^T$:

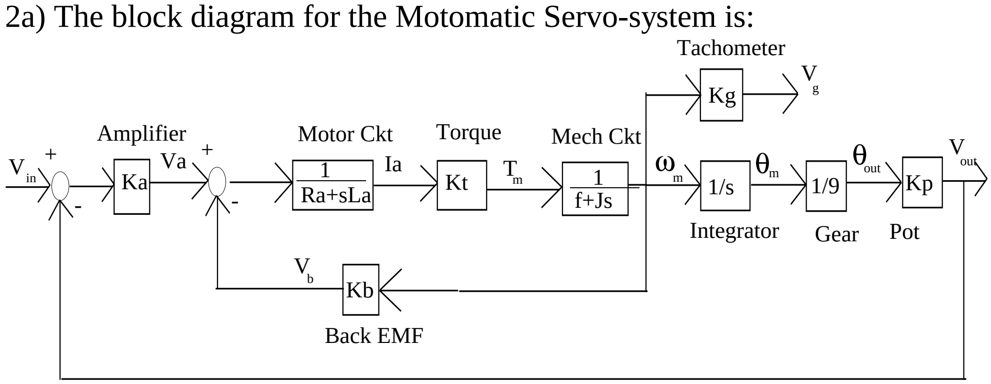
$$\begin{bmatrix}x_1\\ x_2\end{bmatrix}=Tz=T\begin{bmatrix}V_{out}\\ V_g\end{bmatrix}=\begin{bmatrix}1/210.86 & 0\\ 0 & 1/68.0164\end{bmatrix}\begin{bmatrix}V_{out}\\ V_g\end{bmatrix}$$

Plugging this similarity transformation into our state model from part a) produces the following new state variable model in our physical state variables, $[V_{out}\ V_g]^T$:

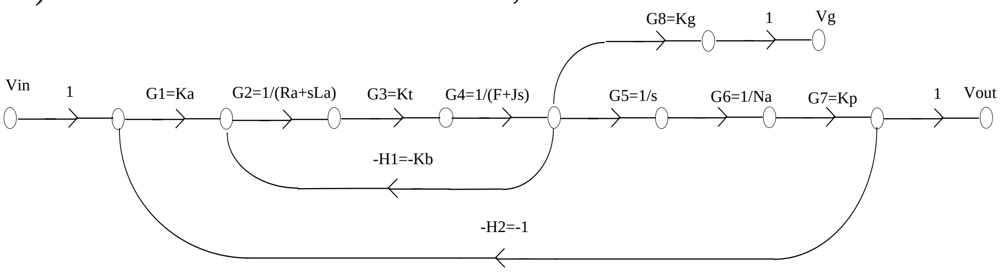
$$\dot{z}=T^{-1}ATz+T^{-1}Bw\Rightarrow\begin{bmatrix}\dot{V}_{out}\\ \dot{V}_g\end{bmatrix}=\begin{bmatrix}0 & 3.1\\ -68.014 & -5.27\end{bmatrix}\begin{bmatrix}V_{out}\\ V_g\end{bmatrix}+\begin{bmatrix}0\\ 68.0164\end{bmatrix}V_{in}$$

$$y=CTz\Rightarrow y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}V_{out}\\ V_g\end{bmatrix}$$

2a) The block diagram for the Motomatic Servo-system is:



2b) To find the overall transfer function, we can first convert this to the following SFG:



By Mason's SFG Formula, the overall transfer function is:

$\dfrac{V_{out}(s)}{V_{in}(s)} = \sum \dfrac{T_n \Delta_n}{\Delta} = \dfrac{T_1 \Delta_1}{\Delta}$ where T_1=G1G2G3G4G5G6G7and

$$\Delta = 1 - \sum L_1's + \sum L_2's - \sum L_3's + \ldots$$

$$= 1 - [G2G3G4(-H1) + G1G2G3G4G5G6G7(-H2)] + 0$$

Thus, $\Delta_1 = 1$ and $\dfrac{V_{out}(s)}{V_{in}(s)} = \dfrac{G1G2G3G4G5G6G7}{1 + G2G3G4H1 + G1G2G3G4HG5G6G7H2}$

or $\dfrac{V_{out}}{V_{in}} = \dfrac{K_a K_T K_P / 9}{L_a J s^3 + (L_a F + J R_a) s^2 + (R_a F + K_b K_t) s + K_a K_T K_P / 9}$

2c) From the SFG and Mason's Gain Formula, we see that:

$\dfrac{V_g(s)}{V_{in}(s)} = \sum \dfrac{T_n \Delta_n}{\Delta} = \dfrac{T_1 \Delta_1}{\Delta}$ where T_1=G1G2G3G4G8 and

$$\Delta = 1 - \sum L_1's + \sum L_2's - \sum L_3's + \ldots = 1 - [G2G3G4(-H1)] + 0$$

Thus, $\Delta_1 = 1$ and $\dfrac{V_g(s)}{V_{in}(s)} = \dfrac{G1G2G3G4G8}{1 + G2G3G4H1}$

or $\dfrac{V_g}{V_{in}} = \dfrac{K_a K_T K_g}{J L_a s^2 + (L_a F + J_a R_a) s + K_b K_T + R_a F}$

2d) From the block diagram, $\dfrac{I_a(s)}{V_a(s) - V_b(s)} = \dfrac{1}{R_a + L_a s}$. If we hold the motor shaft and prevent it from turning, then we force V_b to be zero. Thus, $\dfrac{I_a(s)}{V_a(s)} = \dfrac{1}{R_a + L_a s}$. If V_a is sinusoidal, then we obtain two equations using the magnitude and phase of the response: $\left| \dfrac{I_a(j\omega)}{V_a(j\omega)} \right| = \dfrac{1}{\sqrt{R_a^{\,2} + (L_a \omega)^2}}$ and $\angle \dfrac{I_a(j\omega)}{V_a(j\omega)} = \tan^{-1}(\dfrac{\omega L_a}{R_a})$