

Objective: The objective of lab 2 is to: 1) complete our measurements of the MOTOMATIC internal parameters; 2) To obtain a state variable model of the MOTOMATIC.

Prelab (Counts as HW#9): (due Monday, October 3rd):

1. a) Using the transfer function $V_{out}(s)/V_{in}(s)$ as found in Lab #1, find the SFG for the MOTOMATIC then find the state variable model in phase variable form.
- b) The actual variables (voltages) we can measure are V_{out} (the voltage from the potentiometer wiper arm) and V_g (the voltage from the generator/tachometer). Recall that $V_g = K_g \dot{\theta}_m = K_g N_a \dot{\theta}_{out} = K_g (9) \dot{\theta}_{out}$. Use a similarity transformation relating the state variables, $[V_{out} \ V_g]$ to the state variables you found in part a). Then, find a new state variable model in terms of the state variables, $[V_{out} \text{ and } V_g]$

Hint: recall from class that we can use any similarity transform, $x=Tz$, to transform the given state coordinates (x) into a new state coordinate (z). The result we derived was $\dot{z} = T^{-1}ATz + T^{-1}Bw$
 $y = CTz$

For this problem, x are the variables from your simulation diagram and $z = \begin{bmatrix} V_{out} \\ V_g \end{bmatrix}$. When you find the similarity transformation matrix T where $x=Tz$, T should be a diagonal matrix with the first element relating x_1 to V_{out} and the second diagonal element relating x_2 to V_g .

2. The schematic for the closed-loop unity feedback MOTOMATIC DC servo for which you found the step response is shown in **Figure 1**. Note how the output voltage, V_{out} , is fed-back to form an error (difference) signal with the system input, V_{in} .

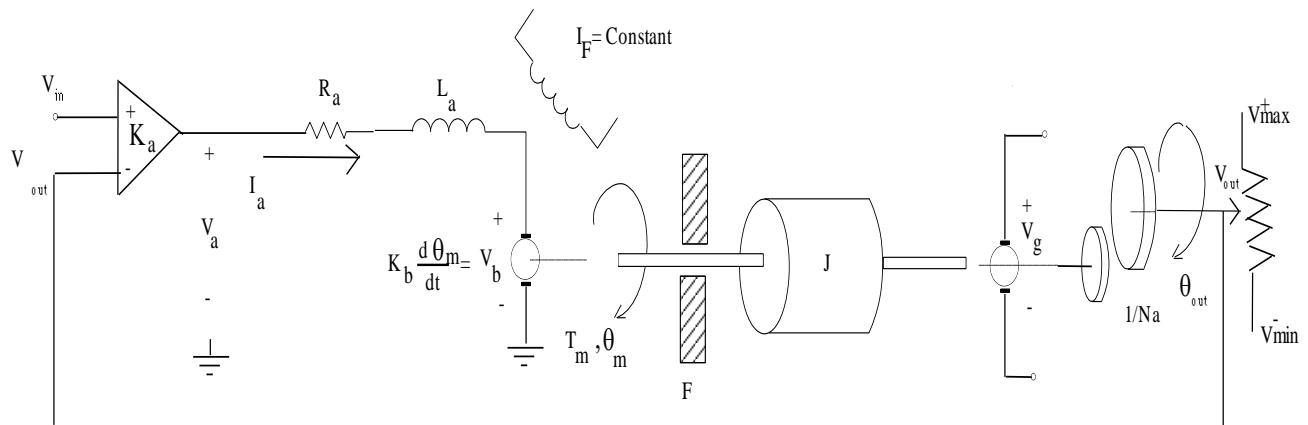


Figure 1. Schematic of MOTOMATIC DC servo motor system.

- a) Find the functional block diagram from the above schematic with outputs V_g and V_{out} .
- b) Find the closed-loop transfer function, $V_{out}(s)/V_{in}(s)$, in terms of the parameters J , F , R , etc.
- c) Now let $L_a = 0$, and repeat part b. You should get a 2nd order system.
- d) Finally, find the open-loop transfer functions, $V_g(s)/V_{in}(s)$ and $V_{out}(s)/V_{in}(s)$ (remember how we do open-loop transfer functions)

- e) Ra and La If we hold the motor shaft so that it cannot move, then the back emf voltage must be zero (i.e., $V_b = 0$). Find the transfer function, $I_a(s)/V_a(s)$, when $V_b = 0$. If we apply a sinusoidal voltage with amplitude A and frequency ω rad/sec at the armature terminals (i.e., $V_a = A \sin \omega t$), then I_a will also be a sinusoid in steady-state. Use your transfer function, $I_a(j\omega)/V_a(j\omega)$ to find two equations involving R_a and L_a if we assume sinusoidal steady-state (Hint: think magnitude and phase).

In Lab (Counts as HW#10) (due Wednesday, October 5th)

In the lab, you will measure R_a , L_a , J , F , and K_a .

1. a) Recall from EE415, that the value for the torque constant of a DC armature controlled motor is the same as the value for the back emf constant. Use your answer from Lab 1 for the back emf constant (K_b) to find the torque constant, K_T (make sure your units are consistent).
- b) Apply a sinusoidal voltage at frequency ω to the motor while holding the motor shaft to insure that $V_b=0$. Take a plot of V_a and I_a versus time and find the magnitude and phase of $I_a(j\omega)/V_a(j\omega)$. Use your answer to part 2e) of the Prelab to obtain values for R_a and L_a .
- c) One of the reasons that our MOTOMATIC is not truly a linear second-order system is that the friction present in the motor contains Coulomb friction as well as viscous friction. Coulomb friction (also known as static friction or "stiction") is approximately a constant force always opposing the motion of the motor. If we call this Coulomb friction constant C , then we can write a new Newton's second law of motion for the Torques as seen at the motor:

$$T_m = K_T I_a = J \dot{\omega}_m + F \omega_m + C \text{sgn}(\omega_m) \quad (1)$$

where $\text{sgn}()$ is the signum function (i.e., $\text{sgn}(x) = +1$ if $x > 0$ or -1 if $x < 0$).

One technique for measuring parameters is called a steady-state plot. This technique is performed by inputting a constant into the system, waiting for the system to reach steady-state, then measure the system's output. By repeating this procedure for a variety of constant inputs, a plot of steady-state output values versus constant input values is obtained. For our particular system, we will input a constant V_{in} which will produce a constant motor speed (i.e., $\omega_m = \text{constant}$). If we measure the generator output voltage, $V_g = K_g \omega_m$, then we can write an equation for I_a in terms of V_g from equation (1). In fact, this equation will be the equation of a straight line ($y = mx + b$). Take a steady-state plot of I_a vs. V_g . Your plot should be fairly linear. Use the slope and the y-intercept of this plot to find the viscous friction coefficient, F , and the Coulomb constant, C .

2. a) J: If we ignore the nonlinear Coulomb friction (assume $C=0$), then equation (1) reduces to:

$$T_m = K_T I_a = J \dot{\omega}_m + F \omega_m = J \dot{V}_g / K_g + F V_g / K_g \quad (2)$$

Run the motor at a constant speed, then set $I_a = 0$ by pulling out the lead of the armature circuit of the MOTOMATIC while the motor is turning. This is accomplished by pulling the red cap at the terminal labelled A (Please pull the cap not the red wire!). Measure the values of V_g and derivative of V_g at the point on the plot where you set $I_a = 0$. Then, use equation (2) to find the motor inertia, J .

- b) Make a plot of V_a vs. V_{in} . Find K_a , the gain of the amplifier, from this plot.

- c) Substitute the values of K_b , K_T , K_p , R_a , L_a , F and J into your answers for Prelab problems 2b) and 2c). Compare your answers from Lab 1 for $V_{out}(s)/V_{in}(s)$ as measured from the step response in experiment 1. Which modelling technique more accurately models how V_{out} responds to V_{in} , the step response method or internal parameter measurement? What are the advantages of each modelling technique?